Coordenadas polares: introdução e relações entre sistemas (13/01/2020)

Lista de Exercícios

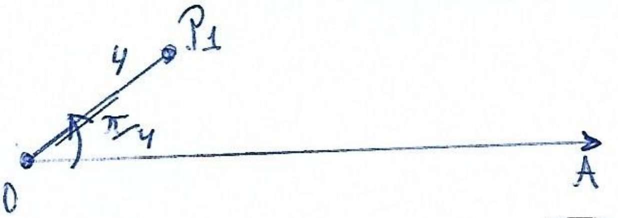
Livro: Cálculo A

Tópico 8.11, página 379.

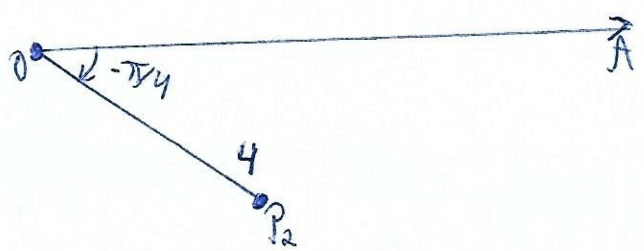
Questões: 1, 4, 5, 7 e 8.

(1) Demarcar os seguintes pontos no sistema de coordenadas polares.

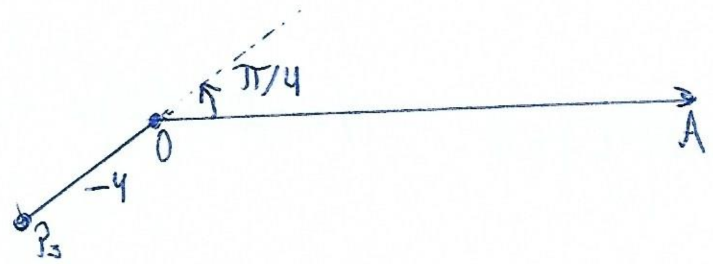
(a) $P_1(4, \pi/4)$



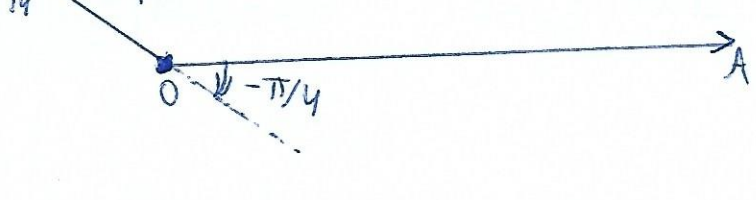
(b) $P_2(4, -\pi/4)$



(c) $P_3(-4, \pi/4)$



(d) $P_4(-4, -\pi/4)$



(4) Encontrar as coordenadas cartesianas dos seguintes pontos dados em coordenadas polares.

$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

$\operatorname{sen}\theta = \frac{y}{r} \Rightarrow y = r\operatorname{sen}\theta$

$r = \pm\sqrt{x^2+y^2}$

(a) $(-2, 2\pi/3)$

$\frac{2\pi}{3} = 120° = \frac{\sqrt{3}}{2}$ $\quad x = -2\cos\left(\frac{2\pi}{3}\right) = 1$ $\quad y = -2\operatorname{sen}\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

$P(1, -\sqrt{3})$

(b) $(4, 5\pi/8)$ $\quad r = 4$

$x = 4\cos\left(\frac{5\pi}{8}\right) = -1{,}53$ $\quad y = 4\operatorname{sen}\left(\frac{5\pi}{8}\right) = 3{,}69$

$P(-1{,}53,\ 3{,}69)$

(c) $(3, 13\pi/4)$ $\quad x = 3\cos\left(\frac{13\pi}{4}\right) = \frac{-3\sqrt{2}}{2}$ $\quad y = 3\operatorname{sen}\left(\frac{13\pi}{4}\right) = \frac{-3\sqrt{2}}{2}$

