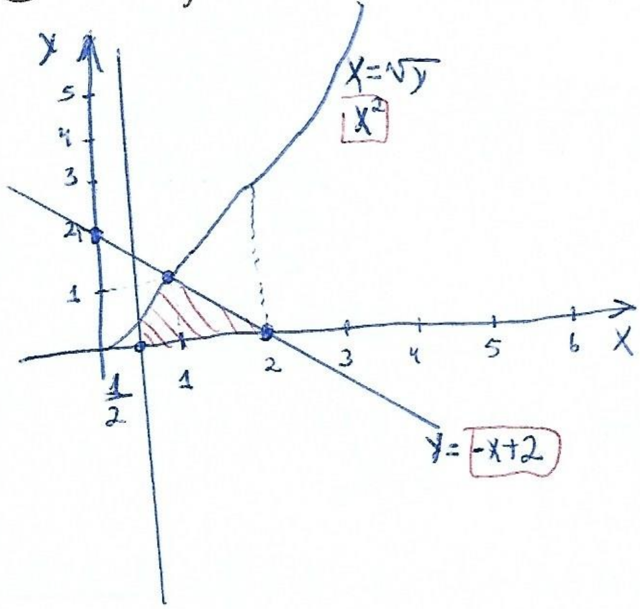
Cálculo de áreas (18/11/2019 - 18/11/2019) Vitor Oliveira Ropke

Livro: Cálculo A.

Tópico 6.13, página 278.

Questões: 1-5, 7, 9, 11-13, 19, 23, 25 e 30.

① $X = \frac{1}{2}, \ X = \sqrt{Y}$ e $Y = -X + 2$



Caso III

$$A = \int_{\frac{1}{2}}^{1} [-x + 2 - x^2]\,dx + \int_{1}^{2} [x^2 - (-x+2)]\,dx$$

$$= \left(-\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Big|_{\frac{1}{2}}^{1} + \left(\frac{x^3}{3} + \frac{x^2}{2} - 2x\right)\Big|_{1}^{2}$$

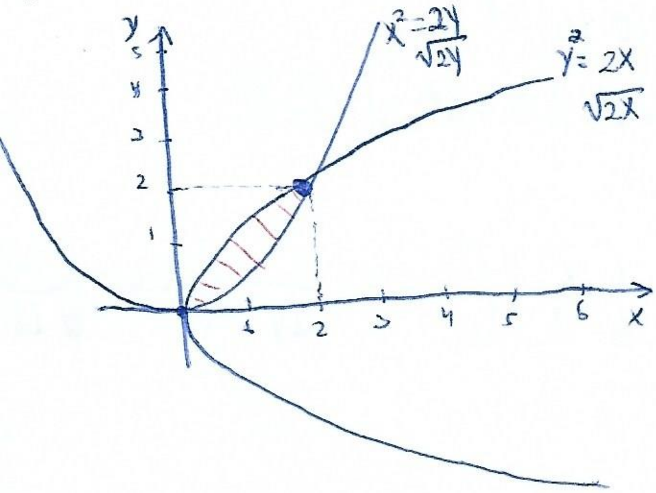
$$= \left(-\frac{(1)^2}{2} + 2\cdot 1 - \frac{1^3}{3}\right) - \left(-\frac{(1/2)^2}{2} + 2(1/2) - \frac{(1/2)^3}{3}\right)$$

$$+ \left(\frac{2^3}{3} + \frac{2^2}{2} - 2\cdot 2\right) - \left(\frac{1^3}{3} + \frac{1^2}{2} - 2\cdot 1\right)$$

$$= \frac{1}{2} + 2 - \frac{1}{3} + \frac{1}{2} - 1 + \frac{3}{2} + \frac{8}{3} + 2 - 4 - \frac{1}{3} - \frac{1}{2} + 2 = 1 + \frac{4}{2} - \frac{6}{3} = 1 + 2 - 2$$

