Integração de funções trigonométricas (25/09/2019)   Vitor Oliveira Rp/K

Livro: Cálculo A.

Tópico 7.4, página 309.

Questões: 1-9.

Página 309

Nos exercícios 1 a 35 (9), calcular a integral indefinida

(1) $\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx = \int \frac{\operatorname{sen} x^{\frac{1}{2}}}{x^{\frac{1}{2}}}\,dx = \int \operatorname{sen} x^{\frac{1}{2}} \cdot \frac{1}{x^{\frac{1}{2}}}\,dx$

$u = x^{\frac{1}{2}} \quad du = \frac{1}{2} x^{-\frac{1}{2}}\,dx$

$du = \frac{1}{2x^{\frac{1}{2}}}\,dx$

$\rightarrow 2du = \frac{1}{x^{\frac{1}{2}}}\,dx$

$\rightarrow \int \operatorname{sen} u\, 2du = 2\int \operatorname{sen} u\, du = 2 . -\cos u + c = \boxed{-2 . \cos\sqrt{x} + c}$

(2) $\int \cos x \cdot \cos(\operatorname{sen} x)\,dx \qquad u = \operatorname{sen} x \quad du = \cos x\,dx$

$= \int \cos(\operatorname{sen} x) \cos x\,dx = \int \cos(u)\,du = +\operatorname{sen}(u) + c = \boxed{\operatorname{sen}(\operatorname{sen} x) + c}$

(3) $\int \frac{\operatorname{sen} 2x}{\cos x}\,dx = \int 2\operatorname{sen}(x)\,dx = 2\int \operatorname{sen} x\,dx = 2 - \cos x = \boxed{-2\cos x + c}$

(4) $\int x\, \operatorname{tg}(x^2+1)\,dx \qquad a = x^2+1 \quad da = 2x\,dx \rightarrow \frac{da}{2} = x\,dx$

$= \int \operatorname{tg} a \frac{da}{2} = \frac{1}{2}\int \operatorname{tg} a\, da = \frac{1}{2}\int \frac{\operatorname{sen} a}{\cos a}\,da \qquad b = \cos a \quad db = -\operatorname{sen} a\, da$

$-db = \operatorname{sen} a\, da$

$= \frac{1}{2}\int \frac{-db}{b} = -\frac{1}{2}\int \frac{db}{b} = -\frac{1}{2}\ln|b| + c = \boxed{-\frac{\ln|\cos x^2+1|}{2} + c}$

(5) $\int \frac{\cot g\left(\frac{1}{x}\right)}{x^2}\,dx$ $\quad a = \frac{1}{x} = x^{-1} \quad da = -x^{-2}dx = -\frac{1}{x^2}dx$

$\to -da = \frac{1}{x^2}dx$

$= \int \cot g\, a \cdot -da = -\int \cot g\, a\, da = -\int \frac{\cos a}{\operatorname{sen} a}\,da \quad b = \operatorname{sen} a \quad db = \cos a\, da$

$= -\int \frac{db}{b} = -\ln|b| + c = \boxed{-\ln\left|\operatorname{sen}\left(\frac{1}{x}\right)\right| + c}$

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(6) $\int \sec(x+1)\,dx \qquad a = x+1 \quad da = dx$

$\int \sec(a)\,da = \int \frac{\sec(a)\left[\sec(a)+tg(a)\right]}{\sec(a)+tg(a)}\,da = \int \frac{\sec^2(a) + \sec(a).tg(a)}{\sec(a)+tg(a)}\,da$

$b = \sec(a) + tg(a) \quad db = \sec(a).tg(a) + \sec^2(a)$ $\qquad \int \frac{db}{b} = \ln|b| + c$

$= \boxed{\ln|\sec(x^2+1) + tg(x^2+1)| + c}$

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(7) $\int \operatorname{sen}(\omega t + \theta)\,dt \qquad u = \omega t + \theta \quad du = \omega\, dt \to \frac{du}{\omega} = dt$

$= \int \operatorname{sen}(u)\frac{du}{\omega} = \frac{1}{\omega}\int \operatorname{sen}(u)\,du = \frac{1}{\omega} - \cos(u) + c = \boxed{-\frac{\cos(\omega t + \theta)}{\omega} + c}$

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(8) $\int x \operatorname{cosec} x^2\, dx \qquad u = x^2 \quad du = 2x\,dx \to \frac{du}{2} = x\,dx$

$= \int \operatorname{cosec}(u)\frac{du}{2} = \frac{1}{2}\int \operatorname{cosec}(u)\,du = \frac{1}{2}\int \frac{\operatorname{cosec}(u)\left[\operatorname{cosec}(u) - \cot g(u)\right]}{\operatorname{cosec}(u) - \cot g(u)}\,du =$

$= \frac{1}{2}\int \frac{\operatorname{cosec}^2(u) - \operatorname{cosec}.\cot g(u)}{\operatorname{cosec}(u) - \cot g(u)}\,du$

$v = \operatorname{cosec}(u) - \cot g(u)$

$dv = \operatorname{cosec}^2(u) - \operatorname{cosec}(u).\cot g(u)\,du$

$= \frac{1}{2}\int \frac{dv}{v} = \frac{1}{2}\ln|v| + c = \boxed{\frac{\ln|\operatorname{cosec}(x^2) - \cot g(x^2)|}{2} + c}$

Página 310

(9) $\int \cos x \cdot tg(\operatorname{sen} x)\, dx \qquad a = \operatorname{sen}(x) \qquad da = \cos x\, dx$

$\int tg(a)\, da = \int \frac{\operatorname{sen} a}{\cos a}\, da \qquad v = \cos a \qquad dv = -\operatorname{sen} a\, da$

$-dv = \operatorname{sen} a\, da$

$= \int \frac{-dv}{v} = -\int \frac{dv}{v} = -\ln|v| + C = \boxed{-\ln|\cos(\operatorname{sen}(x))| + C}$