Transformações lineares Vitor Oliveira Ropke

Livro: Steinbruch

Tópico 4.8, página 211

Questões: 3, 5(j, k, l), 6-9, 11, 14-18 e 22.

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(3) Dentre as transformações $T: \mathbb{R}^2 \to \mathbb{R}^2$ definidas pelas seguintes leis, verificar quais são lineares:

a) $T(x,y) = (x-3y, 2x+5y)$

I) Sejam $u(x_1,y_1)$ e $v=(x_2,y_2)$ vetores do $\mathbb{R}^2$

Logo: $T(u+v) = T(x_1+x_2, y_1+y_2)$

$T(u+v) = ((x_1+x_2)-3(y_1+y_2), 2(x_1+x_2)+5(y_1+y_2))$

$T(u+v) = (x_1+x_2-3y_1-3y_2, 2x_1+2x_2+5y_1+5y_2)$

$T(u)+T(v) = (x_1-3(y_1), 2(x_1)+5(y_1)) + (x_2-3(y_2), 2(x_2)+5(y_2))$

$T(u)+T(v) = (x_1+x_2-3(y_1)-3(y_2), 2(x_1)+2(x_2)+5(y_1)+5(y_2))$

$=$

II) Seja $u=(x_1,y_1) \in \mathbb{R}^2$

$T(\alpha u) = T(\alpha x_1, \alpha y_1) = T(\alpha x_1 - 3\alpha y_1, 2\alpha x_1 + 5\alpha y_1)$

$\alpha T(u) = \alpha(x_1-3y_1, 2x_1+5y_1)$

$= (\alpha x_1 - 3\alpha y_1, 2\alpha x_1 + 5\alpha y_1)$

$=$

Logo, T_1 é linear.

b) $T(x,y)=(y,x)$

I) $u=(x_1,y_1)$
$v=(x_2,y_2)$

$$T(u+v)=T(x_1+x_2,y_1+y_2)$$
$$T(u+v)=(y_1+y_2,x_1+x_2)$$
$$T(u)+T(v)=(y_1,x_1)+(y_2,x_2)$$
$$T(u)+T(v)=(y_1+y_2,x_1+x_2)$$

$=$

II) $u=(x_1,y_1)$

$$T(\alpha u)=(\alpha y_1,\alpha x_1)$$
$$\alpha T(u)=\alpha(y_1,x_1)=(\alpha y_1,\alpha x_1)$$

$=$

Logo, T é linear

c) $T(x,y)=(x^2,y^2)$

I) $u=(x_1,y_1)$
$v=(x_2,y_2)$

$$T(u+v)=(x_1+x_2,y_1+y_2)$$
$$T(u+v)=((x_1+x_2)^2,(y_1+y_2)^2)$$
$$T(u)+T(v)=(x_1^2,y_1^2)+(x_2^2,y_2^2)$$
$$T(u)+T(v)=(x_1^2+x_2^2,y_1^2+y_2^2)$$

$\neq$

Logo, T não é linear

d) $T(x,y)=(x+1,y)$

I) $u=(x_1,y_1)$
$v=(x_2,y_2)$

$$T(u+v)=(x_1+x_2,y_1+y_2)$$
$$T(u+v)=((x_1+x_2)+1,y_1+y_2)$$
$$T(u)+T(v)=T(x_1,y_1)+T(x_2,y_2)$$
$$T(u)+T(v)=(x_1+1,y_1)+(x_2+1,y_2)$$
$$T(u)+T(v)=((x_1+x_2)+2,y_1+y_2$$

$\neq$

Logo, T não é linear.