$$\boxed{A = \boxed{1 \text{ u.a.}}}$$

② $Y^2 = 2X$ e $X^2 = 2Y$



P			
x	0	1	2
y	0	1.4	2

Caso III

$$\int_0^2 [\sqrt{2x} - x^2]\,dx = \int_0^2 \sqrt{2x}\,dx - \int_0^2 x^2\,dx$$

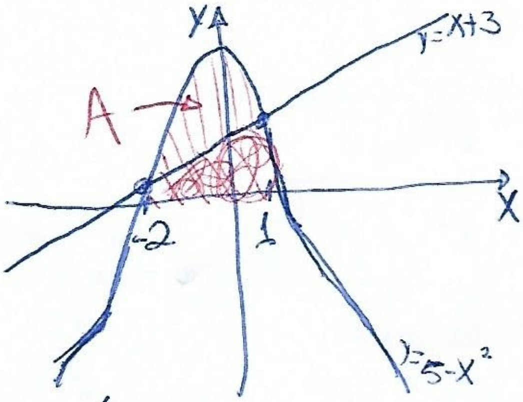
$u = 2x$

$du = 2dx \rightarrow \frac{du}{2} = dx$

$$= \frac{1}{2}\int_0^2 u^{1/2}\,du - \int_0^2 x^2\,dx = \left(\frac{1}{2}\cdot\frac{2u^{3/2}}{3} - \frac{x^3}{3}\right)\Big|_0^2$$

$$= \left(\frac{2u^{3/2}}{6} - \frac{x^3}{3}\right)\Big|_0^2 = \left(\frac{u^{3/2}}{3} - \frac{x^3}{3}\right)\Big|_0^2 = \frac{2^{3/2}}{3} - \frac{2^3}{3} - 0 = \frac{\sqrt{2^3}}{3} - \frac{8}{3} = \frac{\sqrt{8}}{3} - \frac{8}{3} = \boxed{\frac{\sqrt{8} - 8}{3} \text{ u.a.}}$$

(3) $y = 5 - x^2$ e $y = x + 3$



$y = 5 - x^2$

x	0	1	2	3	4	5
y	5	4	1	-4	-11	-20

$y = x + 3$

x	0	1	2	3	4	5
y	3	4	5	6	7	8

$5 - x^2 = x + 3$

$5 - 3 = x + x^2$

$2 = x + x^2$

$0 = x^2 + x - 2$

$\Delta = 1 - 4 \cdot 1(-2)$

$\Delta = 1 + 8$

$\Delta = 9$

$\frac{-1 \pm 3}{2} = x_1 = 1$

$x_2 = -2$

$$A = \int_{-2}^{1} [5 - x^2 - (x+3)]\,dx = \left(5x - \frac{x^3}{3} - \frac{x^2}{2} - 3x\right)\Big|_{-2}^{1} = \left(2x - \frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{-2}^{1}$$

$$= 2 - \frac{1^2}{2} - \frac{1^3}{3} - \left(-4 - \frac{4}{2} - \frac{8}{3}\right) = 2 - \frac{1}{2} - \frac{1}{3} + 4 + 2 + \frac{8}{3} = 8 - \frac{1}{2} + \frac{7}{3} = \frac{48 - 3 + 14}{6}$$

$$= \boxed{\frac{59}{6}\text{ u.a}}$$

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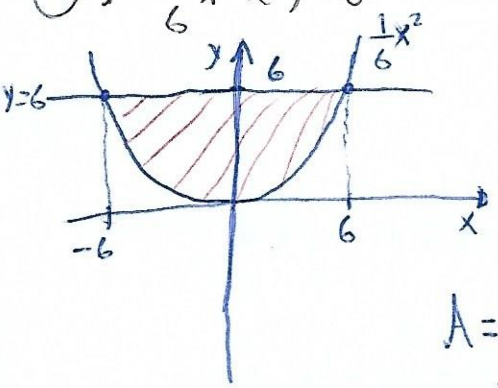
(4) $y = \frac{1}{6}x^2$ e $y = 6$



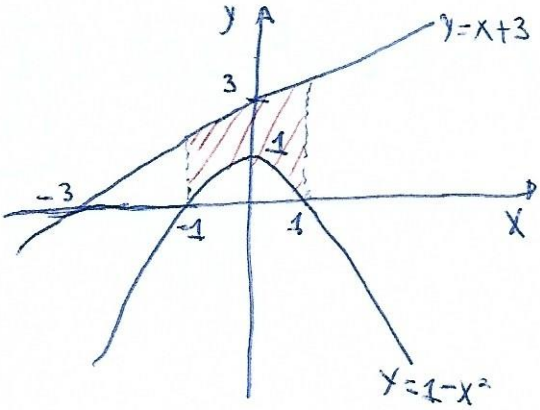
x	-3	-2	-1	0	1	2	3
y	1.5	0.6	0.16	0	0.16	0.6	1.5

