Apresentação da disciplina, primitivas, integral indefinida e integrais imediatas (16/03/2019)

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Livro: Cálculo A

Tópico 6.2, página 246

Questões: 11-13, 16-25, 28, 32, 33, 34 e 36.

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6.2 Exercícios

Nos exercícios de 11 a 31, calcular as integrais indefinidas.

(11) $\int \frac{x^2}{x^2+1}\,dx = \cancel{\int x^2 \cdot \frac{1}{x^2+1}\,dx = x^2\int\frac{1}{x^2+1}\,dx = x^2 \cdot \text{arctg}(x)+c}$

$\int x^2 \cdot \frac{1}{x^2+1}\,dx \quad x^2 = x^2+1-1 \rightarrow \int\left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\right)dx = \int\left(1 - \frac{1}{x^2+1}\right)dx =$

$= \int 1\,dx - \int \frac{1}{x^2+1}\,dx = x + c_1 - \text{arctg}(x) + c_2 = \boxed{x - \text{arctg}(x) + c}$

(12) $\int \frac{x^2+1}{x^2}\,dx = \int\left(1 + \frac{1}{x^2}\right)dx = \int 1\,dx + \int \frac{1}{x^2}\,dx = x + c_1 + \left(-\frac{1}{x}\right)$

$= \boxed{x - \frac{1}{x} + c}$

$\int x^{-2}\,dx \rightarrow \frac{x^{-1}}{-1} \rightarrow -\frac{1}{x}$

(13) $\int \frac{\text{sen}\,x}{\cos^2 x}\,dx = \int \frac{\text{sen}(x)}{\cos(x)} \cdot \frac{1}{\cos(x)}\,dx = \int \text{tg}(x) \cdot \sec(x)\,dx = \boxed{\sec(x) + c}$

Imediato

(16) $\int \frac{8x^4 - 9x^3 + 6x^2 - 2x + 1}{x^2}\,dx = \int \frac{8x^4\,dx}{x^2} - \int \frac{9x^3\,dx}{x^2} + \int \frac{6x^2\,dx}{x^2} - \int \frac{2x\,dx}{x^2} + \int \frac{1}{x^2}\,dx =$

$= \int 8x^2\,dx - \int 9x\,dx + \int 6\,dx - \int 2x^{-1}\,dx + \int \frac{1}{x^2}\,dx = 8\int x^2\,dx - 9\int x\,dx + 6\int dx - 2\int x^{-1}\,dx + \int \frac{1}{x^2}\,dx$

$\int \frac{1}{x} = \ln(x)$

$= \boxed{\frac{8x^3}{3} - \frac{9x^2}{2} + 6x - 2\ln(x) - \frac{1}{x} + c}$

(17) $\int\left(\frac{e^t}{2} + \sqrt{t} + \frac{1}{t}\right) dt = \int \frac{e^t}{2} dt + \int \sqrt{t}\, dt + \int \frac{1}{t} dt = \frac{1}{2}\int e^t dt + \int t^{\frac{1}{2}} dt + \int \frac{1}{t}$

$= \frac{1}{2} e^t + c_1 + \frac{t^{3/2}}{3/2} + c_2 + \ln|t| + c_3 = \boxed{\frac{1}{2} e^t + \frac{2t^{3/2}}{3} + \ln|t| + c}$

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(18) $\int \cos\theta \cdot tg\theta\, d\theta = \int \cancel{\cos\theta} \cdot \frac{sen\theta}{\cancel{\cos\theta}} d\theta = \int sen\theta\, d\theta = \boxed{-\cos\theta + c}$

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(19) $\int(e^x - e^{-x})\, dx = \int 2\, senh(x)\, dx = 2\int senh(x)\, dx$

$= \boxed{2\cosh(x) + 2}$

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(20) $\int(t + \sqrt{t} + \sqrt[3]{t} + \sqrt[4]{t} + \sqrt[5]{t})\, dt = \int t\, dt + \int t^{\frac{1}{2}} dt + \int t^{\frac{1}{3}} dt + \int t^{\frac{1}{4}} dt + \int t^{\frac{1}{5}} dt$

$= \frac{t^2}{2} + c_1 + \frac{t^{3/2}}{3/2} + c_2 + \frac{t^{4/3}}{4/3} + c_3 + \frac{t^{5/4}}{5/4} + c_4 + \frac{t^{6/5}}{6/5} + c_5 =$

$= \boxed{\frac{t^2}{2} + \frac{2t^{3/2}}{3} + \frac{3t^{4/3}}{4} + \frac{4t^{5/4}}{5} + \frac{5t^{6/5}}{6} + c}$

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(21) $\int \frac{x^{-\frac{1}{3}} - 5}{x} dx = \int \frac{x^{-\frac{1}{3}}}{x} dx - \int \frac{-5}{x} dx = \int \frac{\frac{1}{x^{\frac{1}{3}}}}{x} dx - 5\int \frac{1}{x} dx$