$P\left(\frac{-3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right)$

(d) $(-10, \pi/2)$ $\quad x = -10\cos\left(\frac{\pi}{2}\right) = 0$ $\quad y = -10\operatorname{sen}\left(\frac{\pi}{2}\right) = -10$

$P(0, -10)$

(e) $(-10, 3\pi/2)$ $\quad x = -10\cos\left(\frac{3\pi}{2}\right) = 0$ $\quad y = -10\operatorname{sen}\left(\frac{3\pi}{2}\right) = 10$

$P(0, 10)$

(b) $(1,0)$ $\quad x = \cos(0) = 1 \quad y = 1 \operatorname{sen}(0) = 0$

$P(1,0)$

---

(5) Encontrar um par de coordenadas polares dos seguintes pontos:

$$r = \pm\sqrt{x^2+y^2} \qquad \cos\theta = \frac{x}{r} \qquad \operatorname{sen}\theta = \frac{y}{r}$$

(a) $(1,1)$

$$r = \pm\sqrt{1+1} = \pm\sqrt{2} \qquad \cos\theta = \frac{1}{\sqrt{2}} \qquad \operatorname{sen}\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \qquad \theta = \operatorname{arcsen}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$P\left(\sqrt{2}, \frac{\pi}{4}\right)$

(b) $(-1,1)$ $\quad r = \pm\sqrt{(-1)^2+1^2} = \sqrt{1+1} = \sqrt{2} \qquad \cos\theta = \frac{-1}{\sqrt{2}} \qquad \operatorname{sen}\theta = \frac{1}{\sqrt{2}}$

$$\theta = \arccos\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4} \qquad \theta = \operatorname{arcsen}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \qquad \frac{3\pi}{4} = \frac{\pi}{4} = 45°$$

$\theta = 180 - 45$

$\theta = 135$

$\theta = \frac{3\pi}{4}$

$P\left(\sqrt{2}, \frac{3\pi}{4}\right)$

(c) $(-1,-1)$ $\quad r = \pm\sqrt{(-1)^2+(-1)^2} = \sqrt{1+1} = \sqrt{2} \qquad \cos\theta = \frac{-1}{\sqrt{2}} \qquad \operatorname{sen}\theta = \frac{-1}{\sqrt{2}}$

$$\theta = \arccos\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4} \qquad \theta = \operatorname{arcsen}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4} = \frac{5\pi}{4}$$

$\theta = 180 + 45$

$\theta = 225$

$\theta = \frac{5\pi}{4}$

$P\left(\sqrt{2}, \frac{5\pi}{4}\right)$

(d) $(1,-1)$ $\quad r = \pm\sqrt{1^2+(-1)^2} = \sqrt{1+1} = \sqrt{2}$ $\quad \cos\theta = \frac{1}{\sqrt{2}}$ $\quad sen\theta = -\frac{1}{\sqrt{2}}$

$\theta = arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$ $\qquad \theta = arcsen\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4} = \frac{5\pi}{4}$

$\theta = 360 - 45$
$\theta = 315$
$\theta = \frac{7\pi}{4}$

$P\left(\sqrt{2}, \frac{\cancel{3}5\pi}{4}\right)$

---

(7) Transformar as seguintes equações para coordenadas polares.

(a) $\underbrace{x^2+y^2=4}$ $\rightarrow \sqrt{2}^2+\sqrt{2}^2 = 4 \Rightarrow 2+2=4$

$x=\sqrt{2}$ $\quad y=\sqrt{2}$

$x^2+y^2 = \cancel{\quad} 2^2$

$\sqrt{x^2+y^2} = 2 = r$

$\cos\theta = \frac{\sqrt{2}}{2}$ $\qquad sen\theta = \frac{\sqrt{2}}{2}$ $\qquad \theta = arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ $\quad 45°$

$\theta = arcsen\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$P(2, \pi/4)$

