Integrais de funções contínuas por partes (11/12/2019) Vitor Oliveira [illegible]

Livro: Cálculo A

Tópico 6.15, página 290.

Questões: 2 e 3.

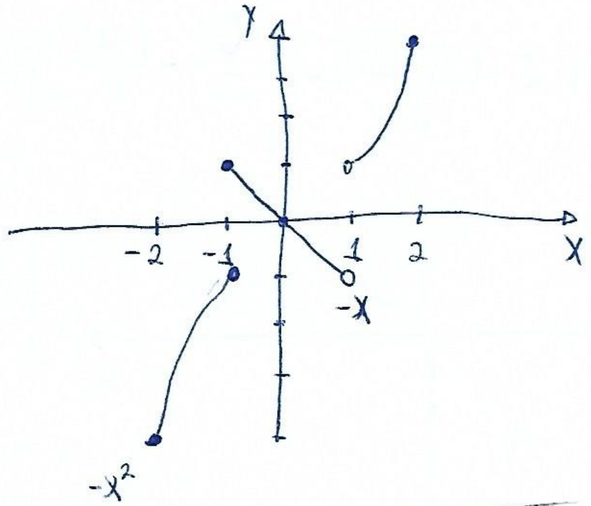
(2) Calcular a integral das seguintes funções contínuas por partes definidas nos intervalos dados. Fazer o gráfico das funções dadas, verificando que os resultados encontrados são corretos.

a) $f(x) = \begin{cases} -x^2, & -2 \leq x \leq -1 \\ -x, & -1 < x \leq 1 \\ x^2, & 1 < x \leq 2 \end{cases}$

$$I = \int_{-2}^{-1} -x^2\,dx + \int_{-1}^{1} -x\,dx + \int_{1}^{2} x^2\,dx = -\frac{x^3}{3}\Big|_{-2}^{-1} + -\frac{x^2}{2}\Big|_{-1}^{1} + \frac{x^3}{3}\Big|_{1}^{2}$$

$$= \left[\frac{-(-1)^3}{3} - \frac{-(-2)^3}{3}\right] + \left[-\frac{1^2}{2} - \frac{-(-1)^2}{2}\right] + \left[\frac{2^3}{3} - \frac{1^3}{3}\right] = \cancel{\frac{1}{3}} - \cancel{\frac{8}{3}} + \left[-\cancel{\frac{1}{2}} + \cancel{\frac{1}{2}}\right] + \cancel{\frac{8}{3}} - \cancel{\frac{1}{3}}$$

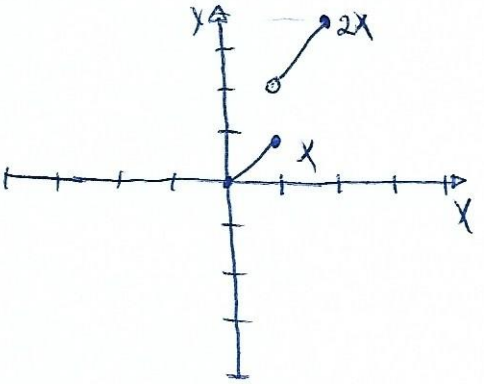
$= 0$



b) $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2x, & 1 < x \leq 2 \end{cases}$

$$I = \int_{0}^{1} x\,dx + \int_{1}^{2} 2x\,dx = \frac{x^2}{2}\Big|_{0}^{1} + \frac{\cancel{2}x^2}{\cancel{2}}\Big|_{1}^{2} = \frac{x^2}{2}\Big|_{0}^{1} + x^2\Big|_{1}^{2} = \frac{1}{2} + 2^2 - 1 = \frac{1}{2} + 4 - 1$$

