

$$A = \begin{pmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{pmatrix}$$

$$l_{11} = 5$$

$$l_{21} = 7$$

$$l_{31} = 3$$

$$l_{11} \cdot u_{12} = -2 \rightarrow u_{12} = -\frac{2}{5} \quad l_{11} \cdot u_{13} = 1 \rightarrow u_{13} = \frac{1}{5}$$

$$l_{21} u_{12} + l_{22} = 1 \rightarrow -\frac{14}{5} + l_{22} = 1 \rightarrow l_{22} = 1 + \frac{14}{5} \rightarrow l_{22} = \frac{5+14}{5}$$

$$l_{22} = \frac{19}{5}$$

$$l_{21} \cdot u_{13} + l_{22} \cdot u_{23} = -5 \rightarrow \frac{7}{5} + \frac{19 u_{23}}{5} = -5 \rightarrow \frac{7 + 19 u_{23}}{5} = -5 \rightarrow 19 u_{23} + 7 = -25 \rightarrow u_{23} = -\frac{32}{19}$$

$$l_{31} u_{12} + l_{32} = 7 \rightarrow -\frac{6}{5} + l_{32} = 7 \rightarrow l_{32} = 7 + \frac{6}{5} \rightarrow l_{32} = \frac{35+6}{5} \rightarrow l_{32} = \frac{41}{5}$$

$$l_{31} \cdot u_{13} + l_{32} \cdot u_{23} + l_{33} = 4$$

$$\rightarrow \frac{3}{5} + -\frac{1312}{95} + l_{33} = 4 \rightarrow l_{33} = 4 - \frac{3}{5} + \frac{1312}{95} \rightarrow l_{33} = \frac{380 - 57 + 1312}{95}$$

$$l_{33} = \frac{1635}{95} \rightarrow l_{33} = \frac{327}{19}$$

$$A = L \cdot U = \begin{pmatrix} 5 & 0 & 0 \\ 7 & 19/5 & 0 \\ 3 & 41/5 & 327/19 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2/5 & 1/5 \\ 0 & 1 & -32/19 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Ax = b; \quad b = \begin{pmatrix} 4 \\ 8 \\ 10 \end{pmatrix}$$

$$L \cdot U \cdot x = b; \quad Ux = z; \quad L \cdot z = b \rightarrow \begin{pmatrix} 5 & 0 & 0 \\ 7 & 19/5 & 0 \\ 3 & 41/5 & 327/19 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 10 \end{pmatrix}$$

L

z

b

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$$5z_1 = 4 \rightarrow \boxed{z_1 = \frac{4}{5}}$$

$$7z_1 + \frac{19z_2}{5} = 8 \rightarrow \frac{28}{5} + \frac{19z_2}{5} = 8 \rightarrow \frac{28 + 19z_2}{5} = 8 \rightarrow 28 + 19z_2 = 40$$

$$\rightarrow 19z_2 = 40 - 28 \rightarrow 19z_2 = 12 \rightarrow \boxed{z_2 = \frac{12}{19}}$$

$$3z_1 + \frac{41z_2}{5} + \frac{327z_3}{19} = 10 \rightarrow \frac{12}{5} + \frac{41 \cdot \frac{12}{19}}{5} + \frac{327z_3}{19} = 10$$

$$\rightarrow \frac{12}{5} + \frac{492}{95} + \frac{327z_3}{19} = 10 \rightarrow \frac{12}{5} + \frac{492}{95} \cdot \frac{1}{5} + \frac{327z_3}{19} = 10$$

$$\rightarrow \frac{12}{5} + \frac{492}{95} + \frac{327z_3}{19} \rightarrow \frac{228 + 492 + 1635z_3}{95} = 10 \rightarrow \frac{720 + 1635z_3}{95} = 10$$

$$\rightarrow 720 + 1635z_3 = 950 \rightarrow 1635z_3 = 950 - 720 \rightarrow 1635z_3 = 230 \rightarrow z_3 = \frac{230}{1635}$$

$$\rightarrow \boxed{z_3 = \frac{46}{327}}$$

$$U \cdot x = z \rightarrow \begin{pmatrix} 1 & -2/5 & 1/5 \\ 0 & 1 & -32/19 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4/5 \\ 12/19 \\ 46/327 \end{pmatrix}$$

$$x_1 + \frac{-2x_2}{5} + \frac{x_3}{5} = \frac{4}{5} \rightarrow \frac{5x_1 - 2x_2 + x_3}{5} = \frac{4}{5} \rightarrow 5x_1 - 2x_2 + x_3 = 4$$

$$\rightarrow 5x_1 - 2x_2 + x_3 = 4$$

$$x_2 + \frac{-32x_3}{19} = \frac{12}{19} \rightarrow \frac{19x_2 - 32x_3}{19} = \frac{12}{19} \rightarrow 19x_2 - 32x_3 = 12$$

$$\boxed{x_3 = \frac{46}{327}}$$

$$\rightarrow 19x_2 - \frac{1472}{327} = 12 \rightarrow \frac{6213x_2 - 1472}{327} = 12$$

$$\rightarrow 5x_1 - 2x_2 + \frac{46}{327} = 4$$

$$\rightarrow 6213x_2 - 1472 = 3924$$

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$$6213x_2 = 5396 \rightarrow x_2 = \frac{5396}{6213} \rightarrow x_2 = \frac{284}{327}$$

$$5x_1 - \cancel{32} \cdot 2 \cdot \frac{284}{327} + \frac{46}{327} = 4 \rightarrow 5x_1 - \frac{568}{327} + \frac{46}{327} = 4 \rightarrow \cancel{1635x_1}$$

$$\frac{1635x_1 - 568 + 46}{327} = 4 \rightarrow 1635x_1 - 568 + 46 = 1308 \rightarrow 1635x_1 = 1830$$

$$\rightarrow x_1 = \frac{1830}{1635} \rightarrow x_1 = \frac{122}{109}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{122}{109} \\ \frac{284}{327} \\ \frac{46}{327} \end{pmatrix}$$