Base e dimensão de um espaço vetorial

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Livro: Steinbruch

Tópico 2.10, página

Questões: 56, 58-62, 72-74.

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(56) Verificar quais dos seguintes conjuntos de vetores formam base do $\mathbb{R}^2$.

a) $\{(1,2),(-1,3)\}$

I) $\frac{1}{-1} \neq \frac{2}{3}$ L.I.

II) $(x,y) = a(1,2) + b(-1,3)$

$(x,y) = (a, 2a) + (-b, 3b)$

$(x,y) = (a-b, 2a+3b)$

$\begin{cases} a-b = x \\ 2a+3b = y \end{cases}$ $\rightarrow a = x+b$

$\rightarrow 2a+3b = y \rightarrow 2(x+b)+3b = y \rightarrow 2x+2b+3b = y$

$2x+5b = y$

$5b = y-2x$

$\boxed{b = \frac{y-2x}{5}}$

$a - x = -b$

$-a + x = b$

$\frac{-2a+y}{3} = b$

$a - x = \frac{y-2x}{5}$

$5a - 5x = y - 2x$

$5a = y - 2x + 5x \rightarrow 5a = y + 3x \rightarrow \boxed{a = \frac{y+3x}{5}}$

$u = \left(\frac{y+3x}{5}\right)v_1 + \left(\frac{y-2x}{5}\right)v_2$

Logo, gera o $\mathbb{R}^2$. Portanto é base do $\mathbb{R}^2$

b) $\{(3,-6),(-4,8)\}$

I) $\frac{3}{-4} = \frac{-6 \div 2}{8 \div 2} = \frac{-3}{4}$ É L.D, logo ~~[illegible]~~ não é base do $\mathbb{R}^2$

c) $\{(0,0),(2,3)\}$

I) $\frac{0}{2} = \frac{0}{3}$ Possui um vetor nulo. Logo é L.D.
Portanto não é base do $\mathbb{R}^2$

d) $\{(3,-1),(2,3)\}$

I) $\frac{3}{2} \neq \frac{-1}{3}$ L.I.

II) $(x,y) = a(3,-1) + b(2,3)$

$(x,y) = (3a,-a) + (2b,3b)$

$(x,y) = (3a+2b, -a+3b)$

$\begin{cases} x = 3a+2b \\ y = -a+3b \quad \times(3) \end{cases}$

$x = 3a+2b$
$3y = -3a+9b$
$x+3y = 11b \rightarrow \boxed{b = \frac{x+3y}{11}}$

$\rightarrow x = 3a + 2\left(\frac{x+3y}{11}\right) \rightarrow x = 3a + \frac{2x+6y}{11} \rightarrow 11x = 33a + 2x + 6y \rightarrow$

$11x - 2x - 6y = 33a \rightarrow \frac{\cancel{9}^{3}x - \cancel{6}^{2}y}{\cancel{33}\,11} = a \rightarrow \boxed{a = \frac{3x-2y}{11}}$

$u = \left(\frac{3x-2y}{11}\right)v_1 + \left(\frac{x+3y}{11}\right)v_2$

Logo, gera o $\mathbb{R}^2$. Portanto é base do $\mathbb{R}^2$

Itens 'a' e 'd' formam base do $\mathbb{R}^2$

58) O conjunto $B=\{(2,-1),(-3,2)\}$ é uma base do $\mathbb{R}^2$. Escrever o vetor genérico do $\mathbb{R}^2$ como combinação linear de B.

$$(x,y)=a(2,-1)+b(-3,2)$$
$$(x,y)=(2a,-a)+(-3b,2b)$$
$$(x,y)=(2a-3b,-a+2b)$$

$$\begin{cases}2a-3b=x\\ -a+2b=y \rightarrow -a=-2b+y \rightarrow \boxed{a=2b-y}\end{cases}$$

$$2a-3b=x$$
$$(\times 2)\ -2a+4b=2y$$
$$\boxed{b=x+2y}$$

$$a=2(x+2y)-y \rightarrow a=2x+4y-y \rightarrow \boxed{a=2x+3y}$$

$$u=(2x+3y)v_1+(x+2y)v_2$$
$$\boxed{(x,y)=(2x+3y)(2,-1)+(x+2y)(-3,2)}$$

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59) Quais dos seguintes conjuntos de vetores formam uma base do $\mathbb{R}^3$?

