Integração de funções envolvendo funções trigonométricas (02/10/2019) - (07/10/2019)

Livro: Cálculo A

Tópico 7.4, página 311.

Questões: 18, 19, 21-24, 26, 27, 28, 33 e 34.

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(18) $\int tg^3 x \cos^4 x\, dx = \int \frac{sen^3 x}{\cos^3 x} \cdot \cos^4 x\, dx = \int sen^3 x \cos x\, dx$ $\quad u = sen\, x$, $du = \cos x\, dx$

$= \int u^3\, du = \frac{u^4}{4} + c = \boxed{\frac{sen^4 x}{4} + c}$

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(19) $\int \cos^4 x\, dx = \int (\cos^2 x)^2 dx = \int \left(\frac{1 + \cos(2x)}{2}\right)^2 dx = \int \left(\frac{1 + 2\cos(2x) + \cos^2(2x)}{4}\right) dx$

$= \frac{1}{4}\int \left(1 + 2\cos(2x) + \frac{1 + \cos(4x)}{2}\right) dx = \frac{1}{4}\int \left(1 + 2\cos(2x) + \frac{1}{2} + \frac{1}{2} \cdot \cos(4x)\right) dx$

$= \frac{1}{4}\int \left(\frac{3}{2} + 2\cos(2x) + \frac{1}{2}\cos(4x)\right) dx = \frac{1}{4}\int \frac{3}{2} dx + \frac{1}{4}\int 2\cos(2x)\, dx + \frac{1}{4}\int \frac{1}{2} \cdot \cos(4x)\, dx$

$u = 2x \quad du = 2dx \rightarrow \frac{du}{2} = dx$

$u' = 4x \quad du' = 4dx \rightarrow \frac{du'}{4} = dx$

$\boxed{= \frac{3}{8} x + \frac{1}{4} sen(2x) + \frac{1}{32} sen(4x) + c}$

$u = tg\, x \quad du = sec^2 x\, dx$

(21) $\int \frac{sen^2 x}{\cos^4 x} dx = \int tg^2 x\, sec^2 x\, dx$

$= \int u^2\, du = \frac{u^3}{3} + c = \boxed{\frac{tg^3 x}{3} + c}$

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(22) $\int 15\, sen^5 x\, dx = 15\int sen^4 x\, sen\, x\, dx = 15\int (sen^2 x)^2 sen\, x\, dx = 15\int (1 - \cos^2 x)^2 sen\, x\, dx$

$u = \cos x \quad du = -sen\, x\, dx$

$-du = sen\, x\, dx$

$= 15\int (1 - u^2)^2 -du = -15\int (1-u^2)^2 du = -15\int (1 - 2u^2 + u^4)\, du = -15\int du - 15\int 2u^2\, du - 15\int u^4\, du$

$= -15u - \frac{30u^3}{3} - \frac{15u^5}{5} + c = \boxed{-15\cos x - 10\cos^3 x - 3\cos^5 x + c}$

(23) $\int 15 \, sen^2 X \cos^3 X \, dX = 15 \int sen^2 X \cos^3 X \, dX = 15 \int sen^2 X \cos^2 X \cos X \, dX$

$u = sen X \quad du = \cos X \, dX$

$= 15 \int sen^2 X (1 - sen^2 X) \cos X \, dX$

$= 15 \int [u^2 (1 - u^2)] \, du = 15 \int (-u^4 + u^2) \, du = -15 \int u^4 \, du + 15 \int u^2 \, du = -\frac{15 u^5}{5} + \frac{15 u^3}{3} + c$

$= -3u^5 + 5u^3 + c = \boxed{-3 \, sen^5 X + 5 \, sen^3 X + c}$

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(24) $\int 48 \, sen^2 X \cos^4 X \, dX = 48 \int sen^2 X (\cos^2 X)^2 \, dX = 48 \int sen^2 X \left(\frac{1 + \cos(2X)}{2} \right)^2 dX$

$= 48 \int sen^2 X \left(\frac{1 + 2\cos(2X) + \cos^2(2X)}{4} \right) dX = 12 \int sen^2 X \left(1 + 2\cos(2X) + \left(\frac{1 + \cos(4X)}{2} \right) \right) dX$

$= 12 \int \left(\frac{1 - \cos(2X)}{2} \right) \left(1 + 2\cos(2X) + \frac{1 + \cos(4X)}{2} \right) dX = 12 \int \frac{1}{2} + \frac{2\cos(2X)}{2} + \frac{1 + \cos(4X)}{4} - \frac{\cos(2X)}{2} - \frac{2\cos^2(2X)}{2}$

$- \frac{\cos(2X)}{4} - \frac{\cos(2X)}{2}$

$+ \frac{\cos(4X)}{2} \, dX$

$12 \int \frac{1}{2} dx + 12 \int \cos(2X) \, dX + 12 \int \frac{1}{4} dX + 12 \int \frac{\cos(4X)}{4} dX - 12 \int \frac{\cos(2X)}{2} - 12 \int \cos^2(2X) - 12 \int \frac{\cos(2X)}{4} - 12 \int \frac{\cos(2X)}{2} + 12 \int \frac{\cos(4X)}{2}$