$$= \frac{1}{2} + 3 = \frac{1+6}{2} = \frac{7}{2}$$



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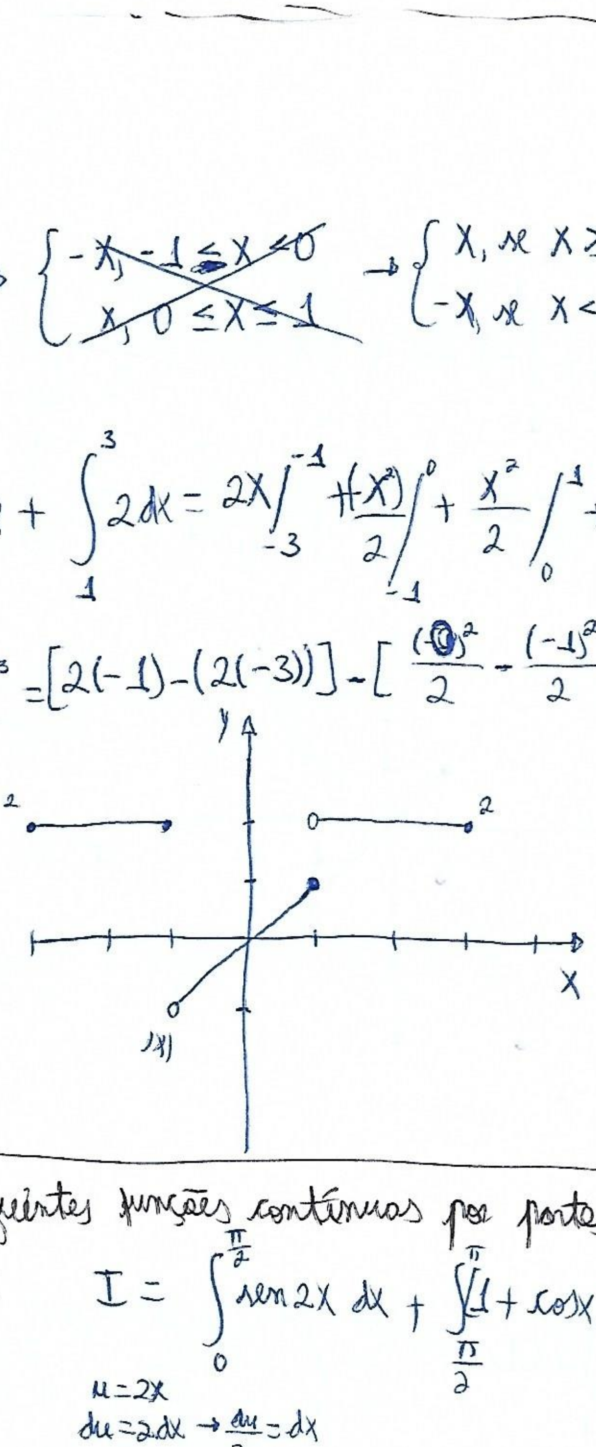
c) $f(x) = \begin{cases} 2, & -3 \leq x \leq -1 \\ |x|, & -1 < x \leq 1 \\ 2, & 1 < x \leq 3 \end{cases} \rightarrow \cancel{\begin{cases} -x, & -1 \leq x < 0 \\ x, & 0 \leq x \leq 1 \end{cases}} \rightarrow \begin{cases} x, & se\ x \geq 0 \\ -x, & se\ x < 0 \end{cases}$

$$I = \int_{-3}^{-1} 2\,dx + \int_{-1}^{0} -x\,dx + \int_{0}^{1} x\,dx + \int_{1}^{3} 2\,dx = 2x\Big|_{-3}^{-1} + \left(\frac{-x^2}{2}\right)\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_{0}^{1} + 2x\Big|_{1}^{3}$$

$$= 2x\Big|_{-3}^{-1} - \frac{x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_{1}^{0} + 2x\Big|_{1}^{3} = \left[2(-1)-(2(-3))\right] - \left[\frac{(0)^2}{2} - \frac{(-1)^2}{2}\right] + \left[\frac{0^2}{2} - \frac{1^2}{2}\right] + [2.3-2$$

$$= -2 + 6 + \frac{1}{2} - \frac{1}{2} + 4 = 8$$



③ - Calcular a integral das seguintes funções contínuas por partes.

a) $f(x) = \begin{cases} sen\,2x, & 0 \leq x \leq \frac{\pi}{2} \\ 1 + \cos x, & \frac{\pi}{2} < x \leq \pi \end{cases}$ $\quad I = \int_{0}^{\frac{\pi}{2}} sen\,2x\,dx + \int_{\frac{\pi}{2}}^{\pi} [1 + \cos x]\,dx$