a) $T(X,Y) = (Y-X, 0)$

I) $u = (X_1, Y_1)$, $v = (X_2, Y_2)$

$$T(u+v) = (X_1+X_2, Y_1+Y_2)$$
$$T(u+v) = ((Y_1+Y_2)-(X_1+X_2), 0)$$
$$T(u+v) = (Y_1+Y_2-X_1-X_2, 0)$$

$$T(u)+T(v) = T(X_1,Y_1)+T(X_2,Y_2)$$
$$= (Y_1-X_1, 0) + (Y_2-X_2, 0)$$
$$= (Y_1+Y_2-X_1-X_2, 0)$$

$=$

II) $u = (X_1, Y_1)$

$$T(\alpha u) = (\alpha X_1, \alpha Y_1)$$
$$T(\alpha u) = (\alpha Y_1 - \alpha X_1, 0)$$
$$\alpha T(u) = \alpha(X_1, Y_1)$$
$$\alpha T(u) = \alpha(Y_1 - X_1, 0)$$
$$= (\alpha Y_1 - \alpha X_1, 0)$$

$=$

Logo, T é linear

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b) $T(X,Y) = (|X|, 2Y)$

I) $u = (X_1, Y_1)$, $v = (X_2, Y_2)$

$$T(u+v) = (X_1+X_2, Y_1+Y_2)$$
$$T(u+v) = (|X_1+X_2|, 2(Y_1+Y_2))$$
$$T(u+v) = (|X_1+X_2|, 2Y_1+2Y_2)$$

$$T(u)+T(v) = T(X_1,Y_1)+T(X_2,Y_2)$$
$$T(u)+T(v) = (|X_1|, 2Y_1) + (|X_2|, 2Y_2)$$
$$= (|X_1|+|X_2|, 2Y_1+2Y_2)$$

$\neq$

$|X_1+X_2|$ nem sempre será $|X_1|+|X_2|$

Logo, T não é linear

g) $T(x,y) = (\operatorname{sen} x, y)$

I) $u = (x_1, y_1)$
$v = (x_2, y_2)$

$T(u+v) = T(x_1 + x_2, y_1 + y_2)$

$T(u+v) = (\operatorname{sen}(x_1 + x_2), y_1 + y_2)$

$T(u) + T(v) = T(x_1, y_1) + T(x_2, y_2)$

$T(u) + T(v) = (\operatorname{sen}(x_1), y_1) + (\operatorname{sen}(x_2), y_2)$

$T(u) + T(v) = (\operatorname{sen} x_1 + \operatorname{sen} x_2, y_1 + y_2)$

$\neq$

$\operatorname{sen}(x+y) \neq \operatorname{sen}(x) + \operatorname{sen}(y)$

Logo, T não é linear

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h) $T(x,y) = (xy, x-y)$

I) $u = (x_1, y_1)$
$v = (x_2, y_2)$

$T(u+v) = T(x_1 + x_2, y_1 + y_2)$

$T(u+v) = ((x_1 + x_2) \cdot (y_1 + y_2), (x_1 + x_2) - (y_1 + y_2))$

$T(u+v) = (x_1 y_1 + x_1 y_2 + x_2 y_1 + x_2 y_2, x_1 + x_2 - y_1 - y_2)$

$T(u) + T(v) = T(x_1, y_1) + T(x_2, y_2)$

$T(u) + T(v) = (x_1 y_1, x_1 - y_1) + (x_2 y_2, x_2 - y_2)$

$= ((x_1 y_1) + (x_2 y_2), x_1 + x_2 - y_1 - y_2)$

$\neq$

Logo, T não é linear

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i) $T(x,y) = (3y, -2x)$

I) $u = (x_1, y_1)$
$v = (x_2, y_2)$

$T(u+v) = T(x_1 + x_2, y_1 + y_2)$

$T(u+v) = (3(y_1 + y_2), -2(x_1 + x_2))$

$= (3y_1 + 3y_2, -2x_1 - 2x_2)$

$T(u) + T(v) = T(x_1, y_1) + T(x_2, y_2)$

$T(u) + T(v) = (3y_1, -2x_1) + (3y_2, -2x_2)$

$= (3y_1 + 3y_2, -2x_1 - 2x_2$

$=$

II) $T_{(\alpha u)} = T(\alpha X_1, \alpha y_1)$

$T_{(\alpha u)} = (3\alpha y_1, -2\alpha X_1)$

$\alpha T_{(u)} = \alpha(X_1, y_1)$

$\alpha T_{(u)} = \alpha(3y_1, -2X_1) = (\alpha 3y_1, -2\alpha X_1)$ =

Logo, T é linear.