a) $(1,1,-1),(2,-1,0),(3,2,0)$

I) $(0,0,0)=a(1,1,-1)+b(2,-1,0)+c(3,2,0)$
$(0,0,0)=(a,a,-a)+(2b,-b,0)+(3c,2c,0)$
$(0,0,0)=(a+2b+3c,\ a-b+2c,\ -a)$

$$\begin{cases}a+2b+3c=0 & c=0\\ a-b+2c=0 & b=0\\ -a=0 \rightarrow a=0\end{cases}$$
L.I.

$-3z-2y+7c=x \rightarrow \boxed{\dfrac{3z+2y+x}{7}=c}$

II) $$\begin{cases}a+2b+3c=x \rightarrow -z+2(-y-z+2c)+3c=x \rightarrow -z-2y-2z+4c+3c=x\\ a-b+2c=y \rightarrow -b+2c=y-a \rightarrow -b=y+z-2c \rightarrow \boxed{b=-y-z+2c}\\ -a=z \rightarrow \boxed{a=-z}\end{cases}$$

$$b=-y-z+2\left(\frac{3z+2y+x}{7}\right) \rightarrow b=-y-z+\frac{6z+4y+2x}{7} \rightarrow 7b=\frac{-7y-7z+6z+4y+2x}{7}$$

$7b = -3y - z + 2x \rightarrow \boxed{b = \dfrac{-3y - z + 2x}{7}}$

$$u = (-z)v_1 + \left(\frac{-3y - z + 2x}{7}\right)v_2 + \left(\frac{3z + 2y + x}{7}\right)v_3$$

$$(x,y,z) = (-z)(1,1,-1) + \left(\frac{-3y - z + 2x}{7}\right)(2,-1,0) + \left(\frac{3z + 2y + x}{7}\right)(3,2,0)$$

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b) $(1,0,1), (0,-1,2), (-2,1,-4)$

I) $(0,0,0) = a(1,0,1) + b(0,-1,2) + c(-2,1,-4)$

$(0,0,0) = (a,0,a) + (0,-b,2b) + (-2c,c,-4c)$

$(0,0,0) = (a-2c, -b+c, a+2b-4c)$

$$\begin{cases} a - 2c = 0 \\ -b + c = 0 \\ a + 2b - 4c = 0 \end{cases} \rightarrow$$

$$\begin{cases} a + 2b - 4c = 0 \\ a = 2b \\ -b + c = 0 \end{cases}$$

$a - 2c = 0$

$-2b + 2c = 0 \quad \times 2$

$a - 2b = 0$

$\boxed{a = 2b}$ É L.D. Portanto não é base de $\mathbb{R}^3$

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c) $(2,1,-1), (-1,0,1), (0,0,1)$

I) $(0,0,0) = a(2,1,-1) + b(-1,0,1) + c(0,0,1)$

$(0,0,0) = (2a, a, -a) + (-b, 0, b) + (0,0,c)$

$(0,0,0) = (2a - b, a, -a + c)$

$$\begin{cases} 2a - b = 0 & \boxed{b=0} \\ a = 0 \rightarrow \boxed{a=0} & \text{Logo } \boxed{b=0} \text{ e } \boxed{c=0} \text{ Então é L.I.} \\ -a + c = 0 & \boxed{c=0} \end{cases}$$