(b) $x=4 = x^2 = 2^2$ $\qquad y=0$ $\qquad r = \pm\sqrt{2^2+0} = \sqrt{4} = 2$

$\cos\theta = \frac{4}{2} = 2$ $\qquad sen\theta = \frac{0}{2} = 0$ $\qquad \theta = arcsen(0) = 0$

$\theta = arccos(\cancel{2}0) = \frac{\pi}{2}$

$P(2, \pi/2)$

Página 380

(c) $y=2$ $\qquad r = \sqrt{2+0} = \sqrt{2}$ $\qquad \cos\theta = \frac{0}{\sqrt{2}} = 0$ $\qquad sen\theta = \frac{2}{\sqrt{2}} = 2 \rightarrow 0$

$\theta = arccos(0) = \frac{\pi}{2}$ $\qquad \theta = arcsen(0) = 0$

$P(\sqrt{2}, \pi/2)$

(d) $y + x = 0 \rightarrow y = -x \quad r = \pm\sqrt{y^2 + (-x)^2} = \pm\sqrt{x^2+y^2} = 0$ $\quad P(0,0)$

$y=0 \quad x=0 \qquad \cos\theta = 0 \qquad \operatorname{sen}\theta = 0$

$\theta = \arccos(0) = 1 \qquad \theta = \operatorname{arcsen}(0) = 0$

(e) $x^2+y^2-2x=0 \rightarrow x^2+y^2 = 2x \rightarrow \sqrt{\frac{x^2+y^2}{2}} = x^2 \Rightarrow r = 2x^2$

$\cos\theta = \frac{x^2}{2x^2} = \frac{1}{2} \qquad \operatorname{sen}\theta = \frac{y^2}{2x^2} \qquad P(2x^2, \pi/3)$

$\theta = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3} \qquad \theta = \operatorname{arcsen}(0) = 0$

(f) $x^2+y^2-6y=0 \rightarrow x^2+y^2 = 6y \rightarrow \sqrt{x^2+y^2} = 6y^2 \rightarrow r = 6y^2$

~~$\theta = \arccos$~~ $\quad \cos\theta = \frac{x^2}{6y^2} = 0 \qquad \operatorname{sen}\theta = \frac{y^2}{6y^2} = \frac{1}{6} \qquad P(6y^2; 0.16)$

$\theta = \arccos(0) = 1 \qquad \theta = \operatorname{arcsen}\left(\frac{1}{6}\right) = 0.16$

---

(8) Transformar os seguintes equações para coordenadas cartesianas.

(a) $r = \cos\theta \rightarrow r = \frac{x}{r} \rightarrow r^2 = x \rightarrow r = \sqrt{x} \qquad P(r^2, 0)$

$r = \pm\sqrt{x^2+y^2} \qquad \rightarrow \sqrt{x} = \sqrt{x^2+y^2}$

(b) $r = 2\operatorname{sen}\theta \rightarrow r = 2\frac{y}{r} \rightarrow r^2 = 2y \rightarrow \frac{r^2}{2} = y \qquad P\left(0, \frac{r^2}{2}\right)$

(c) $r = \frac{1}{\cos\theta + \operatorname{sen}\theta} \rightarrow r = \frac{1}{\frac{x}{r} + \frac{y}{r}} \rightarrow r = \frac{1}{\frac{x+y}{r}} \rightarrow r = \frac{r}{x+y} \rightarrow x+y = \frac{r}{r} \rightarrow x+y = 1$

$x = -y \qquad P(1,-1)$ ou $P(-1,1)$

(d) $r = a$, $a > 0$. $r = 1$ ou 2 ou $3\ldots$ $r = \sqrt{x^2+y^2} \rightarrow$ $1 = \sqrt{x^2+y^2} \rightarrow 1 = x^2+y^2$

$2 = \sqrt{x^2+y^2} \rightarrow 4 = x^2+y^2$

$3 = \sqrt{x^2+y^2} \rightarrow 9 = x^2+y^2$

$P(x, y)$