Teorema fundamental do cálculo (13/11/2019 - 13/11/2019) Vitor Oliveira Rapôso

Livro: Cálculo A

Tópico 6.11, página 271

Questões: 1, 3, 4, 6, 12-21.

Página 269

(1) Calculando as integrais $I_1 = \int_1^2 x^2 dx$, $I_2 = \int_1^2 x dx$ e $I_3 = \int_1^2 dx$, obtemos $I_1 = 7/3$, $I_2 = 3/2$ e $I_3 = 1$. Usando esses resultados, encontrar o valor de:

a) $\int_1^2 (6x-1)\, dx = \int_1^2 6x\, dx - \int_1^2 dx = \left(\frac{6x^2}{2} - x\right)\Big/_1^2 = \left(\frac{6.2^2}{2} - 2\right) - \left(\frac{6.1^2}{2} - 1\right)$

$= \frac{24}{2} - 2 - \frac{6}{2} + 1 = \frac{18}{2} - 1 = \frac{16}{2} = \boxed{8}$

b) $\int_1^2 2x(x+1)\, dx = \int_1^2 (2x^2+2x)\, dx = \int_1^2 2x^2 dx + \int_1^2 2x\, dx = \frac{2x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} = \left(\frac{2x^3}{3} + x^2\right)\Big/_1^2$

$\left(\frac{2.2^3}{3} + 2^2\right) - \left(\frac{2.1^3}{3} + 1^2\right) = \frac{16}{3} + 4 - \frac{2}{3} - 1 = \frac{14}{3} + 3 = \boxed{\frac{23}{3}}$

c) $\int_1^2 (x-1)(x-2)\, dx = \int_1^2 (x^2 - 3x + 3)\, dx = \left(\frac{x^3}{3} - \frac{3x^2}{2} + 3x\right)\Big/_1^2$

$= \left(\frac{2^3}{3} - \frac{3.2^2}{2} + 3.2\right) - \left(\frac{1^3}{3} - \frac{3.1^2}{2} + 3.1\right) = \frac{7}{3} - \frac{15}{2} + 9 = \frac{14 - 45 + 54}{6} = \boxed{\frac{23}{6}}$

d) $\int_1^2 (3x+2)^2 dx$ $\quad u = 3x+2$, $du = 3dx \rightarrow \frac{du}{3} = dx$ $\quad \frac{1}{3}\int_1^2 u^2\, du = \left(\frac{1}{3}.\frac{u^3}{3}\right)\Big/_1^2 = \frac{u^3}{9}\Big/_1^2 = \frac{(3x+2)^3}{9}\Big/_1^2$

$\frac{(3.2+2)^3}{9} - \frac{(3+2)^3}{9} = \frac{512-125}{9} = \frac{387}{9} = \boxed{43}$

③ Se $\int_0^1 \sqrt[5]{x^2}\,dx = \frac{5}{7}$, calcular $\int_1^0 \sqrt[5]{t^2}\,dt = \int_1^0 t^{\frac{2}{5}}\,dt = \frac{5t^{7/5}}{7}\Big|_1^0$

$0 - \frac{5.1^{7/5}}{7} = \boxed{-\frac{5}{7}}$

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④ Se $\int_0^{\frac{\pi}{2}} 9\cos^2 t\,dt = \frac{9\pi}{4}$, calcular $\int_0^{\pi/2} -\cos^2\theta\,d\theta = -\left(\frac{\cos\theta\,\text{sen}\,\theta}{2} + \frac{1}{2}\int 1\,d\theta\right)$

$= -\left(\frac{\cos\theta\,\text{sen}\,\theta}{2} + \frac{\theta}{2}\right) = \left(-\frac{\cos\theta\,\text{sen}\,\theta}{2} - \frac{\theta}{2}\right)\Big|_0^{\pi/2} = \cancel{\frac{\cos(\pi/2)\,\text{sen}(\pi/2)}{2}} - \frac{\pi/2}{2}$

$= \boxed{-\frac{\pi}{4}}$

Página 270

⑥ Determinar as seguintes derivadas:

a) $\frac{d}{dx}\int_2^x \sqrt{t+4}\,dt \qquad \int_2^x (t+4)^{\frac{1}{2}}\,dt \qquad \begin{matrix} u = t+4 \\ du = dt \end{matrix} \qquad \int_2^x u^{\frac{1}{2}}\,du$