II) $\begin{cases} 2a - b = x \rightarrow 2y - b = x \rightarrow -b = x - 2y \rightarrow \boxed{b = -x + 2y} \\ a = y \rightarrow \boxed{a = y} \\ -a + c = z \rightarrow -y + c = z \rightarrow \boxed{c = z + y} \end{cases}$

$$\boxed{(x,y,z) = (y)(2,1,-1) + (-x + 2y)(-1,0,1) + (z + y)(0,0,1)}$$

d) $(1,2,3),(4,1,2)$

I) $\frac{1}{4} \neq \frac{2}{1} \neq \frac{3}{2}$ L.I

II) ~~[illegible]~~

$(x,y,z) = a(1,2,3) + b(4,1,2)$

$(x,y,z) = (a,2a,3a) + (4b,b,2b)$

$(x,y,z) = (a+4b, 2a+b, 3a+2b)$

$$\begin{cases} a+4b=x \xrightarrow{\times(-2)} -2a-8b=-2x \\ 2a+b=y \longrightarrow 2a+b=y \\ 3a+2b=z \end{cases}$$

$-7b = -2x+y \rightarrow \boxed{b = \frac{2x-y}{7}}$

$\rightarrow a + 4\left(\frac{2x-y}{7}\right) = x$

$a + \frac{8x-4y}{7} = x \rightarrow 7a + 8x - 4y = 7x \rightarrow 7a = 7x - 8x - 4y \rightarrow \boxed{a = \frac{-x-4y}{7}}$

$(x,y,z) = \left(\frac{-x-4y}{7}\right)v_1 + \left(\frac{2x-y}{7}\right)v_2 + ?$

A base gerada foi $\mathbb{R}^2$, portanto não é base $\mathbb{R}^3$

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e) $(0,-1,2),(2,1,3),(-1,0,1),(4,-1,-2)$

I) $(0,0,0) = a(0,-1,2) + b(2,1,3) + c(-1,0,1) + d(4,-1,-2)$

$(0,0,0) = (0,-a,2a) + (2b,b,3b) + (-c,0,c) + (4d,-d,-2d)$

$(0,0,0) = (2b-c+4d, -a+b-d, 2a+3b+c-2d)$

$$\begin{cases} 2b-c+4d=0 \\ -a+b-d=0 \\ 2a+3b+c-2d=0 \end{cases}$$

$a = b-d$

$b = a+d$

$d = b-a$

$$\Rightarrow \begin{cases} 2a+3b+c-2d=0 \\ 2a+5b+2d=0 \\ -a+b-d=0 \end{cases} \Rightarrow \begin{cases} 2a+3b+c-2d=0 \\ 4a+8b+c=0 \\ -a+b-d=0 \end{cases} \Rightarrow \begin{cases} 2a+3b+c-2d=0 \quad \cdot(\times 2) \\ 4a+8b+c=0 \\ 12b+c-4d=0 \end{cases}$$

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$$\begin{cases} 12b - c + 4d = 0 \\ 4a + 8b + c = 0 \\ 12b + c - 4d = 0 \end{cases} \Rightarrow \begin{cases} 24b = 0 \\ 4a + 8b + c = 0 \\ 12b + c - 4d = 0 \end{cases} \begin{array}{l} \to \boxed{b = 0} \\ \to a = 0 \\ \to c = 0 \\ \to d = 0 \end{array}$$

B I

II) $\begin{cases} 2b - c + 4d = X \\ -a + b - d = y \\ 2a + 3b + c - 2d = z \end{cases}$

$-a = y - b + d$

$\boxed{a = -y + b - d}$

$\boxed{b = y + a + d}$

$-d = y + a - b$

$\boxed{d = -y - a + b}$

$\to -c = X - 2b - 4d \to \boxed{c = -X + 2b + 4d}$

$\to c = z - 2a - 3b + 2d$

$-X + 2b + 4d = z - 2a - 3b + 2d$

$-X - z = -2b - 4d - 2a - 3b + 2d$

$-X - z = -5b - 2d - 2a$

$\to -X - z = -5(y + a + d) - 2(-y - a + b) - 2a$

$-X - z = -5y - 5a - 5d + 2y + 2a - 2b - 2a$

$-X - z = -3y - 5a - 5d - 2b$

$-X - z + 3y + 5d + 2b = -5a$

$\dfrac{X + z - 3y - 5d - 2b}{5} = a \to c = z - 2\left(\dfrac{X + z - 3y - 5d - 2b}{5}\right) - 3b + 2d$

