Analysis of Irreversible Phenomena via τ-Manifold: A Formal τ-Analysis Approach - Summary

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Abstract

Irreversibility is a fundamental topic in both pure and applied sciences, including mathematics, mathematical physics, and complex systems. Despite extensive efforts by leading researchers, traditional methods often rely on approximations or computationally intensive algorithms. Consequently, many irreversible phenomena — such as entropy evolution or complex economic fluctuations — remain only partially understood.

The introduction of the novel concepts in this paper contained concerns about encoding irreversibility as a dispersive phenomena in a hyperbolic topological manifold - regarding singularities created by the dynamics of the irreversible phenomena - breaking out complex entangled systems (via tensors) to a topological interpretation of the manifold of irreversibility, called τ-space.

1 Introduction

Summary of the Mathematical Foundations

To formalize an irreversible process, we introduce a unit τ satisfying:

$$\sqrt{\tau} = -1$$

Squaring both sides yields $\tau = 1$, however, the original negative sign is lost in the squaring process, indicating an intrinsic loss of information. This property exemplifies the irreversible nature encoded in τ and motivates its use as a foundational unit within the τ-manifold.

A τ-variable in $\mathbb{T}^1$ - the one-dimensional τ-space - is denoted by $q = z + \tau w$; $z, w \in \mathbb{C}^2$; the τ-unit represents the first vector component of the space $\mathbb{T}^1 \cong \mathbb{C} \times \mathbb{C}$.

The extension of $\mathbb{T}^1 \mapsto \mathbb{T}^n$ is naturally done by introducing a τ-tensor $q_{ml} = \chi_m \left| e_m \right\rangle + \psi_l \left| e_l \right\rangle ;\ \chi_m, \psi_l \in \mathbb{C}^{2n}$, where $\left| e_m \right\rangle, \left| e_l \right\rangle$ are the mth complex and lth τ-components, respectively.
Isomorphically, the n-dimensional τ-space is defined as $\mathbb{T}^n \cong \mathbb{C} \ \times \mathbb{C} \ \times \cdots^{[n \text{ times}]} \ \times \mathbb{C} \ \equiv \mathbb{C}^n$.

The extension to the n-dimensional τ-space, conceptually, introduces the called **degree of freedom of irreversibility**, such that the latter increases when the dimensions of the τ-manifold are higher.

2 The τ-sphere: The Eigenvalue Relation and Stable and Holomorphic τ-Functions

τ-Integral & τ-Sphere

The τ-sphere - S_τ - denotes the hyperbolic manifold in which $\mathbb{T}$ is embedded. Such idealized hyperbolic "sphere" possesses a radius $\mathfrak{R}$; poles over the boundaries of S_τ - ∇S_τ - formed by the coordinates $\{P_{lat^\uparrow}(i,0), P_{lat^\downarrow}(-i,0)\}; \{P_{lng^\uparrow}(1,0), P_{lng^\downarrow}(-1,0)\}$. Such sphere has a called scape angle - ξ - such that $-\infty < \xi < \infty$. The name "scape angle" refers to the encoding of irreversibility, such that when $\xi \to \infty$, the process is called totally reversible.

The called τ-integral is realized over the τ-space $\mathbb{T}$, such that an automorphism - defined as a function $f_\tau(q) = \text{ᑎ}(z,w) + \tau\text{۲}(z,w)$ - is interpreted as a field acting over the referred space, which may or not have a rotation, which controls the singularities of such a function by the scape angle- such that if an absence of rotation is observed, no singularity is liberated by the function/field, and then f_τ is said to be stable.