$\frac{1}{6}x^2 = 6 \rightarrow x^2 = \frac{6}{\frac{1}{6}} \rightarrow x^2 = 6 \cdot 6 \rightarrow x^2 = 36 \rightarrow \boxed{x = 6}$

$$A = \int_{-6}^{6} \left[6 - \frac{1}{6}x^2\right] dx = \left(6x - \frac{1}{6} \cdot \frac{x^3}{3}\right)\Big|_{-6}^{6} = 6 \cdot 6 - \frac{6^3}{18} - \left(6(-6) - \frac{(-6)^3}{18}\right)$$

$$= 36 - 12 + 36 - 12 = \boxed{72\text{ u.a.}}$$

(5) $y = 1 - x^2$ e $y = x + 3$



$y = 1 - x^2$

x	-3	-2	-1	0	1	2	3
y	-8	-3	0	1	0	-3	-8

$y = x + 3$

x	-3	-2	-1	0	1	2	3
y	0	1	2	3	4	5	6

$$A = \int_{-1}^{1} [x+3] - [1 - x^2]\,dx = \left(\frac{x^2}{2} + 3x - x + \frac{x^3}{3}\right)\Big|_{-1}^{1}$$

$$= \frac{1^2}{2} + 3 - 1 + \frac{1^3}{3} - \frac{(-1)^2}{2} + 3(-1) - (-1) + \frac{(-1)^3}{3} = \cancel{\frac{1}{2}} + 2 + \frac{1}{3} - \cancel{\frac{1}{2}} = 2 + \frac{1}{3} = \frac{6+1}{3} = \frac{7}{3}$$

$\boxed{\frac{7}{3} \text{ u.a.}}$

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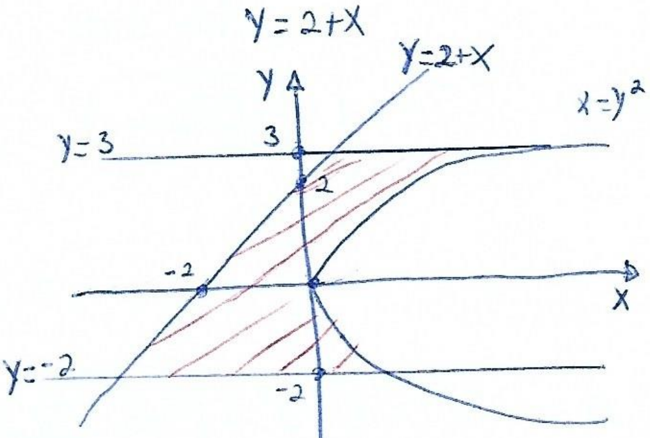
(7) $x = y^2$, $y - x = 2$, $y = -2$ e $y = 3$

$y = 2 + x$



$x = y^2$

x	9	4	1	0	1	4	9
y	-3	-2	-1	0	1	2	3

$y = 2 + x \rightarrow y - 2 = x$

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

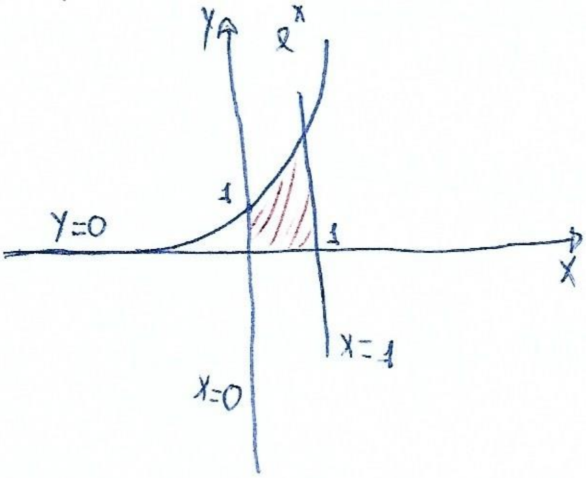
$$A = \int_{-2}^{3} [y^2 - (y-2)]\,dy = \left(\frac{y^3}{3} - \frac{y^2}{2} + 2y\right)\Big|_{-2}^{3}$$

$$= \frac{3^3}{3} - \frac{3^2}{2} + 2.3 - \left(\frac{(-2)^3}{3} - \frac{(-2)^3}{2} + 2(-2)\right)$$

$$= \frac{27}{3} - \frac{9}{2} + 6 - \left(-\frac{8}{3} - \frac{-8}{2} - 4\right) = 9 - \frac{9}{2} + 6 + \frac{8}{3} + 4 - 4 = 15 - \frac{9}{2} + \frac{8}{3} = \frac{90 - 27 + 16}{6}$$