$= \frac{2u^{3/2}}{3}\Big|_2^x = \frac{2(t+4)^{3/2}}{3}\Big|_2^x \rightarrow \frac{d}{dx}\left(\frac{2(t+4)^{3/2}}{3}\right)\Big|_2^x = \boxed{t+4}$

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b) $\frac{d}{dy}\int_3^y \frac{2x}{x^2+9}\,dx \qquad G'(x) = \frac{2x}{x^2+9}$

$\downarrow$

$G(y) - G(3) \Rightarrow \frac{d}{dy} = G'(y) - G'(3) = \frac{2y}{y^2+9} - \frac{2.3}{3^2+9} = \boxed{\frac{2y}{y^2+9} - \frac{6}{18}}$

c) $\underbrace{\frac{d}{d\theta}\int_{-1}^{\theta} t\,\text{sen}\,t\,dt}_{} = G(\theta) - G(-1)$ onde $G'(t) = t\,\text{sen}\,t$

$F'(\theta) = G'(\theta) + G'(-1)$

$= \theta\,\text{sen}\,\theta \; \cancel{+(-1)\,\text{sen}(1)}$

$= \boxed{\theta\,\text{sen}\,\theta - \text{sen}\,1}$

Página 271

Nos exercícios 12 a 34, calcular as integrais.

12) $\int_{-1}^{2} x(1+x^3)\,dx = \int_{-1}^{2}(x + x^4)\,dx = \left(\frac{x^2}{2} + \frac{x^5}{5}\right)\Big|_{-1}^{2} = \frac{2^2}{2} + \frac{2^5}{5} - \frac{(-1)^2}{2} - \frac{(-1)^5}{5}$

$= 2 + \frac{32}{5} - \frac{1}{2} + \frac{1}{5} = 2 - \frac{1}{2} + \frac{33}{5} = \frac{20-5+66}{10} = \frac{81}{10}$

13) $\int_{-3}^{0}(x^2 - 4x + 7)\,dx = \left(\frac{x^3}{3} - 2x^2 + 7x\right)\Big|_{-3}^{0} = 0 - \frac{(-3)^3}{3} + 2(-3)^2 - 7(-3)$

$= \frac{27}{3} + 18 + 21 = \frac{27+54+63}{3} = \frac{144}{3} = \boxed{48}$

14) $\int_{1}^{2}\frac{dx}{x^6} = \int_{1}^{2} x^{-6}\,dx = -\frac{x^{-5}}{5}\Big|_{1}^{2} = -\frac{1}{5x^5}\Big|_{1}^{2} = -\frac{1}{5.2^5} + \frac{1}{5.1^5} = -\frac{1}{160} + \frac{1}{5}$

$\frac{1+32}{160} = \boxed{\frac{31}{160}}$

15) $\int_{4}^{9} 2t\sqrt{t}\,dt = 2\int_{4}^{9} t.t^{\frac{1}{2}}\,dt = 2\int_{4}^{9} t^{3/2}\,dt = \frac{2.2.t^{5/2}}{5} = \frac{4t^{5/2}}{5}\Big|_{4}^{9}$

$= \frac{4.\sqrt{9^5}}{5} - \frac{4.\sqrt{4^5}}{5} = \frac{4.243}{5} - \frac{4.32}{5} = \frac{972}{5} - \frac{128}{5} = \boxed{\frac{844}{5}}$

(16) $\displaystyle\int_0^1 \frac{dy}{\sqrt{3y+1}}$

$u = 3y+1$

$du = 3\,dy \rightarrow \frac{du}{3} = dy$

$\displaystyle\frac{1}{3}\int_0^1 \frac{du}{u^{1/2}} = \frac{1}{3}\int_0^1 u^{-1/2}\,du = \frac{1}{3}\cdot 2u^{1/2}\Big|_0^1$

$\displaystyle= \frac{2u^{1/2}}{3}\Big|_0^1 = \frac{2\sqrt{3y+1}}{3}\Big|_0^1 = \frac{2.2}{3} = \boxed{\frac{4}{3}}$