The τ-integral, then, is an integral realized over $\mathbb{T}$, embedded in the hyperbolic geometry of S_τ. Then, by the tensor interpretation developed in Section 3, and the Generalized Stokes' Theorem for irreversible generic manifolds *explored in the complete version of the preprint (under request)*, the evaluation of the τ-integral based on its rotation, takes the form:

$$\oint_{S_\tau} f_\tau(q)dq = \oiint_{\mathbb{T}} \langle (z,w) + \tau(z,w) | dz + \tau dw \rangle$$

$$= \oiint_{\mathbb{T}} \langle \text{ᑎ}(z,w) + \tau \text{۲}(z,w) \, | dz + \tau dw \rangle$$

$$= \oiint_{\mathbb{T}\mapsto\mathbb{C}^2} (\nabla \times f_\tau) dz dw = \oiint_{\mathbb{T}\mapsto\mathbb{C}^2} \left(\frac{\partial \text{۲}(z,w)}{\partial z} - \frac{\partial \text{ᑎ}(z,w)}{\partial w} \right) dz dw$$

- τ-trivial functions are those in which $\dfrac{\partial \text{ᚠ}(z,w)}{\partial z} = \dfrac{\partial \text{ᚢ}(z,w)}{\partial w}$, such that the τ-integral is zero.

  This interpretation is also naturally extendable to $\mathbb{T}^n$ by the Generalized Stokes' Theorem - explored in the *complete version of the preprint*.

τ-trivial functions led to an instantaneous understanding about the rotation the field f_τ enforces – the rotation is zero, therefore the angle ξ has an absence of deviation, then, τ-trivial functions represent the called irreversible static functions.

The Eigenvalue Relation and Stable Functions

The operator theory and its applications to quantum mechanics permits the interpretation of dispersion of information to assume a well regarded relation from linear algebra - the eigenvalue relation.

The eigenvalue relation, in τ-analysis, attributes a multiplication of the operators $\hat{\text{ᛉ}}\hat{\text{ᛉ}}^{-1} = \hat{\text{ᛗ}}$ - so that the latter are called irreversibility operators - to a τ-eigenfunction $f_\tau : \mathbb{T} \to \mathbb{T}$, and affirms its equivalence to the product of an eigenvalue ю with the eigenfunction itself.

Such interpretation, then, affirms that when ю $= 1$, then the irreversibility operator applied to the τ-eigenfunction is the eigenfunction itself, therefore, such operator is equivalent the identity operator, and then, f_τ **is called a stable function**.

The eigenvalue relation follows the form:

$$\hat{\text{ᛗ}} \left| f_\tau \right\rangle = \text{ю} \left| f_\tau \right\rangle$$

Such that, as previously defined, when ю $= 1$, f_τ is an stable function, denoted by $f_\tau^{\blacksquare}$.

The PDE Conditions for the Holomorphicity of f_τ

The Conditions for the Holomorphicity of functions denotes one of the basics of complex analysis. In τ-analysis such conditions also exist; they are derived using a slightly different approach, technically guided in the *complete version of the preprint*:

For a given endofunction $f_\tau : \mathbb{T}^n \to \mathbb{T}^n$ in its tensor form - $(f_\tau(q))_{j\sigma} = \text{ɧ}_j(q) \left| e_j \right\rangle + \text{ɰ}_\sigma(q) \left| e_\sigma \right\rangle$, and a stable variable $q_{ml}^{\blacksquare} = \chi_m^{\blacksquare} \left| e_m \right\rangle + \psi_l^{\blacksquare} \left| e_l \right\rangle$:

$$\frac{\partial \text{ɧ}_j}{\partial \chi_m^{\blacksquare}} = \frac{\partial \text{ɰ}_\sigma}{\partial \psi_l^{\blacksquare}}$$

$$;\ \tau^2 \frac{\partial \text{ɧ}_j}{\partial \psi_l^{\blacksquare}} = \frac{\partial \text{ɰ}_\sigma}{\partial \chi_m^{\blacksquare}}$$

