Substituição simples ou mudança de variável para integração (18/09/2013)
Livro: Cálculo A.
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Tópico 6.4, página 250.
Questões: 1-12, 18-21, 26, 28, 30, 33 e 39.

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Calcular as integrais seguintes usando o método da substituição.

1) $\int (2x^2+2x-3)^{10}(2x+1)\,dx$

$u = 2x^2+2x-3$

$du = 4x+2\,dx \rightarrow \frac{du}{2} = 2x+1\,dx$

$$\int (u)^{10}\frac{du}{2} = \frac{1}{2}\int u^{10}\,du = \frac{1}{2}\frac{u^{11}}{11}+c = \boxed{\frac{1}{2}\cdot\frac{(2x^2+2x-3)^{11}}{11}+c}$$

2) $\int (x^3-2)^{1/7}x^2\,dx$ $\quad u = x^3-2 \quad du = 3x^2\,dx \rightarrow \frac{du}{3} = x^2\,dx$

$$\int u^{1/7}\frac{du}{3} = \frac{1}{3}\int u^{1/7}\,du = \frac{1}{3}\cdot\frac{u^{8/7}}{8/7}+c = \boxed{\frac{1}{3}\cdot\frac{7u^{8/7}}{8}+c} = \boxed{\frac{1}{3}\cdot\frac{7(x^3-2)^{8/7}}{8}+c}$$

3) $\int \frac{x\,dx}{\sqrt[5]{x^2-1}}$ $\quad u = x^2-1 \quad du = 2x\,dx \rightarrow \frac{du}{2} = x\,dx$

$$\int \frac{du}{2u^{1/5}} = \frac{1}{2}\int\frac{du}{u^{1/5}} = \frac{1}{2}\int u^{-1/5}\,du = \frac{1}{2}\cdot\frac{5u^{4/5}}{4}+c = \boxed{\frac{1}{2}\cdot\frac{5(x^2-1)^{4/5}}{4}+c}$$

4) $\int 5x\sqrt{4-3x^2}\,dx$ $\quad u = 4-3x^2 \quad du = -6x\,dx \rightarrow \frac{du}{-6} = x\,dx\ (\times 5) \rightarrow \frac{5\,du}{-6} = 5x\,dx$

$$\int \sqrt{u}\,\frac{5\,du}{-6} = \frac{5}{-6}\int u^{1/2}\,du = \frac{5}{-6}\cdot\frac{2u^{3/2}}{3}+c = \boxed{\frac{5}{-6}\cdot\frac{2(4-3x^2)^{3/2}}{3}+c}$$

5) $\int \sqrt{x^2+2x^4}\,dx = \int\sqrt{1+2x^2}\,x\,dx$ $\quad u = 2x^2+1 \quad du = 4x\,dx \rightarrow \frac{du}{4} = x\,dx$

$$\int \sqrt{u}\,\frac{du}{4} = \frac{1}{4}\int u^{1/2}\,du = \frac{1}{4}\cdot\frac{2u^{3/2}}{3}+c = \boxed{\frac{1}{4}\cdot\frac{2(1+2x^2)}{3}+c}$$

(6) $\int (e^{2t}+2)^{1/3} e^{2t}\,dt \qquad u = e^{2t}+2 \qquad du = 2e^{2t}\,dt \to \frac{du}{2} = e^{2t}\,dt$

$\int u^{1/3}\frac{du}{2} = \frac{3u^{4/3}}{8} + c = \boxed{\frac{3(e^{2t}+2)^{4/3}}{8} + c}$

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(7) $\int \frac{e^t\,dt}{e^t+4} \qquad u = e^t+4 \qquad du = e^t\,dt$

$= \int \frac{du}{u} = \ln|u| + c = \boxed{\ln|e^t+4| + c}$

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(8) $\int \frac{e^{1/x}+2}{x^2}\,dx \qquad u = e^{1/x}+2 \qquad du = -\frac{e^{1/x}}{x^2}\,dx \to -du+2 = \frac{e^{1/x}}{x^2}\,dx + 2$

$= \int u \cdot -du + 2 = -1 + 2\int u \cdot du = \frac{u^2}{2} + c = \frac{e^{1/x}+2}{\cancel{2}} + c = \boxed{e^{1/x} + c}$

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(9) $\int tg(x)\sec^2 x\,dx \qquad u = tg(x) \qquad du = \sec^2 x\,dx$

$\int u\,du = \frac{u^2}{2} + c = \boxed{\frac{(tg(x))^2}{2} + c}$

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(10) $\int sen^4 x \cos x\,dx \qquad u = sen(x) \qquad du = \cos(x)\,dx$

$\int u^4\,du = \frac{u^5}{5} + c = \boxed{\frac{sen^5(x)}{5} + c}$

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(11) $\int \frac{sen\,x}{\cos^5 x}\,dx \qquad u = \cos x \qquad du = -sen\,x\,dx \to -du = sen\,x\,dx$

$= \int \frac{-du}{u^5} = -\int \frac{du}{u^5} = -\int u^{-5}\,du = -\frac{u^{-4}}{-4} + c = -\frac{\cos^{-4} x}{-4} + c = \boxed{\frac{1}{4\cos^4 x} + c}$