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(17) $\displaystyle\int_{\pi/4}^{3\pi/4} \operatorname{sen} x \cos x\,dx$

$u = \operatorname{sen} x$

$du = \cos x\,dx$

$\displaystyle\int_{\pi/4}^{3\pi/4} u\,du = \frac{u^2}{2}\Big|_{\pi/4}^{3\pi/4} = \frac{\operatorname{sen}^2 x}{2}\Big|_{\pi/4}^{3\pi/4}$

$\displaystyle\frac{\operatorname{sen}^2\left(\frac{3\pi}{4}\right)}{2} - \frac{\operatorname{sen}^2\left(\frac{\pi}{4}\right)}{2} = \frac{3/2}{2} - \frac{1/2}{2} = \frac{6}{2} - \frac{2}{2} = \frac{4}{2} = \boxed{2}$

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(18) $\displaystyle\int_{-1}^1 \frac{x^2\,dx}{\sqrt{x^3+9}}$

$u = x^3+9$

$du = 3x^2\,dx \rightarrow \frac{du}{3} = x^2\,dx$

$\displaystyle\frac{1}{3}\int_{-1}^1 \frac{du}{u^{1/2}} = \frac{1}{3}\int_{-1}^1 u^{-1/2}\,du$

$\displaystyle= \frac{1}{3}\cdot 2u^{1/2} = \frac{2\sqrt{x^3+9}}{3}\Big|_{-1}^1 = \boxed{\frac{2\sqrt{10}}{3} - \frac{2\sqrt{8}}{3}}$

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(19) $\displaystyle\int_0^{2\pi} |\operatorname{sen} x|\,dx = \frac{\operatorname{sen} x}{|\operatorname{sen} x|}\int_0^{2\pi} \operatorname{sen} x\,dx = \frac{\operatorname{sen} x}{|\operatorname{sen} x|}\cdot(-\cos x)\Big|_0^{2\pi} = \frac{\operatorname{sen} 2\pi\,(-\cos 2\pi)}{|\operatorname{sen} 2\pi|} - \frac{\operatorname{sen} 0\,(-\cos}{|\operatorname{sen} 0|}$

$\cancel{0 = 0 - 0}$ $0 - 0 = \boxed{0}$

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(20) $\displaystyle\int_{-2}^5 |2t-4|\,dt = \frac{2t-4}{|2t-4|}\int_{-2}^5 (2t-4)\,dt = \frac{2t-4}{|2t-4|}\; t^2 - 4t\Big|_{-2}^5$

$\displaystyle= \left(\frac{6}{|6|}\cdot 25 - 20\right) - \left(\frac{-8}{|-8|}\cdot 4 + 8\right) = \frac{150}{|6|} - 20 - \left(-\frac{32}{|-8|} + 8\right) = 25 - 20 + 4 - 8$

$= 29 - 28 = \boxed{1}$

(21) $\int_0^4 |x^2-3x+2|\,dx = \frac{x^2}{|x^2|}\int_0^4 x^2\,dx - \frac{3x}{|3x|}\int_0^4 3x\,dx + \frac{2}{|2|}\int_0^4 2\,dx$

$= \left(\frac{x^2}{|x^2|}\cdot\frac{x^3}{3} - \frac{3x}{|3x|}\cdot\frac{3x^2}{2} + \frac{2}{|2|}\cdot 2x\right)\Big/_0^4 = \frac{4^2}{|4^2|}\cdot\frac{4^3}{3} - \frac{3.4}{|3.4|}\cdot\frac{3.4^2}{2} + \frac{2}{|2|}\cdot 2.4$

$= \frac{16}{|16|}\cdot\frac{64}{3} - \frac{12}{|12|}\cdot\frac{48}{2} + \frac{2}{|2|}\cdot 8 = \frac{1024}{3|16|} - \frac{576}{2|12|} + \frac{16}{|2|} = \frac{1024}{48} - \frac{576}{24} + \frac{16}{2} =$

$= \frac{1024}{48} - 24 + 8 = \frac{1024}{48} - 16 = \frac{1024-768}{48} = \frac{256}{48} = \boxed{\frac{16}{3}}$