The Normalization of the τ-function

- Given two τ-functions $\text{ᚺ}_\alpha(q); \text{ᚺ}_\beta(q)$ in hyperbolic coordinates - $\text{ᚺ}(\text{Я}, \xi_\alpha); \text{ᚺ}(\text{Я}, \xi_\beta)$, their inner product is defined as follows:

$$\langle \text{ю}_\alpha \text{ᚺ}(\text{Я}, \xi_\alpha) | \text{ю}_\beta \text{ᚺ}(\text{Я}, \xi_\beta) \rangle = \text{ю}_\alpha \text{ю}_\beta \delta_{\alpha\beta}$$

; $\delta_{\alpha\beta}$ is the Kronecker Delta, defined as follows: $\delta_{\alpha\beta} = \begin{cases} 1, \alpha = \beta \\ 0, \alpha \neq \beta \end{cases}$

Therefore, the inner product is just different of zero when the angles are equal. Then:

$$\langle \text{ю}_\alpha \text{ᚺ}(\text{Я}, \xi_\alpha) | \text{ю}_\beta \text{ᚺ}(\text{Я}, \xi_\beta) \rangle = \oiint_{\mathbb{T}^2} \overline{(\text{ю}_\alpha \text{ᚺ}(\text{Я}, \xi_\alpha)}(\text{ю}_\beta \text{ᚺ}(\text{Я}, \xi_\beta)) d\text{Я} d\xi = \text{ю}_\alpha \text{ю}_\beta \delta_{\alpha\beta}$$

$$\Rightarrow \oiint_{\mathbb{T}^2} \overline{(\text{ю}_\alpha \text{ᚺ}(\text{Я}, \xi_\alpha)}(\text{ю}_\beta \text{ᚺ}(\text{Я}, \xi_\beta)) d\text{Я} d\xi =_{eigenvalue} \oiint_{\mathbb{T}^2} \overline{(\hat{\text{ᛗ}}_\alpha \text{ᚺ}(\text{Я}, \xi_\alpha)}(\hat{\text{ᛗ}}_\beta \text{ᚺ}(\text{Я}, \xi_\beta)) d\text{Я} d\xi$$

Theorem I: If $\text{ᚺ}_\alpha^{\blacksquare}$ and $\text{ᚺ}_\beta^{\blacksquare}$ are the stable form of ᚺ_α and ᚺ_β, respectively, $\hat{\text{ᛉ}}^{-1}$ and $\hat{\text{ᛉ}}$ are homomorphisms and anti-homomorphisms, respectively ($\hat{\text{ᛉ}}_\alpha^{-1}; \hat{\text{ᛉ}}_\beta^{-1} : \mathbb{T}^n \twoheadrightarrow \mathbb{C}^n; \hat{\text{ᛉ}}_\alpha, \hat{\text{ᛉ}}_\beta : \mathbb{C}^n \twoheadrightarrow \mathbb{T}^n$) – (*as proved in Theorem II in the complete version of the preprint*), then the stable function of the product of ᚺ_α and ᚺ_β is also stable.

Proof. If $\hat{\text{ᛉ}}^{-1}$ and $\hat{\text{ᛉ}}$ are homomorphisms and anti-homomorphisms, respectively, then:

$$\hat{\text{ᛉ}}^{-1} |\varsigma\Phi\rangle = \hat{\text{ᛉ}}^{-1} |\varsigma\rangle \, \hat{\text{ᛉ}}^{-1} |\Phi\rangle$$
$$; \hat{\text{ᛉ}} |\varsigma\Phi\rangle = \hat{\text{ᛉ}} |\Phi\rangle \, \hat{\text{ᛉ}} |\varsigma\rangle$$

For any two τ-functions $\varsigma; \Phi \in \mathbb{T}^n$.

$$\Rightarrow \hat{\text{ᛉ}} \hat{\text{ᛉ}}^{-1} |\varsigma\Phi\rangle = \hat{\text{ᛉ}} \left| \hat{\text{ᛉ}}^{-1} |\varsigma\rangle \, \hat{\text{ᛉ}}^{-1} |\Phi \right. = \hat{\text{ᛉ}} \hat{\text{ᛉ}}^{-1} |\Phi\rangle \, \hat{\text{ᛉ}} \hat{\text{ᛉ}}^{-1} |\varsigma\rangle$$

$$\Rightarrow \hat{\text{ᛗ}} |\varsigma\Phi\rangle = \hat{\text{ᛗ}} |\Phi\rangle \, \hat{\text{ᛗ}} |\varsigma\rangle$$