$= \boxed{\frac{79}{6} \text{ u.a.}}$

(9) $y = e^x$, $x = 0$, $x = 1$ e $y = 0$



$$A = \int_0^1 e^x\,dx = e^x \Big|_0^1 = e^1 - e^0 = 2{,}71 - 1 = \boxed{1{,}71 \text{ u.a.}}$$

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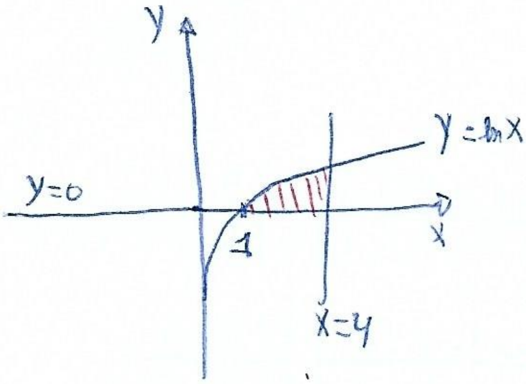
(11) $y = \ln x$, $y = 0$ e $x = 4$



$$A = \int_1^4 \ln x\,dx = x(\ln(x) - 1)\Big|_1^4 = 4(\ln(4) - 1) - (\ln(1) - 1)$$

$$= 4(1{,}38) - (0 - 1) = 5{,}52 + 1 = \boxed{6{,}52 \text{ u.a.}}$$

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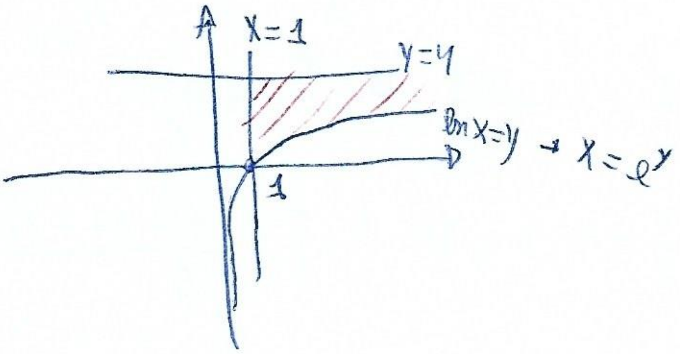
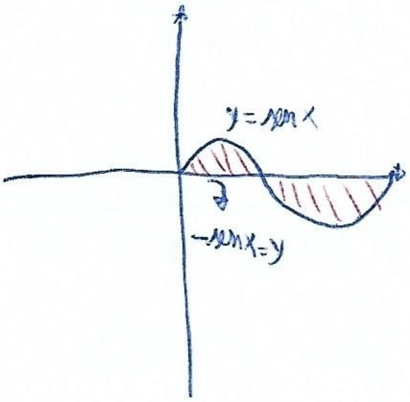
(12) $y = \ln x$, $x = 1$ e $y = 4$



$$A = \int_0^4 e^y\,dy = e^y \Big|_0^4 = e^4 - e^0 = \boxed{53{,}59 \text{ u.a.}}$$

(18) $y = \operatorname{sen} x$ e $y = -\operatorname{sen} x$, $x \in [0, 2\pi]$



$$A = \int_0^{\pi} \operatorname{sen} x - (-\operatorname{sen} x)\,dx + \int_{\pi}^{2\pi} -\operatorname{sen} x - \operatorname{sen} x\,dx = \int_0^{\pi} [\operatorname{sen} x + \operatorname{sen} x]\,dx + \int_{\pi}^{2\pi} -2\operatorname{sen} x\,dx$$

$$= -2\cos x \Big|_0^{\pi} + 2\cos x \Big|_{\pi}^{2\pi} = -2\cos\pi - (-2\cos 0) + 2\cos 2\pi - 2\cos\pi$$

$$= 2 + (-2) + 2 - 2 = \boxed{0 \text{ u.a.}}$$

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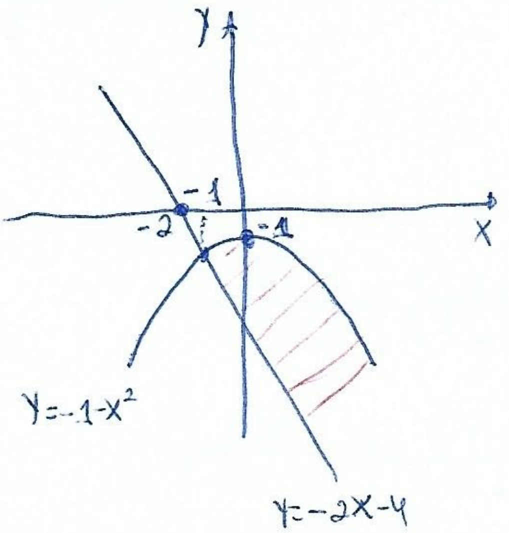
(19) $y = -1 - x^2$; $y = -2x - 4$