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(12) $\int \frac{2\,sen\,x - 5\cos x}{\cos x} = \int \frac{2\,sen\,x\,dx}{\cos x} - \int \frac{5\cos x\,dx}{\cos x} = 2\int \frac{sen\,x\,dx}{\cos x} - 5\int \frac{\cos x}{\cos x}\,dx$

$u = \cos x$

$du = -sen\,x\,dx \to -du = sen\,x\,dx$

$= 2\int \frac{-du}{u} - 5\int dx = -2\ln|u| - 5x + c = \boxed{-2\ln|\cos x| - 5x + c}$

(18) $\displaystyle\int \frac{dx}{16+x^2}$ $\quad u = \frac{x}{4} \quad du = \frac{1}{4}dx \to 4\,du = dx$

$$\int \frac{4\,du}{16+16u^2} = \frac{4}{16}\int \frac{du}{1+u^2} = \frac{1}{4}\cdot \text{arctg}(u) + c = \boxed{\frac{1}{4}\cdot \text{arctg}\left(\frac{x}{4}\right) + c}$$

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(19) $\displaystyle\int \frac{dy}{y^2-4y+4} = \int \frac{dy}{(y-2)^2}$ $\quad u = y-2 \quad du = dy$

$$\int \frac{du}{u^2} = \int u^{-2}\,du = \frac{u^{-1}}{-1} + c = -\frac{1}{u} + c = \boxed{-\frac{1}{y-2} + c}$$

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(20) $\displaystyle\int \sqrt[3]{\text{sen}\,\theta}\,\cos\theta\,d\theta = \int \text{sen}\,\theta^{\frac{1}{3}} \cos\theta\,d\theta$ $\quad u = \text{sen}\,\theta \quad du = \cos\theta\,d\theta$

$$\int u^{\frac{1}{3}}\,du = \frac{3u^{4/3}}{4} + c = \boxed{\frac{3\,\text{sen}^{4/3}\theta}{4} + c}$$

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(21) $\displaystyle\int \frac{\ln x^2}{x}\,dx = \int \frac{2\ln x}{x}\,dx$ $\quad u = \ln x \quad du = \frac{1}{x}dx \to x\,du = dx$

$$\int 2u\,du = 2\int u\,du = 2\frac{u^2}{2} + c = \frac{\cancel{2}\ln^2 x}{\cancel{2}} + c = \boxed{\ln^2 x + c}$$

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(26) $\displaystyle\int \frac{e^x\,dx}{e^{2x}+16}$ $\quad u = \frac{e^x}{4} \quad du = \frac{e^x}{4}dx \to 4\,du = e^x\,dx \to \frac{4\,du}{e^x} = dx \to 4e^{-x}\,du = dx$

$$= \int \frac{4\,du}{16u^2+16} = \frac{1}{4}\int \frac{1}{u^2+1}\,du = \frac{1}{4}\cdot \text{arctg}\,u + c = \boxed{\frac{1}{4}\cdot \text{arctg}\left(\frac{e^x}{4}\right) + c}$$

(28) $\displaystyle\int \frac{3\,dx}{x\ln^2 3x}$ $\quad u = \ln 3x$

$du = \frac{1}{x}dx \rightarrow x\,du = dx$

$$= 3\int \frac{dx}{x\ln^2 3x} = 3\int \frac{x\,du}{x u^2} = 3\int \frac{du}{u^2} = 3\int u^{-2}\,du = 3\cdot \frac{u^{-1}}{-1} + c$$

$$= 3\cdot -\frac{1}{u} + c = \boxed{3\cdot -\frac{1}{\ln 3x} + c}$$

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(30) $\displaystyle\int 2^{x^2+1} x\,dx$ $\quad u = x^2+1 \quad du = 2x\,dx \rightarrow \frac{du}{2} = x\,dx$

$$= \int 2^u \frac{du}{2} = \frac{1}{2}\int 2^u\,du = \frac{1}{2}\cdot\frac{2^{u-1}}{\ln|2|} + c = \frac{1}{2}\cdot\frac{2^{x^2+1-1}}{\ln|2|} + c = \boxed{\frac{1}{2}\cdot\frac{2^{x^2}}{\ln|2|} + c}$$

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(33) $\displaystyle\int \frac{dt}{t\ln t}$ $\quad u = \ln|t|$

$du = \frac{1}{t}dt \rightarrow t\,du = dt$

$$= \int \frac{t\,du}{t u} = \int \frac{du}{u} = \ln|u| + c = \boxed{\ln|\ln|t|| + c}$$

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(39) $\displaystyle\int x^2\sqrt{1+x}\,dx = \int x^2(1+x)^{\frac{1}{2}}\,dx$ $\quad u = 1+x$

$du = dx$

$$\int x^2 u^{1/2}\,du = \int (u-1)^2\sqrt{u}\,du = \int \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + \sqrt{u}\right)du = \int u^{5/2}\,du - \int 2u^{3/2}\,du + \int \sqrt{u}\,du$$

$$= \frac{2u^{\frac{7}{2}}}{7} - 2\cdot\frac{2u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3} + c = \boxed{\frac{2(1+x)^{7/2}}{7} - \frac{4(1+x)^{5/2}}{5} + \frac{2(1+x)^{3/2}}{3} + c}$$