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⑤ Dentre as seguintes funções, verificar quais são lineares:

g) $T: \mathbb{R}^2 \to M(2,2)$, $T(x,y) = \left(\begin{bmatrix} 2y & 3x \\ -y & x+2y \end{bmatrix}\right)$

I) $u = (x_1, y_1)$

$v = (x_2, y_2)$

$T_{(u+v)} = (x_1+x_2, y_1+y_2)$

$$T_{(u+v)} = \begin{pmatrix} 2(y_1+y_2) & 3(x_1+x_2) \\ -(y_1+y_2) & x_1+x_2+2(y_1+y_2) \end{pmatrix}$$

$$T_{(u+v)} = \begin{bmatrix} 2y_1+2y_2 & 3x_1+3x_2 \\ -y_1-y_2 & x_1+x_2+2y_1+2y_2 \end{bmatrix}$$ =

$T_{(u)}+T_{(v)} = (x_1, y_1)+(x_2, y_2)$

$$T_{(u)}+T_{(v)} = \begin{bmatrix} 2y_1 & 3x_1 \\ -y_1 & x_1+2y_1 \end{bmatrix} + \begin{bmatrix} 2y_2 & 3x_2 \\ -y_2 & x_2+2y_2 \end{bmatrix} = \begin{bmatrix} 2y_1+2y_2 & 3x_1+3x_2 \\ -y_1-y_2 & x_1+x_2+2y_1+2y_2 \end{bmatrix}$$

II) $$T_{(\alpha u)} = \begin{bmatrix} 2\alpha y_1 & 3\alpha x_1 \\ -\alpha y_1 & \alpha x+2\alpha y_1 \end{bmatrix}$$ =

$$\alpha T_{(u)} = \alpha\begin{bmatrix} 2y_1 & 3x_1 \\ -y_1 & x+2y_1 \end{bmatrix} = \begin{bmatrix} 2\alpha y_1 & 3\alpha x_1 \\ -\alpha y_1 & \alpha x+2\alpha y_1 \end{bmatrix}$$

Logo, T é linear

k) $T: M(2,2) \to \mathbb{R}^2$, $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a-c, b+c)$

I) $u = (a_1, b_1, c_1, d_1)$

$v = (a_2, b_2, c_2, d_2)$

$T(u+v) = T(a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2)$

$T(u+v) = (a_1+a_2-(c_1+c_2), b_1+b_2+c_1+c_2)$

$T(u+v) = (a_1+a_2-c_1-c_2, b_1+b_2+c_1+c_2)$

$T(u)+T(v) = (a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2)$

$T(u)+T(v) = (a_1-c_1, b_1+c_1) + (a_2-c_2, b_2+c_2)$

$T(u)+T(v) = (a_1+a_2-c_1-c_2, b_1+b_2+c_1+c_2)$

$=$

II) $T(\alpha u) = (\alpha a_1, \alpha b_1, \alpha c_1, \alpha d_1)$

$T(\alpha u) = (\alpha a_1 - \alpha c_1, \alpha b_1 + \alpha c_1)$

$=$

$\alpha T(u) = \alpha(a_1, b_1, c_1, d_1)$

$\alpha T(u) = \alpha(a_1 - c_1, b_1+c_1) = (\alpha a_1 - \alpha c_1, \alpha b_1 + \alpha c_1)$

Logo, T é linear

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l) $T: M(2,2) \to \mathbb{R}$, $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (ad - bc)$

I) $u = (a_1, b_1, c_1, d_1)$

$v = (a_2, b_2, c_2, d_2)$

$T(u+v) = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2)$

$T(u+v) = ((a_1+a_2)(d_1+d_2) - (b_1+b_2)(c_1+c_2))$

$= (a_1d_1 + a_1d_2 + a_2d_1 + a_2d_2 - b_1c_1 - b_1c_2 - b_2c_1 - b_2c_2)$

$T(u)+T(v) = T(a_1, b_1, c_1, d_1) + T(a_2, b_2, c_2, d_2)$

$T(u)+T(v) = (a_1 d_1 - b_1 c_1) + (a_2 d_2 - b_2 c_2)$

$= (a_1 d_1 + a_2 d_2 - b_1 c_1 - b_2 c_2)$ $\neq$

Logo, T não é linear

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⑥ Seja a aplicação $T: \mathbb{R}^2 \to \mathbb{R}^3$