$\to c = z - \dfrac{2X - 2z + 6y + 5d + 2b}{5} - 3b + 2d \to 5c = 5z - 2X - 2z + 6y + 5d + 2b - 15b + 10d$

$5c = 3z - 2X + 6y + 15d - 13b \to c = \dfrac{3z - 2X + 6y + 15d - 13b}{5}$

a, b, c e d formam a base $\mathbb{R}^4$. Portanto não é base $\mathbb{R}^3$

60) Quais dos seguintes conjuntos de vetores formam base de P_2?

a) $2t^2+t-4,\ t^2-3t+1$

I) $(0,0,0) = a(2t^2+t-4)+b(t^2-3t+1)$

$(0,0,0) = (2at^2+at-4a)+(bt^2-3bt+b)$

$(0,0,0) = (2at^2+bt^2+at-3bt-4a+b)$

$(0,0,0) = (2a+b)t^2+(a-3b)t+(-4a+b)$

$\begin{cases} 2a+b=0 \\ a-3b=0 \\ -4a+b=0 \end{cases}$ $\rightarrow$ $\begin{array}{l} 2a+b=0 \\ -2a+3b=0 \\ \hline 4b=0 \rightarrow \boxed{b=0} \end{array}$ $\boxed{a=0}$ L.I

II) $\begin{cases} 2a+b=x \\ a-3b=y \rightarrow a=y+3b \\ -4a+b=z \end{cases}$

$2(y+3b+b)=x \rightarrow 2y+6b+2b=x \rightarrow 2y+8b=x$

$8b=x-2y \rightarrow \boxed{b=\dfrac{x-2y}{8}}$

$a=y+3\left(\dfrac{x-2y}{8}\right) \rightarrow a=y+\dfrac{3x-6y}{8} \rightarrow 8a=8y+3x-6y$

$\rightarrow 8a=2y+3x \rightarrow \boxed{a=\dfrac{2y+3x}{8}}$

Gera P_1

$\boxed{\dim S=2}$ $\boxed{\text{Não gera } P_2}$ Logo não é base do P_2

b) $1, t, t^2$

I) $(0,0,0) = a(1)+b(t)+c(t^2)$

$(0,0,0) = (a)+(bt)+(ct^2)$

$(0,0,0) = (a)+(b)t+(c)t^2$

$\begin{cases} a=0 \\ b=0 \\ c=0 \end{cases}$ L.I

II) $\begin{cases} a=x \\ b=y \\ c=z \end{cases}$

Gera P_2

É base do P_2

$(x,y,z) = (x)(1)+(y)(t)+(z)(t^2)$

c) $2,\ 1-X,\ 1+X^2$

I) $(0,0,0) = a(2) + b(1-X) + c(1+X^2)$

$(0,0,0) = (2a) + (b - bX) + (c + cX^2)$

~~$(0,0,0) = (2a) + (b - bX) + (c + cX^2)$~~

$(0,0,0) = (2a+b+c) + (-b)X + (c)X^2$

$$\begin{cases} 2a+b+c=0 & a=0 \\ -b=0 \\ c=0 \end{cases}$$

L.I

II) $2a+b+c=X \to 2a-y+z=X$

$-b = y$

$c = z$

$2a = X+y-z$

$$a = \frac{X+y-z}{2}$$

$$u = \left(\frac{X+y-z}{2}\right)p_0 + (-y)p_1 + (z)p_2$$

Portanto é base de P_2

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d) $1+X+X^2,\ X+X^2,\ X^2$

I) $(0,0,0) = a(1+X+X^2) + b(X+X^2) + c(X^2)$

$(0,0,0) = (a + aX + aX^2) + (bX + bX^2) + (cX^2)$

$(0,0,0) = (a) + (a+b)X + (a+b+c)X^2$

$$\begin{cases} a=0 \\ a+b=0 & b=0 \\ a+b+c=0 & c=0 \end{cases}$$

L.I

II) $\begin{cases} a=X \\ a+b=y \to X+b=y \to b=y-X \\ a+b+c=z \to X+y-X+c=z \end{cases}$

