Comprimento de arco de uma curva plana usando a sua equação cartesiana (20/11/2019)
Livro: Cálculo A.
Tópico 8.4, página 344.
Questões: 1, 3, 5, 6, 10, 11 e 13.

(1) $y = 5x - 2, \; -2 \leq x \leq 2$

$$L = \int_{-2}^{2} \sqrt{1 + [(5x-2)']^2}\, dx \qquad (5x-2)' = 5$$

$$L = \int_{-2}^{2} \sqrt{1 + 5^2}\, dx = \int_{-2}^{2} \sqrt{26}\, dx = \sqrt{26}\, x \Big|_{-2}^{2} = \sqrt{26}\,2 - (\sqrt{26}(-2)) = \sqrt{26}\,2 + \sqrt{26}\,2 =$$

$$= \boxed{\sqrt{26}\,4} \text{ u.c.}$$

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(3) $y = \frac{1}{3}(2+x^2)^{3/2}, \; 0 \leq x \leq 3$

$$g(y) = \frac{1}{3}(2+x^2)^{3/2} \qquad g'(y) = \frac{1}{3} \cdot \frac{3}{2}(2+x^2)^{\frac{1}{2}} \cdot 2x$$

$$= \frac{3}{6}(2+x^2)^{1/2}\, 2x$$

$$= x(2+x^2)^{1/2}$$

$$L = \int_0^3 \sqrt{1 + (x(2+x^2)^{1/2})^2}\, dx = \int_0^3 \sqrt{1 + x^2(2+x^2)}\, dx$$

$$= \int_0^3 \sqrt{1 + 2x^2 + x^4}\, dx = \int_0^3 \sqrt{1} + 2x + x^2\, dx = \left(\sqrt{1}\,x + \frac{2x^2}{2} + \frac{x^3}{3}\right)\Big|_0^3$$

$$\left(\sqrt{1}\,x + x^2 + \frac{x^3}{3}\right)\Big|_0^3 = \sqrt{1}.3 + 3^2 + \frac{3^3}{3} - 0 = 3\sqrt{1} + 9 + \frac{27}{3} = 3\sqrt{1} + 9 + 9 = 3\sqrt{1} + 18$$

$$\boxed{21 \text{ u.c.}}$$

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(5) $y = \frac{1}{4}x^4 + \frac{1}{8x^2}, \; 1 \leq x \leq 2$

$$g(y) = \frac{1}{4}x^4 + \frac{1}{8x^2} = \frac{1}{4}x^4 + \frac{1x^{-2}}{8}$$

$$g'(y) = x^3 + \frac{(-2)x^{-3}}{8} = x^3 - \frac{1}{4x^3}$$

$$L = \int_1^2 \sqrt{1 + \left(x^3 - \frac{1}{4x^3}\right)^2}\, dx = \int_1^2 \sqrt{1 + x^6 - \frac{1}{16x^6}}\, dx = \int_1^2 \sqrt{\frac{16x^6 + 16 - 1}{16x^6}}\, dx$$

$$= \int_1^2 \sqrt{\frac{1}{x^6} - \frac{1}{16x^6}}\, dx = \int_1^2 \sqrt{x^{-6} - 16x^{-6}}\, dx = \int_1^2 \sqrt{-15x^{-6}}\, dx = \int_1^2 \sqrt{-15}\, x^{-3}\, dx = \sqrt{-15}\, \frac{x^{-2}}{-2}$$

$$= \sqrt{-15}\, \frac{1}{-2x^2} \Big|_1^2 = \sqrt{-15}\, \frac{1}{-2.2^2} - \left(\sqrt{-15}\, \frac{1}{-2}\right) = \frac{\sqrt{-15}}{-8} - \frac{\sqrt{-15}}{-2} = \boxed{\frac{\sqrt{-15}}{2} - \frac{\sqrt{-15}}{8}} \text{ u.c.}$$

(6) $x = \frac{1}{3}y^3 + \frac{1}{4y}, \; 1 \leq y \leq 3$

$$g(y) = \frac{1}{3}y^3 + \frac{1}{4y} = \frac{1}{3}y^3 + \frac{y^{-1}}{4}$$

$$g'(y) = y^2 + \frac{(-1)\,y^{-2}}{4} = y^2 - \frac{1}{4y^2}$$

$$L = \int_1^3 \sqrt{1+\left(y^2 - \frac{1}{4y^2}\right)^2}\,dx = \int_1^3 \sqrt{1 + y^4 - \frac{y^2}{4y^2} - \frac{y^2}{4y^2} + \frac{1}{16y^4}}\,dx$$

$$= \int_1^3 \sqrt{1 + y^4 - \frac{2}{4} + \frac{1}{16y^4}}\,dx = \int_1^3 \sqrt{\frac{1}{2} + y^4 + \frac{1}{16y^4}}\,dx = \int_1^3 \sqrt{\frac{8y^4 + 16y^{16} + 1}{16y^4}}\,dx$$