$= \int \frac{1}{x^{\frac{1}{3}}} \cdot x\, dx - 5\ln|x| + c_2 = \int \frac{x}{x^{4/3}} dx - 5\ln|x| + c_2 = \int \frac{1}{x^{4/3}} dx\ 5\ln|x| + c_2$

$= \int x^{-\frac{4}{3}} dx - 5\ln|x| + c_2 = \frac{x^{-\frac{1}{3}}}{-1/3} + c_1 - 5\ln|x| + c_2 = -\frac{3x^{-\frac{1}{3}}}{1} + c_1 - 5\ln|x| + c_2$

$= -\frac{3}{x^{\frac{1}{3}}} + c_1 - 5\ln|x| + c_2 = \boxed{-\frac{3}{\sqrt[3]{x}} - 5\ln|x| + c}$

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(22) $\int(2^t - \sqrt{2}\, e^t + \cosh t)\, dt = \int 2^t dt - \int \sqrt{2}\, e^t dt + \int \cosh t\, dt =$

$= \frac{2^t}{\ln(2)} + c_1 - \sqrt{2}\int e^t dt + senh + c_3 = \boxed{\frac{2^t}{\ln(2)} - \sqrt{2}\, e^t + senh + c}$

(23) $\int \sec^2 x(\cos^3 x + 1)\,dx = \int \sec^2(x)\cdot\cos^3(x)\,dx + \int \sec^2 x\,dx$

$= \int \frac{1}{\cancel{\cos^2(x)}}\cdot\cos^{\cancel{3}\,1}(x)\,dx + tg(x) + c_2 = \int \cos(x)\,dx + tg(x) + c_2 = \boxed{sen(x) + tg(x) + c}$

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(24) $\int \frac{dx}{(ax)^2 + a^2}$, $a \neq 0$, constante. $= \int \frac{dx}{a^2(x^2+1)} = \int \underset{\substack{\uparrow \\ \text{constante}}}{\frac{1}{a^2}} \cdot \frac{dx}{(x^2+1)} = \frac{1}{a^2}\int \frac{dx}{(x^2+1)}$

$= \frac{1}{a^2}\cdot arctg(x) + c$

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(25) $\int \frac{x^2 - 1}{x^2+1}\,dx = \int \frac{\cancel{x^2}}{\cancel{x^2}+1}\,dx + \int \frac{-1}{x^2+1}\,dx = \int dx - \int \frac{dx}{x^2+1} = \boxed{x - arctg(x) + c}$

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(28) $\int \frac{\ln x}{x \ln x^2}\,dx = \int \frac{1}{x \ln x}\,dx = \int \frac{1}{x2}\,dx = \frac{1}{2}\int \frac{dx}{x} = \boxed{\frac{1}{2}\cdot \ln|x| + c}$

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(32) Encontrar uma primitiva F, da função $f(x) = x^{\frac{2}{3}} + x$, que satisfaça $F(1) = 1$.

Se $F' = x^{\frac{2}{3}} + x$, então $\int (x^{\frac{2}{3}} + x)\,dx = \int x^{\frac{2}{3}}dx + \int x\,dx = \boxed{\frac{3x^{5/3}}{5} + \frac{x^2}{2} + c}$

Tal que c é $1 = \frac{3\sqrt[3]{1^5}}{5} + \frac{1^2}{2} + c \to 1 = \frac{3}{5} + \frac{1}{2} + c \to 1 = \frac{6+5}{10} + c$ $\boxed{c = \frac{-1}{10}}$

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(33) Determinar a função $f(x)$ tal que $\int f(x)\,dx = x^2 + \frac{1}{2}\cos 2x + c$.

$\left[x^2 + \frac{1}{2}\cos 2x + c\right]' = \boxed{2x - sen(2x)}$

constantes

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(34) Encontrar uma primitiva da função $f(x) = \frac{1}{x^2} + 1$ que se anule no ponto $x = 2$

$\int \left(\frac{1}{x^2} + 1\right)dx = \int x^{-2}dx + \int dx = \frac{x^{-1}}{-1} + c_1 + x + c_2 = \boxed{-\frac{1}{x} + x + c}$

Tal que c é $0 = -\frac{1}{2} + 2 + c \to -c = \frac{-1+4}{2} \to -c = \frac{3}{2} \to c = \frac{-3}{2}$

(36) Encontrar uma função f tal que $\underbrace{f'(x) + \text{sen}\, x = 0}$ e $f(0) = 2$

$f'(x) = -\text{sen}(x) \rightarrow \int -\text{sen}(x) = \boxed{\cos(x) + C}$

Sendo que C é $\cos(0) + C = 2 \rightarrow 1 + C = 2 \rightarrow C = 2 - 1 \rightarrow \boxed{C = 1}$