$6X + 6 \, sen(2X) + 3X + \frac{3}{4} sen(4X) - 3 \, sen(2X) - 12 \int 1 - sen^2(2X) - \frac{3}{2} sen(2X) - 3 \, sen(2X) + \frac{3}{2} sen(4X)$

$\boxed{9X + 3 \, sen(2X) + \frac{9}{4} sen(4X) + c}$

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(26) $\int \frac{-3\cos^2 X}{sen^4 X} dX = -3 \int \frac{\cos^2 X}{(sen^2 X)^2} dX = -3 \int \frac{sec^2 X}{tg^4 X} dX \qquad u = tg X \quad du = sec^2 X \, dX$

$= -3 \int \frac{du}{u^4} = -3 \int u^{-4} = -3 \frac{u^{-3}}{-3} = \frac{1}{u^3} = \boxed{\frac{1}{tg^3 X} + c}$

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(27) $\int sen 3X \cos 5X \, dX = \int \frac{[sen(3X + 5X) - sen(3X - 5X)]}{2} dX = \int \frac{sen(8X) - sen(-2X)}{2} dX \qquad u = -2X \quad du = -2 \, dX \quad -\frac{du}{2} = dX$

$-\int \frac{sen(-4u) - sen(u)}{2} \cdot \frac{du}{2} = -\frac{1}{4} \int sen(-4u) - sen(u) \, du = -\frac{1}{4} \int sen(-4u) \, du + \frac{1}{4} \int sen(u) \, du$

$u' = -4u$

$-\frac{du'}{4} = du$

$= -\frac{1}{4} \int sen(u') \cdot -\frac{du'}{4} + \frac{1}{4} \int sen(u) \, du = -\frac{1}{16} \cdot -\cos(8X) + \frac{1}{4} \cdot -\cos(-2X) + c = \boxed{\frac{1}{16} \cdot \cos(8X) + \frac{1}{4} \cdot \cos(2X) + c}$

(28) $\int tg^2 5X\,dX \quad u=5X \quad \frac{du}{5}=dx \quad \frac{1}{5}\int tg^2 u\,du = \frac{1}{5}\int sec^2(u)-$

$= \frac{1}{5}\int sec^2(u)\,du - \frac{1}{5}\int -1\,du = \frac{1}{5} tg(5X) + \frac{5X}{5} + c = \boxed{\frac{1}{5}\cdot tg(5X)+x+c}$

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(33) $\int sec^3(1-4X)\,dX \quad a=1-4X \quad da=-4dx \rightarrow -\frac{da}{4}=dX \quad -\frac{1}{4}\int sec^3(a)\,da$

$= -\frac{1}{4}\int sec(a).sec^2(a)\,da \quad u=sec(a) \rightarrow du = sec(a).tg(a)\,da$
$dv = sec^2(a)da \rightarrow v = tg(a)$

$= -\frac{1}{4}\int sec(a).sec^2(a)\,da = sec(a)\,tg(a) - \int tg(a)\,sec(a).tg(a)\,da$

$= -\frac{1}{4}\int sec^3(a)\,da \quad = sec(a).tg(a) - \int tg^2(a)\,sec(a)\,da$

$= -\frac{1}{4}\int sec^3(a)\,da \quad = sec(a).tg(a) - \int[sec^2(a)-1]\,sec(a)\,da$

$= -\frac{1}{4}\int sec^3(a)\,da \quad = sec(a).tg(a) - \int[sec^3(a) - sec(a)]\,da$

$= -\frac{1}{4}\int sec^3(a)\,da \quad = sec(a).tg(a) - \int sec^3(a)\,da + \int sec(a)\,da$

$= -\frac{1}{4}\int sec^3(a)\,da \quad = sec(a).tg(a) + ln|sec(a)+tg(a)| + c$

$= -\frac{5}{4}\int sec^3(a)\,da$

$= sec^3(1-4x)dx \quad = \boxed{-\frac{4}{5}\cdot sec(1-4X).tg(1-4X) + ln|sec(1-4X)+tg(1-4X)| + c}$

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(34) $\int cosec^4(3-2X)\,dX \quad u=3-2X \quad -\frac{du}{2}=dX \quad -\frac{1}{2}\int(cosec^2(u))^2\,du$

$= -\frac{1}{2}\int[cosec^2(u).cosec^2(u)]\,du = -\frac{1}{2}\int(cotg^2(u)+1)\,cosec^2(u)\,du$

$u' = cotg(u)$
$du' = -cosec^2(u)\,du$
$-du' = -cosec^2(u)du$

$= \frac{1}{2}\int[u'^2+1]\,du' = \frac{1}{2}\int u'^2\,du' + \frac{1}{2}\int du' = \frac{1}{2}\cdot\frac{u'^3}{3} + \frac{1}{2}u' = \boxed{\frac{1}{6}\cdot cotg^3(3-2X) + \frac{1}{2}\cdot cotg(3-2X)+c}$