$\to y+c=z \to c=z-y$

$$u = (X)p_0 + (y-X)p_1 + (z-y)p_2$$

Portanto é base de P_2

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e) $1+X,\ X-X^2,\ 1+2X-X^2$

$(0,0,0) = a(1+X) + b(X-X^2) + c(1+2X-X^2)$

$(0,0,0) = (a+aX) + (bX - bX^2) + (c + 2cX - cX^2)$

$(0,0,0) = (a+c) + (a+b+2c)X + (-b-c)X^2$

$$\begin{cases} a+c=0 \\ a+b+2c=0 \\ -b-c=0 \end{cases} \to \begin{cases} a=-c \\ a+b=-2c \\ b=-c \end{cases}$$

$a=b$

L.D

Não é base P_2

(61) Mostrar que o conjunto $\left\{\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix}\begin{bmatrix}1 & -1\\0 & -2\end{bmatrix},\begin{bmatrix}-3 & -2\\1 & -1\end{bmatrix},\begin{bmatrix}3 & -7\\-2 & 5\end{bmatrix}\right\}$ é uma base de $M(2,2)$.

I) $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = a\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix} + b\begin{bmatrix}1 & -1\\0 & -2\end{bmatrix} + c\begin{bmatrix}-3 & -2\\1 & -1\end{bmatrix} + d\begin{bmatrix}3 & -7\\-2 & 5\end{bmatrix}$

$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = \begin{bmatrix}2a & 3a\\-a & 0\end{bmatrix} + \begin{bmatrix}b & -b\\0 & -2b\end{bmatrix} + \begin{bmatrix}-3c & -2c\\c & -c\end{bmatrix} + \begin{bmatrix}3d & -7d\\-2d & -5d\end{bmatrix}$

$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = \begin{bmatrix}2a+b-3c+3d & 3a-b-2c-7d\\-a+c-2d & -2b-c-5d\end{bmatrix}$

$\begin{cases}2a+b-3c+3d=0\\-a+c-2d=0\\3a-b-2c-7d=0\\-2b-c-5d=0\end{cases}$ $\times 5$

$+ \rightarrow$ $5a-5c-4d=0$
$-5a+5c-10d=0$ $+$

$\boxed{-14d=0} \rightarrow \boxed{d=0}$

$\boxed{c=0}$
$\boxed{b=0}$
$\boxed{a=0}$

B.I

II) $\begin{cases}2a+b-3c+3d=x\\-a+c-2d=y\\3a-b-2c-7d=z\\-2b-c-5d=w\end{cases}$

$-a+c-2d=y \rightarrow \boxed{c=a+2d+y}$
$-a=-c+2d+y$
$\boxed{a=c-2d-y}$

$-2d=a-c+y$
$\boxed{d=\dfrac{-a+c-y}{2}}$

$\rightarrow -c=5d+2b+w$
$\boxed{c=-5d-2b-w} \rightarrow -5d-2b-w=a+2d+y$
$-5d-2d-2b-w=a+y$
$\boxed{-7d-2b-w-y=a}$