$(x,y) \to (x+ky, x+k, y)$

Verificar em que caso(s) T é linear:

a) $K = X$

I) $u = (x_1, y_1)$
$v = (x_2, y_2)$

$T(u+v) = (x_1+x_2+(x_1+x_2)(y_1+y_2), x_1+x_2+x_1+x_2, y_1+y_2)$

$T(u+v) = (x_1+x_2+x_1y_1+x_1y_2+x_2y_1+x_2y_2, 2x_1+2x_2, y_1+y_2)$

$T(u)+T(v) = (x_1+x_1y_1, x_1+x_1, y_1) + (x_2+x_2y_2, x_2+x_2, y_2)$

$= (x_1+x_2+x_1y_1+x_2y_2, x_1+x_1+x_2+x_2, y_1+y_2)$ $\neq$

Logo, T não é linear onde $K = X$

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b) $K = 1$

I) $u = (x_1, y_1)$
$v = (x_2, y_2)$

$T(u+v) = (x_1+x_2+y_1+y_2, (x_1+x_2)+1, y_1+y_2)$

$T(u+v) = (x_1+x_2+y_1+y_2, x_1+x_2+1, y_1+y_2)$

$T(u)+T(v) = (x_1+y_1, x_1+1, y_1) + (x_2+y_2, x_2+1, y_2)$

$T(u)+T(v) = (x_1+x_2+y_1+y_2, x_1+x_2+2, y_1+y_2)$ $\neq$

Logo, T não é linear onde $K = 1$

I) $K=0$

$u=(x_1,y_1)$ $\quad T_{(u+v)}=(x_1+x_2+0,\ x_1+x_2+0,\ y_1+y_2)$

$v=(x_2,y_2)$ $\quad =(x_1+x_2,\ x_1+x_2,\ y_1+y_2)$

$T_{(u)}+T_{(v)}=(x_1,x_1,y_1)+(x_2,x_2,y_2)$

$=(x_1+x_2,\ x_1+x_2,\ y_1+y_2)$ $\quad =$

II) $T_{(\alpha u)}=(\alpha x_1,\alpha x_1,\alpha y_1)$ $\quad =$

$\alpha T_{(u)}=\alpha(x_1,x_1,y_1)=(\alpha x_1,\alpha x_1,\alpha y_1)$

Logo, T é linear quando $K=0$

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7) a) Determinar a transformação linear $T:\mathbb{R}^2\to\mathbb{R}^3$ tal que $T(-1,1)=(3,2,1)$ e $T(0,1)=(1,1,0)$.

$(0,0)=a(-1,1)+b(0,1)$

$(0,0)=(-a,a)+(0,b)$

$(0,0)=(-a,a+b)$

$\begin{cases}-a=0 \to \boxed{a=0}\\ a+b=0 \quad \boxed{b=0}\end{cases}$ B.I.

$\begin{cases}-a=x \to \boxed{a=-x}\\ a+b=y \to -x+b=y \to \boxed{b=x+y}\end{cases}$

$(x,y)=(-x)(-1,1)+(x+y)(0,1)$

Logo: $T(x,y)=aT(-1,1)+bT(0,1)$

$T(x,y)=(-x)(3,2,1)+(x+y)(1,1,0)$

$T(x,y)=(-3x,-2x,-x)+(x+y,x+y,0)$

$\boxed{T(x,y)=(-2x+y,\ -x+y,\ -x)}$

b) Encontrar $v \in \mathbb{R}^2$ tal que $T(v) = (-2, 1, -3)$

$$(-2, 1, -3) = (-2x + y, -x + y, -x)$$

$$-2x + y = -2$$
$$-x + y = 1 \rightarrow -3 + y = 1 \rightarrow y = 1 + 3 \rightarrow \boxed{y = 4}$$
$$-x = -3 \rightarrow \boxed{x = 3}$$

$$\boxed{v = (3, 4)}$$

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8) a) Determinar a transformação linear $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ tal que $T(1, -1, 0) = (1, 1)$
$T(0, 1, 1) = (2, 2)$ e $T(0, 0, 1) = (3, 3)$.

