Área de uma superfície de revolução (02/12/2019 - 04/12/2019)

Livro: Cálculo A.

Tópico 8.7, página 360.

Questões: 22, 23, 25-27.

Nos exercícios 22 a 27, calcular a área da superfície gerada pela rotação do arco de curva dado.

(22) $y = 2x^3, 0 \le x \le 2$; eixo dos X

$f(x) = 2x^3 \quad f'(x) = 6x^2 \quad a = 0 \quad b = 2$

$[f'(x)]^2 = 36x^4$

$A = 2\pi \int_0^2 2x^3 \sqrt{1+36x^4}\, dx$

$u = 1+36x^4$

$du = 144x^3\, dx$

$= 2\pi\, 2\int x^3\sqrt{1+36x^4}\, dx \rightarrow \dfrac{du}{144} = x^3\, dx \rightarrow 4\pi \int_0^2 \sqrt{u}\, \dfrac{du}{144} = \dfrac{4\pi}{144}\int u^{\frac{1}{2}}\, du$

$= \dfrac{\pi}{36} \cdot \dfrac{2u^{3/2}}{3} \Big|_0^2 = \dfrac{2\pi\sqrt{(1+36x^4)^3}}{108}\Big|_0^2 = \dfrac{2\pi\sqrt{(1+36.2^4)^3}}{108} - \dfrac{2\pi}{108}$

$= \dfrac{2\pi\sqrt{577}}{108} - \dfrac{2\pi}{108} = \boxed{\dfrac{\pi\sqrt{577} - \pi}{54}\ u.a.} = \dfrac{\pi}{54}(\sqrt{577} - 1)$

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(23) $x = \sqrt{y}, 1 \le y \le 4$; eixo dos Y

$f(y) = \sqrt{y} = y^{\frac{1}{2}} \quad f'(y) = \dfrac{y^{-\frac{1}{2}}}{2} = \dfrac{1}{2\sqrt{y}} \quad [f'(y)]^2 = \dfrac{1}{4y} \quad c = 1 \quad d = 4$

$A = 2\pi \int_1^4 \sqrt{y}\, \sqrt{1 + \dfrac{1}{4y}}\, dy$

$u = \sqrt{y} = y^{\frac{1}{2}}$

$du = \dfrac{1}{2\sqrt{y}}\, dx \Rightarrow 2\sqrt{y}\, du = dx$

$= \pi \int_1^4 \sqrt{y}\, \sqrt{4 + \dfrac{1}{y}}\, dy \quad \dfrac{1}{y} = \dfrac{1}{u^2} \rightarrow y = u^2$

$= 2\pi \int_1^4 u\sqrt{4u^2+1}\, du \quad v = 4u^2+1$

$dv = 8u\, du \rightarrow \dfrac{dv}{8} = u\, du \quad = \dfrac{2\pi}{8}\int_1^4 \sqrt{v}\, dv = \dfrac{\pi}{4}\int_1^4 \sqrt{v}$

$= \dfrac{\pi}{4} \cdot \dfrac{2v^{3/2}}{3} = \dfrac{2\pi\sqrt{v^3}}{12} = \dfrac{\pi\sqrt{v^3}}{6}\Big|_1^4 = \dfrac{\pi\sqrt{(4(\sqrt{y})^2+1)^3}}{6}\Big|_1^4 = \dfrac{\pi\sqrt{(4y+1)^3}}{6}\Big|_1^4$

$= \dfrac{\pi\sqrt{(4.4+1)^3}}{6} - \dfrac{\pi\sqrt{(4.1+1)^3}}{6} = \dfrac{\pi}{6}\left(\sqrt{17^3} - \sqrt{5^3}\right) = \boxed{\dfrac{\pi}{6}\left(\sqrt{4913} - \sqrt{125}\right)\ u.a.}$