$-7d-2b-w-y=c-2d-y$

$\rightarrow 3(-7d-2b-w-y)-b-2(-5d-2b-w)-7d=z$

$\rightarrow -21d-6b-3w-3y-b+10d+4b+2w-7d=z$

$-18d-3b-w-3y=z \rightarrow -18d=z+3b+w+3y$

$2a+b-3c+3d=x$

$b=x-2a+3c-3d \xrightarrow{\times 6} 6b=6x-12a+18c-18d$

$6b=6x-12a+18c+z+3b+w+3y$

$6b-3b=-12a+18c+6x+3y+z+w$

$3b=-12a+18c+6x+3y+z+w$

$3b=-12(-7d-2b-w-y)+18(-5d-2b-w)+6x+3y+z+w$

$3b=84d+24b+12w+12y-90d-36b-18w+6x+3y+z+w$

$3b=-6d-12b-5w+15y+6x+z$

$15b=-6d-5w+15y+6x+z \rightarrow 15b=-\overset{3}{\cancel{6}}\left(\frac{-a+c-y}{2}\right)-5w+15y+6x+z$

$\rightarrow 15b=3a-3c+3y-5w+15y+6x+z \rightarrow 15b=3a-3c+6x+18y+z-5w$

$\rightarrow b=\dfrac{3a-3c+6x+18y+z-5w}{15}$

$a=-7d-2\left(\dfrac{3a-3c+6x+18y+z-5w}{15}\right)-w-y$

$a=-7d-\dfrac{6a+6c-12x-36y-2z+5w}{15}-w-y$

$15a=-105d-6a+6c-12x-36y-2z+5w-15w-15y$

$21a=6c-105d-12x-51y-2z-10w$

$\boxed{a=\dfrac{6c-105d-12x-51y-2z-10w}{21}}$

(b)

$$c = a + 2d + y \rightarrow c = \frac{6c - 105d - 12x - 51y - 2z - 10w}{21} + 2d + y$$

$$6c + 21c = -105d - 12x - 51y - 2z - 10w + 42d + 21y$$

$$15c = -63d - 12x - 30y - 2z - 10w \rightarrow c = \frac{-63d - 12x - 30y - 2z - 10w}{15}$$

(c) $$d = -\frac{a + c - y}{2} \rightarrow d = -\frac{\left(\frac{6c - 105d - 12x - 51y - 2z - 10w}{21}\right) + \left(\frac{-63d - 12x - 30y - 2z - 10w}{15}\right) - y}{2}$$

$$d = \frac{-\frac{6c + 105d + 12x + 51y + 2z + 10w}{21} + \frac{-63d - 12x - 30y - 2z - 10w}{15}}{2}$$

62) Mostrar que o conjunto $\{(1,1,0,0),(0,0,1,1),(1,0,0,3),(0,0,0,5)\}$ é base do $\mathbb{R}^4$

I) $(0,0,0,0) = a(1,1,0,0) + b(0,0,1,1) + c(1,0,0,3) + d(0,0,0,5)$

$(0,0,0,0) = (a,a,0,0) + (0,0,b,b) + (c,0,0,3c) + (0,0,0,5d)$

$(0,0,0,0) = (a+c, a, b, b+3c+5d)$

$$\begin{cases} a+c=0 & c=0 \\ a=0 \\ b=0 \\ b+3c+5d=0 & d=0 \end{cases}$$

LI

II)
$$\begin{cases} a + c = x & \rightarrow \boxed{c = x-y} \\ a = y & \rightarrow \boxed{a=y} \\ b = z & \rightarrow \boxed{b=z} \\ b+3c+5d = w & \rightarrow z+3(x-y)+5d = w \rightarrow z+3x-3y-w = -5d \end{cases}$$

$$\boxed{d = \frac{-3x+3y-z+w}{5}}$$

$(x,y,z,w) = (y)v_1 + (z)v_2 + (x-y)v_3 + \left(\frac{-3x+3y-z+w}{5}\right)v_4$

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72) Determinar a dimensão e uma base para cada um dos seguintes espaços vetoriais:

a) $\{(x,y,z) \in \mathbb{R}^3 / y = 3x\}$

$(x,y,z) = (x,3x,z)$

$(x,3x,z) = (x,3x,0) + (0,0,z)$

$(x,y,z) = x(1,3,0) + z(0,0,1)$ $\boxed{\dim V = 2}$

$B = \{(1,3,0),(0,0,1)\}$

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b) $\{(x,y,z) \in \mathbb{R}^3 / y = 5x \text{ e } z = 0\}$