$$(0,0,0) = a(1, -1, 0) + b(0, 1, 1) + c(0, 0, 1)$$
$$(0,0,0) = (a, -a, 0) + (0, b, b) + (0, 0, c)$$
$$(0,0,0) = (a, -a + b, b + c)$$

$$\begin{cases} \boxed{a = 0} \\ -a + b = 0 \quad \boxed{b = 0} \\ b + c = 0 \quad \boxed{c = 0} \end{cases} \quad \text{b I}$$

$$a = x \rightarrow a = x$$
$$-a + b = y \rightarrow -x + b = y \rightarrow \boxed{b = x + y}$$
$$b + c = z \rightarrow x + y + c = z \rightarrow \boxed{c = -x - y + z}$$

$$(x, y, z) = (x)(1, -1, 0) + (x + y)(0, 1, 1) + (-x - y + z)(0, 0, 1)$$

Logo:
$$T(x, y, z) = aT(1, -1, 0) + bT(0, 1, 1) + cT(0, 0, 1)$$
$$T(x, y, z) = (x)(1, 1) + (x + y)(2, 2) + (-x - y + z)(3, 3)$$
$$T(x, y, z) = (x, x) + (2x + 2y, 2x + 2y) + (-3x - 3y + 3z, -3x - 3y + 3z)$$
$$T(x, y, z) = (-y + 3z, -y + 3z)$$

b) Achar $T(1,0,0)$ e $T(0,1,0)$.

$$(-y+3z) = (-1)y + 3(z)$$

$$T(1,0,0) = a(0) + b(-1) + c(3)$$

$$T(1,0,0) = -b+3c$$

$$-b=0 \qquad T(1,0,0)=(0,0)$$
$$c=0$$

$$T(0,1,0) = (-1,-1)$$

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(9) Seja $T: \mathbb{R}^3 \to \mathbb{R}^2$ uma transformação linear definida por $T(1,1,1)=(1,2)$, $T(1,1,0)=(2,3)$ e $T(1,0,0)=(3,4)$

a) Determinar $T(x,y,z)$

$$(x,y,z) = a(1,1,1)+b(1,1,0)+c(1,0,0)$$
$$(x,y,z)=(a,a,a)+(b,b,0)+(c,0,0)$$
$$(x,y,z)=(a+b+c,\ a+b,\ a)$$

$$a+b+c = x \to z+y-z+c=x \to \boxed{c=x-y}$$
$$a+b = y \to z+b=y \to \boxed{b=y-z}$$
$$a = z \to \boxed{a=z}$$

$$(x,y,z)=(z)(1,1,1)+(y-z)(1,1,0)+(x-y)(1,0,0)$$

$$T(x,y,z) = aT(1,1,1)+bT(1,1,0)+cT(1,0,0)$$
$$T(x,y,z)=(z)(1,2)+(y-z)(2,3)+(x-y)(3,4)$$
$$T(x,y,z)=(z,2z)+(2y-2z,3y-3z)+(3x-3y,4x-4y)$$
$$\boxed{T(x,y,z)=(3x-y-z,\ 4x-y-z)}$$

b) Determinar $v \in \mathbb{R}^3$ tal que $T(v) = (-3, 2)$

$(-3,2) = (3x - y - z, 4x - y - z)$

$\begin{cases} 3x - y - z = -3 \\ 4x - y - z = 2 \end{cases}$

$3x = -3 + y + z \rightarrow x = \frac{-3+y+z}{3}$

$y = 3 + 3x - z$

$y = 3 + 3.5 - z$

$y = 18 - z$

$4x - (3 + 3x - z) - z = 2 \rightarrow 4x - 3 - 3x + \cancel{z} - \cancel{z} = 2 \rightarrow x - 3 = 2 \rightarrow \boxed{x = 5}$

$4.5 - y - z = 2 \rightarrow 20 - y - z = 2 \rightarrow z = -2 - y + 20 \rightarrow z = 18 - y$