- Applying $\text{ᚻ}_\alpha ; \text{ᚻ}_\beta$:

$$\hat{\text{ᛗ}} \left| \text{ᚻ}_\alpha \text{ᚻ}_\beta \right\rangle = \hat{\text{ᛗ}} \left| \text{ᚻ}_\beta \right\rangle \hat{\text{ᛗ}} \left| \text{ᚻ}_\alpha \right\rangle = \text{ᚻ}^{\blacksquare}_\alpha \text{ᚻ}^{\blacksquare}_\beta, \text{ which is stable.}$$

This completes our proof, and provides the necessary information to proceed with the main derivation □

Applying *Theorem I* to the double integral:

$$\oiint_{\mathbb{T}^2} \overline{(\hat{\text{ᛗ}}_\alpha \text{ᚻ}(\mathfrak{A}, \xi_\alpha))} (\hat{\text{ᛗ}}_\beta \text{ᚻ}(\mathfrak{A}, \xi_\beta)) d\mathfrak{A} d\xi = \oiint_{\mathbb{T}^2} \overline{(\text{ᚻ}^{\blacksquare}(\mathfrak{A}, \xi_\beta))} (\text{ᚻ}^{\blacksquare}(\mathfrak{A}, \xi_\alpha)) d\mathfrak{A} d\xi$$

- By *Corollaries I & II*, that prove the τ-integration of a stable function can be reduced to a complex integration over $\mathbb{C}^2$ (*again, a more detailed proof of them is presented in the complete version of the preprint*):

$$\oiint_{\mathbb{T}^2} \overline{(\text{ᚻ}^{\blacksquare}(\mathfrak{A}, \xi_\beta))} (\text{ᚻ}^{\blacksquare}(\mathfrak{A}, \xi_\alpha)) d\mathfrak{A} d\xi = \oiint_{\Gamma \in \mathbb{C}^2} \overline{(\text{ᚻ}^{\blacksquare}(\mathfrak{A}, \xi_\beta))} (\text{ᚻ}^{\blacksquare}(\mathfrak{A}, \xi_\alpha)) d\mathfrak{A} d\xi = \text{ю}_\alpha \text{ю}_\beta \delta_{\alpha\beta}$$

- And for $\beta = \alpha$:

$$\oiint_{\Gamma \in \mathbb{C}^2} \overline{(\text{ᚻ}^{\blacksquare}(\mathfrak{A}, \xi_\beta))} (\text{ᚻ}^{\blacksquare}(\mathfrak{A}, \xi_\alpha)) d\mathfrak{A} d\xi = \oiint_{\Gamma \in \mathbb{C}^2} \mid \text{ᚻ}^{\blacksquare}(\mathfrak{A}, \xi_\alpha) \mid^2 d\mathfrak{A} d\xi = \text{ю}^2_\alpha$$

- The index α can be removed without any loss of generality:

$$\therefore \oiint_{\Gamma \in \mathbb{C}^2} \mid \text{ᚻ}^{\blacksquare}(\mathfrak{A}, \xi) \mid^2 d\mathfrak{A} d\xi = \text{ю}^2$$