$(x,y,z) = (x,5x,0)$

$(x,y,z) = (x,5x,0) + (0,0,0)$

$(x,y,z) = x(1,5,0) + \cancel{z(0,0,0)}$ $\boxed{\dim V = 1}$

$B = \{(1,5,0)\}$

c) $\{(x,y) \in \mathbb{R}^2 / x+y=0\}$

$(x,y) = (x,-x)$

$(x,y) = x(1,-1)$ $\boxed{\dim V = 1}$

$\boxed{B=\{(1,1)\}}$

d) $\{(x,y,z) \in \mathbb{R}^3 / x=3y \text{ e } z=-y\}$

$(x,y,z) = (3y, y, -y)$

$(x,y,z) = y(3,1,-1)$ $\boxed{\dim V = 1}$

$\boxed{B=\{(3,1,-1)\}}$

e) $\{(x,y,z) \in \mathbb{R}^3 / 2x - y + 3z = 0\}$

$-y = -3z - 2x$

$y = 2x + 3z$

$(x,y,z) = (x, 2x+3z, z)$

$(x,y,z) = (x,2x,0) + (0,3z,z)$

$(x,y,z) = \underline{x}(1,2,0) + \underline{z}(0,3,1)$ $\boxed{\dim V = 2}$

$\boxed{B=\{(1,2,0),(0,3,1)\}}$

f) $\{(x,y,z) \in \mathbb{R}^3 / z=0\}$

$(x,y,z) = (x,y,0)$

$(x,y,z) = (x,0,0) + (0,y,0) + (0,0,0)$

$(x,y,z) = \underline{x}(1,0,0) + \underline{y}(0,1,0)$ $\boxed{\dim V = 2}$

$\boxed{B=\{(1,0,0),(0,1,0)\}}$

73) Determinar a dimensão e uma base para cada um dos seguintes subespaços vetoriais de $M(2,2)$;

a) $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix};\ b = a+c \text{ e } d = c\right\}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & a+c \\ c & c \end{bmatrix} \Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} a & a \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & c \\ c & c \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \underline{a}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + \underline{c}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \qquad \boxed{\dim S = 2}$$

$$B = \left\{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}\right\}$$

b) $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix};\ b = a+c\right\}$ $\quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & a+c \\ c & d \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & a \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & c \\ c & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \underline{a}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + \underline{c}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \underline{d}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{\dim S = 3} \qquad B = \left\{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

c) $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix};\ c = a-3b \text{ e } d = 0\right\}$ $\quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ a-3b & 0 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 0 \\ a & 0 \end{bmatrix} + \begin{bmatrix} 0 & b \\ -3b & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \underline{a}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + \underline{b}\begin{bmatrix} 0 & 1 \\ -3 & 0 \end{bmatrix}$$

$$\boxed{\dim S = 2} \qquad B = \left\{\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -3 & 0 \end{bmatrix}\right\}$$

d) $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix};\ a+d = b+c\right\}$ $\quad a = b+c-d$ $\quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} b+c-d & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} b & b \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} c & 0 \\ c & 0 \end{bmatrix} + \begin{bmatrix} -d & 0 \\ 0 & d \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = b\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{\dim S = 3} \qquad B = \left\{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

74) Seja o subespaço S de M(2,2): $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}; c = a+b \text{ e } d = a \right\}$

a) Qual a dimensão de S?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ a+b & a \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 0 \\ a & a \end{bmatrix} + \begin{bmatrix} 0 & b \\ b & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \underline{a} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + \underline{b} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$\boxed{\dim S = 2}$

b) O conjunto $\left\{ \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \right\}$ é uma base de S? Justifique.

① $a = 1$ $b = -1$ $c = 0$ $d = 1$

② $a = 2$ $b = 1$ $c = 3$ $\boxed{d = 2}$ $\neq$

Logo, não é base