25) $y = \frac{1}{2}x,\ 0 \leq x \leq 4$; eixo dos X

$f(x) = \frac{x}{2} \qquad f'(x) = \frac{1}{2} \qquad [f'(x)]^2 = \frac{1}{4} \qquad a=0 \quad b=4$

$$A = 2\pi\int_0^4 \frac{x}{2}\sqrt{1+\frac{1}{4}}\,dx = \pi\int_0^4 x\sqrt{\frac{5}{4}}\,dx \qquad \begin{matrix} u = x \\ du = dx \end{matrix} \qquad \pi\int_0^4 \sqrt{\frac{5}{4}}\,u\,du = \sqrt{\frac{5}{4}}\,\pi\int_0^4 u\,du$$

$$= \pi\sqrt{\frac{5}{4}}\,\frac{u^2}{2}\Big|_0^4 = \pi\sqrt{\frac{5}{4}}\,\frac{x^2}{2}\Big|_0^4 = \pi\sqrt{\frac{5}{4}}\,\frac{4^2}{2} - 0 = \pi\sqrt{\frac{5}{4}}\,\frac{16}{2} = \boxed{\pi\sqrt{\frac{5}{4}}\ 8 \text{ u.a.}}$$

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26) $y = \sqrt{4-x^2},\ 0 \leq x \leq 1$; eixo dos X

$f(x) = \sqrt{4-x^2} \qquad f'(x) = \frac{1}{\cancel{2}\sqrt{4-x^2}} \cdot -\cancel{2}x = -\frac{x}{\sqrt{4-x^2}} \qquad [f'(x)]^2 = \left[\frac{-x}{\sqrt{4-x^2}}\right]\cdot\left[\frac{-x}{\sqrt{4-x^2}}\right]$

$a=0 \quad b=1$

$$= \frac{x^2}{4-x^2}$$

$$A = 2\pi\int_0^1 \cancel{\sqrt{4-x^2}}\sqrt{1+\frac{x^2}{\cancel{4-x^2}}}\,dx = 2\pi\int_0^1 2\,dx = 4\pi\int_0^1 dx = 4\pi x\Big|_0^1 = \boxed{4\pi \text{ u.a.}}$$

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27) $y = \sqrt{16-x^2};\ -3 \leq x \leq 3$; eixo dos X

$f(x) = \sqrt{16-x^2} = (16-x^2)^{\frac{1}{2}} \qquad f'(x) = \frac{1}{2}(16-x^2)^{-\frac{1}{2}} \cdot -2x = \frac{1}{\cancel{2}\sqrt{16-x^2}} \cdot -\cancel{2}x = \frac{-x}{\sqrt{16-x^2}}$

$[f'(x)]^2 = \frac{x^2}{16-x^2} \qquad a=-3 \quad b=3$

$$A = 2\pi\int_{-3}^{3}\sqrt{16-x^2}\sqrt{1+\frac{x^2}{16-x^2}}\,dx = 2\pi\int_{-3}^{3}\sqrt{(16-x^2)\left(1+\frac{x^2}{16-x^2}\right)}\,dx$$

$$= 2\pi\int_{-3}^{3}\sqrt{16+\frac{16x^2}{16-x^2}-x^2-\frac{x^4}{16-x^2}}\,dx = 2\pi\int_{-3}^{3}\sqrt{16-x^2+\frac{16x^2-x^4}{16-x^2}}\,dx \qquad \frac{x^2\cancel{(16-x^2)}}{\cancel{16-x^2}}$$

$$= 2\pi\int_{-3}^{3}\sqrt{16-\cancel{x^2}+\cancel{x^2}} = 2\pi\int_{-3}^{3}\sqrt{16} = 2\pi\int_{-3}^{3}4 = 2\pi\,4x\Big|_{-3}^{3} = 2\pi\,4.3 - 2\pi\,4(-3) = 2\pi(12+12)$$

$$= 2\pi(24)$$

$$= \boxed{48\pi \text{ u.a.}}$$