$z = 3 - y + 3x \rightarrow z = 3 - y + 3.5 \rightarrow z = 3 - y + 15 \rightarrow z = 18 - y$

$v = (5, 18 - z, z)$

c) Determinar $v \in \mathbb{R}^3$ tal que $T(v) = (0,0)$

$(0,0) = (3x - y - z, 4x - y - z)$

$\begin{cases} 3x - y - z = 0 \\ 4x - y - z = 0 \end{cases}$

$3x = y + z \rightarrow 0 = y + z \rightarrow y = -z$

$z = -y$

$-y = z - 3x \rightarrow y = 3x - z$

$z = -y + 3x$

$4x = y + z$

$y = 4x - z$

$z = -y + 4x$

$4x - z = 3x - z$

$4x - 3x = z - z$

$\boxed{x = 0}$

$3x - y - z = 0 \quad \times(-4)$

$4x - y - z = 0 \quad \times(3)$

$-12x + 4y + 4z = 0$

$12x - 3y - 3z = 0$

$y + z = 0$

$\boxed{y = -z}$

$\boxed{v = (0, -z, z)}$

(11) Determinar a transformação linear $T: P_2 \rightarrow P_2$ tal que $T(1) = x$, $T(x) = 1 - x^2$ e $T(x^2) = x + 2x^2$.

$(m, n, o) = a(1) + b(x) + c(x^2)$

$(m, n, o) = (a) + (bx) + (cx^2)$

$a = m$

$bx = n \rightarrow \boxed{b = n/x}$

$cx^2 = o \rightarrow \boxed{c = \frac{o}{x^2}}$

$T(m, n, o) = (a)x + (b)(1 - x^2) + (c)(x + 2x^2)$

$T(m,n,o) = ax + b - bX^2 + cX + 2cX^2$

$T(m,n,o) = (a+c)X + b + (-b+2c)X^2$

$T(m,n,o) = b + (a+c)X + (-b+2c)X^2$

~~14) $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(X,Y) = (3X-Y, -3X+Y)$~~

Nos problemas 14 a 18 são apresentadas transformações lineares. Para cada uma delas:

a) Determinar o núcleo, uma base para esse subespaço e sua dimensão. T é injetora? Justificar.

b) Determinar a imagem, uma base para esse subespaço e sua dimensão. T é sobrejetora? Justificar.

14) $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(X,Y) = (3X-Y, -3X+Y)$

a) Núcleo

$N(T) = \{(X,Y) \in \mathbb{R}^2 / T(X,Y) = (0,0)\}$

Logo:

$$(3X-Y, -3X+Y) = (0,0) \qquad \begin{cases} 3X - Y = 0 \to \boxed{X = \frac{Y}{3}} \\ -3X + Y = 0 \to \boxed{Y = 3X} \end{cases} \qquad 3X = Y$$

$N(T) = \{(X, 3X) / X \in \mathbb{R}\} = \{X(1,3) / X \in \mathbb{R}\}$

ou ainda

$\boxed{N(T) = [(1,3)]}$ $\boxed{\dim N(T) = 1}$

Como $\{(1,3)\}$ gera $N(T)$ e é L.I, temos que

$\boxed{B = \{(1,3)\} \text{ é base de } N(T)}$

T não é injetora, pois $N(T) \neq \{(0,0)\}$

b) Imagem

$$(3X - Y, -3X + Y) = (3X, -3X) + (-Y, Y)$$
$$= X(3,-3) + Y(-1,1)$$

$Im(T) = [(3,-3), (-1,1)]$

$\dim N(T) + \dim Im(T) = \dim V$

$1 + \dim Im(T) = 2 \to \boxed{\dim Im(T) = 1}$

~~Base~~

$\boxed{\text{Base de } Im(T) = \{(3,-3),(-1,1)\}}$

$Im(T) = W$ logo é sobrejetora.