3 Generic Functions and Tensor Interpretation

Definition of Generic Functions

A generic n-dimensional space $\{\mathcal{G}\}^n$, by definition, denotes a space where whether or not might exist irreversibility, so that it is unknown, and then called generic. $\{\mathcal{G}\}$ has an attached variable (already in its tensor form) κ, such that $\kappa_{cd} = \rho_c \left| e_c \right\rangle + \lambda_d \left| e_d \right\rangle$, where ρ_c, λ_d are variables in a space called desirable - $\{S\}^{2n}$, such that it is **a proper subset** of $\{\mathcal{G}\}^n$, as well as the relation betwixt $\mathbb{C} \subset \mathbb{T}$ or $\mathbb{R} \subset \mathbb{C}$. Then, following the isomorphical interpretation of a tensor field in the desirable space, similarly to what has been developed for $\mathbb{T}^n$, it is logical that $\{\mathcal{G}\}^n \cong \{S\} \times \{S\} \times \cdots^{[n \text{ times}]} \times \{S\} \equiv \{S\}^{2n}$.

A generic endofunction $f(\kappa) : \{\mathcal{G}\}^n \to \{\mathcal{G}\}^n$ is defined similarly to a τ-function in $\mathbb{T}^n$, despite the irreversibility uncertainty - characteristic of the generic space.
Then, conceptually, the n-dimensions of the generic space does not affirm that

$\{\mathcal{G}\}$ possesses n degrees of freedom of irreversibility, but just that $\deg_\tau(\{\mathcal{G}\}) \leq n$.

Remark: Note that the notation $\deg_\tau(\{\mathcal{G}\}^n)$ refers to the degree of irreversibility of the space $\{\mathcal{G}\}^n$.

The Tensor Interpretation and Generic Topological Manifolds

A generic space $\{\mathcal{G}\}^{\mathcal{X}}$ can be thought of as a generic **irreversible** topological manifold $\mathcal{G}^{\mathcal{X}}$ (the notation of the manifold just changes by the exclusion of the curly brackets), since the latter attends to three basic conditions:
(**Remark**: $\mathcal{G}^{\mathcal{X}}$ is a generic manifold of order $\mathcal{X}$ (whose tensors have order $\mathcal{X}$).)

I – $\mathcal{G}^{\mathcal{X}}$ is a **Hausdorff Space**: $\forall \kappa \in \mathcal{G}^{\mathcal{X}}$, neighbor points κ_1 and κ_2 can be separated into two disjoint subsets U^n and V^n, such that $U^n \cap V^n = \varnothing$; $U^n, V^n \subseteq \mathcal{G}^n$

II – $\mathcal{G}^{\mathcal{X}}$ is locally an Euclidean Space: Each point of $\mathcal{G}^{\mathcal{X}}$ has an assigned homeomorphic element at an open subset $\mathbb{R}^n$.

III – $\mathcal{G}^{\mathcal{X}}$ is second-countable.

(The same conditions are true for τ-manifolds)

Condition I ensures each point in $\mathcal{G}^{\mathcal{X}}$ is uniquely defined; condition II allows the subsequent tensor interpretation of the generic variable by setting that locally, $\mathcal{G}^{\mathcal{X}} \cong \mathbb{R}_1^n \times \mathbb{R}_2^n \times \cdots^{[\mathcal{X} \text{ times}]} \cdots \times \mathbb{R}_{\mathcal{X}}{}^n$; condition III reaffirms condition II and condition I by ensuring the existence of a countable basis of sets homomorphic to $\mathbb{R}^n$, such that their union forms all the open subsets of $\mathcal{G}^{\mathcal{X}}$.

Therefore, by condition two, conventionally we can assign a matrix/tensor representation of a generic variable, such that the columns represent the mth generic component; the rows the nth desirable component, such that the called position co-tensor κ_{dc} is defined as follows:

$$\kappa_{dc} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1(m+1)} \\ y_{21} & y_{22} & \cdots & y_{2(m+1)} \\ \vdots & \vdots & \ddots & \vdots \\ y_{(n+1)1} & y_{(n+1)2} & \cdots & y_{(n+1)(m+1)} \end{bmatrix}$$

Where $y_{(n+1)(m+1)}$ denotes **a co-tensor**, therefore, the $(m+1)$th and the $(n+1)$ components are attached to the generic and desirable components, respectively, although they are distributed with exchanged lines and columns.