$y = -1 - x^2$

x	-3	-2	-1	0	1	2	3
y	-10	-5	-2	-1	-2	-5	-10

$y = -2x - 4$

x	-3	-2	-1	0	1	2	3
y	2	0	-2	-4	-6	-8	-10

$-1 - x^2 = -2x - 4 \rightarrow -x^2 + 2x + 3 = 0$

$x_1 + x_2 = -\frac{b}{a} = -\frac{2}{-1} = 2$ $\qquad \Delta = 4 - 4(-1).3$

$\Delta = 4 + 12$

$x_1 * x_2 = \frac{c}{a} = \frac{3}{-1} = -3$ $\qquad \Delta = 16$

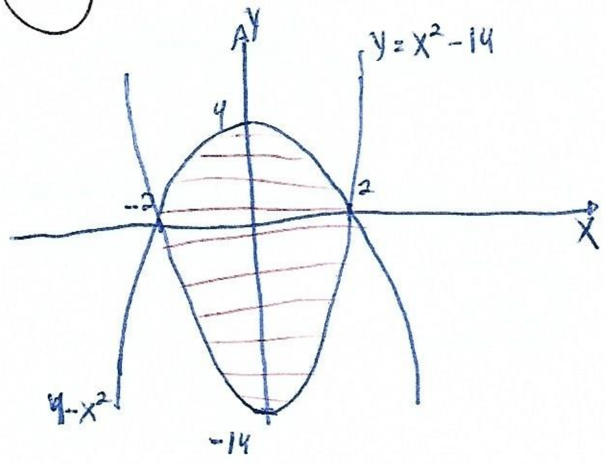
$\frac{-2 \pm 4}{-2} = \begin{array}{|c|} x_1 = -1 \\ \hline x_2 = 3 \end{array}$ limites

$$A = \int_{-1}^{3} [-1 - x^2 - (-2x - 4)]\,dx = \int_{-1}^{3} [-1 - x^2 + 2x + 4]\,dx = \left(-x - \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + 4x\right)\Big|_{-1}^{3}$$

$$= \left(3x + x^2 - \frac{x^3}{3}\right)\Big|_{-1}^{3} = 3.3 + 3^2 - \frac{3^3}{3} - \left(3(-1) + (-1)^2 - \frac{(-1)^3}{3}\right)$$

$$= \cancel{9} + 9 - \cancel{9} - \left(-3 + 1 - \frac{(-1)}{3}\right) = 9 + 3 - 1 - \frac{1}{3} = 11 - \frac{1}{3} = \frac{33 - 1}{3} = \boxed{\frac{32}{3} \text{ u.a.}}$$

23) $Y = 4 - X^2$ e $Y = X^2 - 14$



$y = 4 - x^2$

x	-3	-2	-1	0	1	2	3
y	-5	0	3	4	3	0	-5

$y = x^2 - 14$

x	-3	-2	-1	0	1	2	3	4
y	-5	-10	-13	-14	-13	-10	-5	2

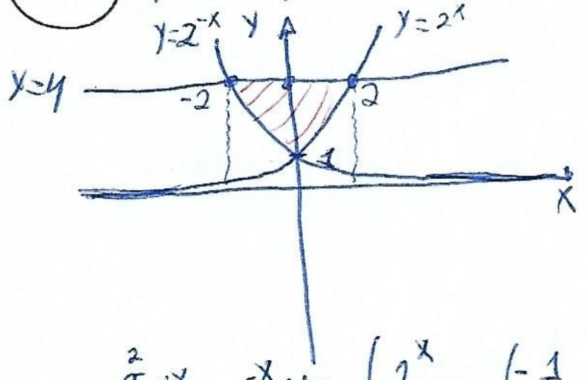
$$A = \int_{-2}^{2} [4 - x^2 - (x^2 - 14)]\,dx = \int_{-2}^{2} [4 - x^2 - x^2 + 14]\,dx$$