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(15) $T: \mathbb{R}^2 \to \mathbb{R}^3$, $T(x,y) = (x+y, x, 2y)$

a) Núcleo

$N(T) = \{(x,y) \in \mathbb{R}^2 / T(x,y) = (0,0,0)\}$

Logo: $(x+y, x, 2y) = (0,0,0)$ $\begin{cases} x+y=0 \\ x=0 \\ 2y=0 \quad \boxed{y=0} \end{cases}$

$N(T) = [(0,0)]$, logo T é injetora

$\boxed{\dim N(T) = 0}$

---

b) ~~Nú~~

Imagem

$(x+y, x, 2y) = (x,x)+(y,2y)+(0)$

$= x(1,1)+y(1,2)+z(0)$

$= x(1,1)+y(1,2)$

$\boxed{Im(T) = [(1,1)+(1,2)]}$

$\underset{0}{\dim N(T)} + \dim Im(T) = \underset{2}{\dim V} \to \boxed{\dim Im(T) = 2}$

$B = \{(1,1)+(1,2)\}$ é base de $Im(T)$

T não é sobrejetora pois $\dim Im(T) \neq W$

16) $T: \mathbb{R}^2 \to \mathbb{R}^2,\ T(x,y) = (x-2y, x+y)$

a) Núcleo

$N(T) = \{(x,y) \in \mathbb{R}^2 / T(x,y) = (0,0)\}$

Logo:

$(x-2y,\ x+y) = (0,0)$ $\qquad \begin{cases} x-2y=0 \to -y-2y=0 \to -3y=0 \to \boxed{y=0} \\ x+y=0 \to x=-y \to \boxed{x=0} \end{cases}$

$\boxed{N(T) = [(0,0)]}$, logo T é injetora.

E $\boxed{\dim N(T) = 0}$

b) Imagem

$(x-2y, x+y) = (x,x) + (-2y,y)$
$\qquad = x(1,1) + y(-2,1)$

$Im(T) = [(1,1), (-2,1)]$

$\dim N(T) + \dim Im(T) = \dim V \quad \to \quad \dim Im(T) = 2$
$\quad 0 \qquad\qquad\qquad\qquad\qquad 2 \qquad\qquad \dim W = 2$

Logo, T é sobrejetora

---

17) $T: \mathbb{R}^3 \to \mathbb{R}^2,\ T(x,y,z) = (x+2y-z,\ 2x-y+z)$

a) Núcleo

$N(T) = \{(x,y,z) \in \mathbb{R}^3 / T(x,y,z) = (0,0)\}$

$(x+2y-z,\ 2x-y+z) = (0,0)$ $\qquad \begin{cases} x+2y-z=0 \to x+2(-3x)-z=0 \to x-6x-z=0 \\ 2x-y+z=0 \end{cases}$ $\to -5x-z=0$

$3x+y=0 \to \boxed{y=-3x} \qquad\qquad \to \boxed{z=-5x}$

$N(T) = \{(x,-3x,-5x) / x \in \mathbb{R}\} = \{x(1,-3,-5) / x \in \mathbb{R}\}$

$\boxed{N(T) = [(1,-3,-5)]} \qquad \boxed{\dim N(T) = 1}$

Como $\{(1,-3,-5)\}$ gera $N(T)$ e é L.I temos que $\boxed{\beta = \{(1,-3,-5)\}}$ é base de $N(T)$.

T não é injetora pois $N(T) \neq 0_V$

b) Imagem

$$(x+2y-z, 2x-y+z) = (x, 2x) + (2y, -y) + (-z, z)$$

$$= x(1,2) + y(2,-1) + z(-1,1)$$

$$\boxed{Im(T) = [(1,2), (2,-1), (-1,1)]}$$

Logo $\boxed{\gamma = \{(1,2), (2,-1)\}}$ é base da $Im(T)$ pois gera $Im(T)$ e é L.I

$\dim N(T) + \dim Im(T) = \dim V$

$1 \qquad\qquad 3$ $\rightarrow$ $\boxed{\dim Im(T) = 2}$

Logo, T é sobrejetora pois $\dim Im(T) = \dim W = 2$

---

(18) $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $T(x,y,z) = (x-y-2z, -x+2y+z, x-3z)$