$u = 2x$

$du = 2dx \rightarrow \frac{du}{2} = dx$

$$= \int_{0}^{\frac{\pi}{2}} sen(u)\,\frac{du}{2} + \int_{\frac{\pi}{2}}^{\pi} dx + \int_{\frac{\pi}{2}}^{\pi} \cos x\,dx = \frac{1}{2}(-\cos(2x))\Big|_{0}^{\frac{\pi}{2}} + x\Big|_{\frac{\pi}{2}}^{\pi} + sen(x)\Big|_{\frac{\pi}{2}}^{\pi}$$

$$= \left[\frac{1}{2}\left(-\cos\left(2\cdot\frac{\pi}{2}\right)\right) - \frac{1}{2}(-\cos(2\cdot 0))\right] + \left[\pi - \frac{\pi}{2}\right] + \left[\operatorname{sen}(\pi) - \operatorname{sen}\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{-\cos(\pi)}{2} + \frac{\cos(0)}{2} + \pi - \frac{\pi}{2} + \operatorname{sen}(\pi) - \operatorname{sen}\left(\frac{\pi}{2}\right) = \frac{-(-1)}{2} + \frac{1}{2} + \pi - \frac{\pi}{2} + 0 - 1$$

$$= \cancel{1} + \pi - \frac{\pi}{2} - \cancel{1} = \boxed{\pi - \frac{\pi}{2}}$$

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b) $f(x) = \begin{cases} \dfrac{1}{x+1}, & 0 \le x \le 2 \\ (x-1)^2, & 2 < x \le 4 \end{cases}$

$$I = \underbrace{\int_0^2 \frac{1}{x+1}\,dx}_{\substack{u = x+1 \\ du = dx}} + \underbrace{\int_2^4 (x-1)^2\,dx}_{\substack{v = x-1 \\ dv = dx}} = \int_0^2 \frac{du}{u} + \int_2^4 v^2\,dv$$

$$= \ln|x+1|\Big|_0^2 + \frac{(x-1)^3}{3}\Big|_2^4 = [\ln|3| - \ln|1|] + \left[\frac{(4-1)^3}{3} - \frac{(2-1)^3}{3}\right]$$

$$= \ln|3| + 9 - \frac{1}{3} = \frac{3\ln|3| + 27 - 1}{3} = \frac{3\ln|3| + 26}{3} = \ln|3| + \frac{26}{3}$$

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c) $f(x) \begin{cases} \operatorname{tg} x, & 0 \le x \le \dfrac{\pi}{4} \\ \cos 3x, & \dfrac{\pi}{4} < x < \dfrac{\pi}{3} \end{cases}$

$$I = \underbrace{\int_0^{\frac{\pi}{4}} \operatorname{tg} x\,dx}_{\frac{\operatorname{sen} x}{\cos x} \quad \substack{u = \cos x \\ du = -\operatorname{sen} x\,dx \\ -du = \operatorname{sen} x\,dx}} + \underbrace{\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 3x}_{\substack{v = 3x \\ dv = 3\,dx \\ \frac{dv}{3} = dx}} = -\int_0^{\frac{\pi}{4}} \frac{du}{u} + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos(v)\,\frac{dv}{3}$$

$$= -\ln|\cos x|\Big|_0^{\frac{\pi}{4}} + \frac{\operatorname{sen} 3x}{3}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \ln|\sec x|\Big|_0^{\frac{\pi}{4}} + \frac{\operatorname{sen} 3x}{3}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left[\ln\left|\sec\frac{\pi}{4}\right| - \cancel{\ln|\sec 0|}^{\,0}\right] + \left[\cancel{\frac{\operatorname{sen} 3\frac{\pi}{3}}{3}}^{\,0}\right.$$

$$\left. - \frac{\operatorname{sen} 3\frac{\pi}{4}}{3}\right] = \ln\left|\sec\left(\frac{\pi}{4}\right)\right| - \frac{\operatorname{sen}\left(\frac{3\pi}{4}\right)}{3} = \frac{3\ln\left|\sec\left(\frac{\pi}{4}\right)\right| - \operatorname{sen}\left(\frac{3\pi}{4}\right)}{3}$$