$$= \int_{-2}^{2} [-2x^2 + 18]\,dx = \left(-2\frac{x^3}{3} + 18x\right)\Big|_{-2}^{2}$$

$$= -2\cdot\frac{2^3}{3} + 18\cdot 2 - \left(-2\frac{(-2)^3}{3} + 18(-2)\right) = -\frac{16}{3} + 36 - \frac{16}{3} + 36 = -\frac{32}{3} + 72 = \frac{-32+216}{3}$$

$$= \boxed{\frac{184}{3}\text{ u.a.}}$$

25) $y = 2^x$, $y = 2^{-x}$ e $y = 4$



$y = 2^x$

x	-3	-2	-1	0	1	2	3
y	0.125	0.25	0.5	1	2	4	8

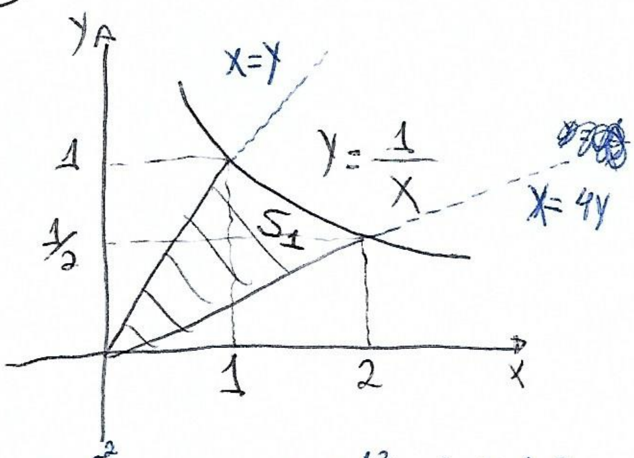
$y = 2^{-x}$

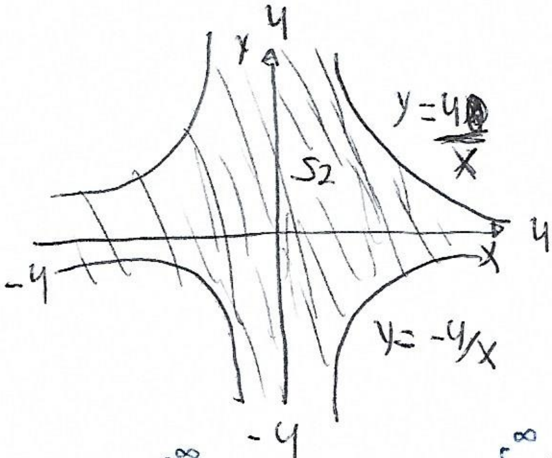
x	-3	-2	-1	0	1	2	3
y	8	4	2	1	0.5	0.25	0.125

$$A = \int_{-2}^{2} [2^{+x} - 2^{-x}]\,dx = \left(\frac{2^x}{\ln(2)} - \left(-\frac{1}{\ln(2).2^x}\right)\right)\Big|_{-2}^{2} = \left(\frac{2^x}{\ln 2} + \frac{1}{\ln 2.2^x}\right)\Big|_{-2}^{2}$$

$$= \frac{2^2}{\ln 2} + \frac{1}{\ln 2.2^2} - \frac{2^{-2}}{\ln 2} - \frac{1}{\ln 2.2^{-2}} = \frac{4}{\ln 2} + \frac{1}{\ln 2.4} - \frac{1}{\ln 2.4} - \frac{4}{\ln 2} = \boxed{0\text{ u.a.}}$$

(30) Encontrar a área das regiões S_1 e S_2 vistas na figura a seguir:





$$A_{S_1} = \int_0^2 \frac{1}{x}\,dx = \ln x \Big|_0^2 = \ln 2 - \ln 0$$

$$= 0.69 - ?$$

$$A_{S_1} \text{ em } y = \int_0^1 \frac{1}{y}\,dy = \ln y \Big|_0^1 = \ln 1 - \ln 0$$

$$= 0 - ?$$

$$A_{S_2} = \int_{-\infty}^{\infty} [4/x - (-4/x)]\,dx = \int_{-\infty}^{\infty} 8/x \, dx$$

$$= 8 \ln x \Big|_{-\infty}^{\infty} = 8\ln(\infty) - 8\ln(-\infty) = \boxed{0 \text{ u.a.}}$$