$N(T) = \{(x,y,z) \in \mathbb{R}^3 / T(x,y,z) = (0,0,0)\}$

$(x-y-2z, -x+2y+z, x-3z) = (0,0,0)$

$$\begin{cases} x-y-2z=0 \\ -x+2y+z=0 \\ x \quad -3z=0 \rightarrow \boxed{x=3z} \end{cases}$$

$x-y-2z=0 \rightarrow 3z-y-2z=0 \rightarrow z-y=0 \rightarrow \boxed{y=z}$

$N(T) = \{(3z, z, z) / z \in \mathbb{R}\} = \{z(3,1,1) / z \in \mathbb{R}\}$

$N(T) = [(3,1,1)]$ $\boxed{\dim N(T) = 1}$

como $\{(3,1,1)\}$ gera $N(T)$ é L I temos que $\beta = \{(3,1,1)\}$ é base de $N(T)$.

T não é injetora pois $N(T) \neq 0_V$

---

b) Imagem $(x-y-2z, -x+2y+z, x-3z) = (x, -x, x) + (-y, 2y, 0) + (-2z, z, -3z)$

$$= x(1,-1,1) + y(-1,2,0) + z(-2,1,-3)$$

$$\boxed{Im(T) = [(1,-1,1), (-1,2,0), (-2,1,-3)]}$$

$\dim N(T) + \dim Im(T) = \dim V$

$1 \qquad\qquad\qquad = 3 \quad \rightarrow \boxed{\dim Im(T) = 2}$

$\dim Im(T) \neq \dim W$ logo, T não é sobrejetora.

$\gamma = \{(1,-1,1), (-1,2,0)\}$ é base da $Im(T)$

---

22) Seja a transformação linear $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ tal que $T(-2,3) = (-1,0,1)$ e $T(1,-2) = (0,-1,0)$.

a) Determinar $T(x,y)$.

$(x,y) = a(-2,3) + b(1,-2)$

$(x,y) = (-2a, 3a) + (b, -2b)$

$(x,y) = (-2a+b, 3a-2b)$

$\begin{cases} -2a + b = x \rightarrow b = x + 2a \rightarrow b = x + 2(y-2x) \\ 3a - 2b = y \end{cases}$

$b = x + 2y - 4x$

$\boxed{b = 2y - 3x}$

$\rightarrow 3a - 2(x + 2a) = y$

$\rightarrow 3a - 2x - 4a = y$

$\rightarrow -a - 2x = y \rightarrow \boxed{a = -y - 2x}$

$\rightarrow (x,y) = (y-2)(-2,3) + (2y-x)(1,-2)$

$T(x,y) = aT(-2,3) + bT(1,-2)$

$T(x,y) = (y-2x)(-1,0,1) + (2y-x)(0,-1,0)$

$T(x,y) = (2x-y, 0, -2x+y) + (0, x-2y, 0)$

$\boxed{T(x,y) = (2x+y, 3x+2y, -2x-y)}$

---

b) Determinar $N(T)$ e $Im(T)$.

$N(T) = \{(x,y) \in \mathbb{R}^2 \,/\, T(x,y) = (0,0,0)\}$

$(2x+y, 3x+2y, -2x-y) = (0,0,0)$

$= \begin{cases} 2x+y=0 \\ 3x+2y=0 \\ \cancel{-2x-y=0} \end{cases} \rightarrow \begin{cases} 2x+y=0 \rightarrow y = 2x \rightarrow \boxed{y=0} \\ 3x+2y=0 \rightarrow 3x + 4x = 0 \end{cases}$

$7x = 0$

$\boxed{x = 0}$

$N(T) = [(0,0)]$ $\quad$ T é injetora pois $N(T) = \{(0,0)\}$

$\dim N(T) = 0$

Imagem

$(2x+y, 3x+2y, -2x-y) =$

$= (2x, 3x, -2x) + (y, 2y, -y)$

$= x(2,3,-2) + y(1,2,-1)$

$Im(T) = [(2,3,-2), (1,2,-1)]$

$\dim N(T) = 0 \qquad \dim Im(T) = 2$

$\dim V = 2$

T não é sobrejetora pois $\dim Im(T) \neq \dim W$