$$= \int_1^3 \sqrt{\frac{1}{16y^4}} + 1 + \frac{1}{2}\,dx = \int_1^3 \left[\frac{1}{4y^2} + 1 + \sqrt{\frac{1}{2}}\right]dx = \int_1^3 4y^{-2} + 1 + \sqrt{1/2}\,dx$$

$$= \left(\frac{1}{4y} + y + \sqrt{0.5}\,y\right)\Big|_1^3 = -\frac{1}{12} + 3 + 3\sqrt{0.5} - \left(-\frac{1}{4} + 1 + \sqrt{0.5}\right) =$$

$$= -\frac{1}{12} + \frac{1}{4} - 2 + 2\sqrt{0.5} = \frac{-1+3-24+24\sqrt{0.5}}{12} = \frac{-22+24\sqrt{0.5}}{12}$$

$$= \boxed{-\frac{11}{6} + 12\sqrt{0.5} \text{ u.c.}}$$

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(10) $y = \sqrt{x^3}$, de $P_0(0,0)$ até $P_1(4,8)$
$\quad\quad\quad\quad\quad\quad\quad\quad x \; y \quad\quad\quad\quad x \; y$

$$f(x) = x^{3/2}$$

$$f'(x) = \frac{3}{2}\sqrt{x}$$

$$L = \int_0^4 \sqrt{1 + \left(\frac{3}{2}\sqrt{x}\right)^2}\,dx = \int_0^4 \sqrt{1 + \frac{9}{4}x}\,dx$$

$$u = 1 + \frac{9}{4}x$$

$$du = \frac{9}{4}dx \rightarrow \frac{4}{9}du = dx$$

$$= \int_0^4 \sqrt{u}\,\frac{4}{9}\,du = \frac{4}{9}\int_0^4 \sqrt{u}\,du = \left(\frac{4}{9}\cdot\frac{2}{3}u^{3/2}\right)\Big|_0^4 = \left(\frac{8}{27}u^{3/2}\right)\Big|_0^4 = \left(\frac{8}{27}\cdot\left(1 + \frac{3}{4}x\right)^{3/2}\right)\Big|_0^4$$

$$= \frac{\sqrt{(8+18x)^3}}{27}\Big|_0^4 = \frac{\sqrt{(8+18.4)^3}}{27} - \cancel{\frac{\sqrt{(8+18.0)^3}}{27}} \to 0 = \frac{\sqrt{(8+72)^3}}{27} = \frac{\sqrt{80^3}}{27} = \frac{\sqrt{512000}}{27} = \frac{715}{27}$$

$$= \boxed{26.50 \text{ u.c.}}$$

(11) $y = 4\sqrt{x^3} + 2$, de $P_0(0,2)$ até $P_1(1,6)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x\ y \quad\quad\quad\quad x\ y$

$f(x) = 4\sqrt{x^3} + 2$

$f'(x) = 4 \cdot \frac{3}{2}\sqrt{x} = 6\sqrt{x}$

$$b = \int_0^1 \sqrt{1 + (6\sqrt{x})^2}\, dx = \int_0^1 \sqrt{1 + 12x}\, dx$$

$u = 12x + 1$

$du = 12\,dx \rightarrow \frac{du}{12} = dx$

$$= \frac{1}{12}\int_0^1 u^{1/2}\, du = \frac{1}{12} \cdot \frac{2}{3} u^{3/2} \Big/_0^1 = \frac{2}{36}\sqrt{(1+12x)^3} = \frac{1}{18} \cdot (1 + 6x + 20x^2 + 41x^3) \Big/_0^1$$

$$\frac{1+6+20+41}{18} = \frac{68}{18} = \boxed{4 \text{ u.c.}}$$

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(13) $(y-1)^2 = (x+4)^3$, de $P_0(-3,2)$ até $P_1(0,9)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x\ y \quad\quad\quad\quad x\ y$

$(y-1)(y-1) = y^2 - 2y + 1$

$f(x) = (x+4)^3$

$f'(x) = 3(x+4)^2 \cdot 1 = (3x+12)^2$

$$b = \int_{-3}^{0} \sqrt{1 + (3x+12)^2}\, dx = \int_{-3}^{0} \sqrt{1 + 144 + 72x + 9x^2}\, dx = \int_{-3}^{0} \sqrt{9x^2 + 72x + 145}\, dx$$

$u = 9x^2 + 72x + 145$

$du = (18x + 72)\, dx$

$\frac{du}{18x+72} = dx$

$$b = \frac{1}{18x+72}\int_{-3}^{0} \sqrt{u}\, dx = \frac{2}{54x+216} \cdot u^{3/2} \Big/_{-3}^{0} = \frac{2}{54x+216} \cdot \sqrt{(3x^2+72x+145)^3} \Big/_{-3}^{0}$$

$$= -\left(\frac{2}{54(-3)+216} \cdot \sqrt{(9(-3)^2 + 72(-3) + 145)^3}\right) = \frac{2}{54} \cdot \sqrt{(81 - 216 + 145)^3} =$$

$$= \frac{1}{27}\sqrt{10} \approx \frac{3}{27} \approx \boxed{\frac{1}{9} \text{ u.c.}}$$