Analysis of Irreversible Phenomena via τ-Manifold: A Formal τ-Analysis Approach

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Section 0 – Introduction:

Irreversibility is a fundamental topic in both pure and applied sciences, including mathematics, mathematical physics, and complex systems. Despite extensive efforts by leading researchers, traditional methods often rely on approximations or computationally intensive algorithms. Consequently, many irreversible phenomena — such as entropy evolution or complex economic fluctuations — remain only partially understood.
In this preprint I will introduce a novel formalism for treating irreversibility not merely as a unavoidable loss of information, but as a structured domain – a τ manifold, systematically designed to contain and control information dissipation.

To formalize an irreversible process, we introduce a unit τ satisfying:

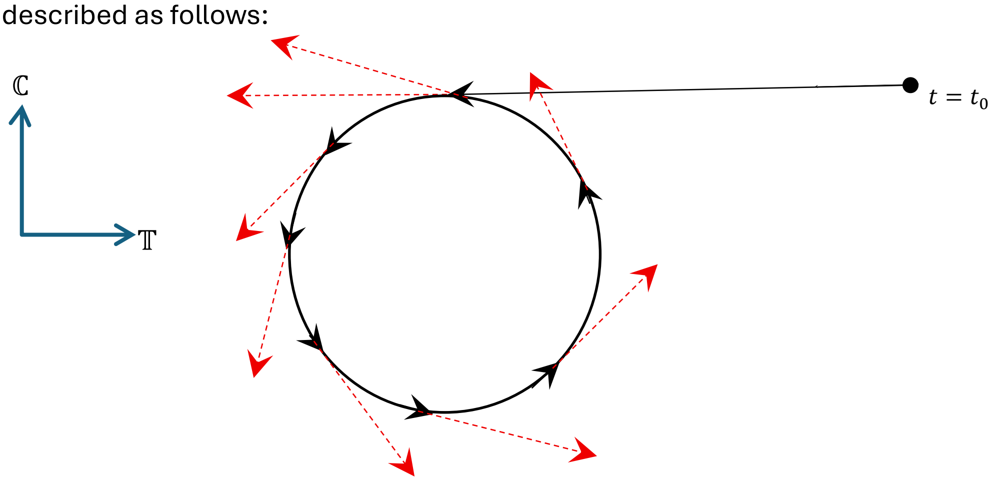
$$\sqrt{\tau} = -1$$

Squaring both sides yields $\tau = 1$, however the original negative sign is lost in the squaring process, indicating an intrinsic loss of information. This property exemplifies the irreversible nature encoded in τ and motivates its use as a foundational unit within the τ-manifold.

Section 1 – The τ-Sphere :

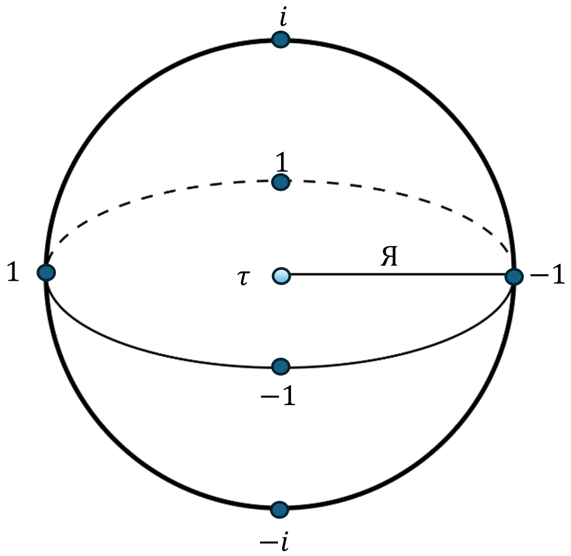
To follow the chronological sequence of the theory, for an advance on mathematical abstraction I shall introduce the idea of time-current. The time is one of the most logical and direct ways to identify irreversibility, in the sight that, a change in some parameter in some point in a given time position, may affect future events chaotically.

The interpretation of time-current is directly connected with the definition of circulation - described as follows:



Once a perturbation at $t_0$is done, the time current takes the path towards a point t; the path walked by the current passes along a circumference, which represents geometrically the τ space. According to the degree of reversibility of the analyzed system, the current scapes by an angle ξ compared with the original route, and such an angle describes the degree of irreversibility of the system. By convention, because the degree of irreversibility is not periodic, the angle ξ is denoted to be hyperbolic, $-\infty < \xi < \infty$, such that, when $\xi = 0$, there is absence of deviation, and when $\xi \to \infty$ the complete irreversibility happens (in hyperbolic coordinates it's similar to affirm orthogonality).

The τ-sphere - S_τ, is a remarkable way to interpret the τ-space: It consists of six poles representing the supremum of S_τ ($\sup(S_\tau)$): (-1,1); (-i, i); (-1,1). The previously seen circulation consists in a sub circumference of S_τ. Inside the sphere the pure τ domain is evident; the center of the sphere is the τ-unit, far from a distance Я of the vertexes (that changes according to the chosen direction).



- *Theorem I: The τ-unit $\{\tau\}$ is a basis for S_τ and $\mathbb{C}$ concomitantly, but not for $\mathbb{R}$.*

Let $\sqrt{\tau} = -1$, then:
1) $\sqrt[4]{\tau} \equiv i \in \mathbb{C}$

2) $\tau^2 \neq 1$, otherwise, information dispersion occurs.

Therefore, rational powers of the τ-unit are represented as complex numbers, but the same is not true for real numbers.

Section 2 – Mathematical Framework:

This section will be entirely dedicated to analyze what these simple properties of S_τ imply in a more systematic mathematical framework.

The current situation requires an introduction of operators; their function in τ-analysis is extremely important and provides a direct and clear interpretation and application using the concepts explored hitherto.

Let Υ: $\mathbb{C} \to \mathbb{T}$ be the called τ-inversion operator, which diverse meanings depending the analysed situations – I shall present some of them in detail further.

For every operator, there must be a function in which the same can be applied:

Remark: We will be using Dirac notation for inner products; bra's $\langle\ |$ and ket's $|\ \rangle$, due to many similarities our approach has with Quantum Mechanics.

Let $\boldsymbol{q} = \boldsymbol{z} + \boldsymbol{\tau w}$; $z, w \in \mathbb{C}$ be a τ-variable – this is, a well defined variable inside S_τ. Then, let $\boldsymbol{f_\tau(q)}$ be a function $f_\tau: \mathbb{T} \to \mathbb{T}$. The following process will be, with caution, to adopt a similar approach to what is done in Complex Analysis and derive precisely the two kinds of τ-functions:

1) **τ-trivial functions**
2) **τ-analytic functions.**

The τ-integration:

- For a well behaved τ-function $f_\tau: \mathbb{T} \to \mathbb{T}$ in which is integrable; differentiable by each one of its components (z, w); continuous under the period of integration we define a τ-integral of f_τ as follows:

$$\oint_{S_\tau} f_\tau(q)dq$$

Where f_τ is identified as a **field**, and S_τ as a **path** in which the field acts.

1) Definition of τ-trivial functions:

- Given $f_\tau(q) = \text{Ꞃ}(z, w) + \tau \text{Ᵽ}(z, w)$:

$$\oint_{S_\tau} f_\tau(q)dq = \oiint \langle \text{Ꞃ}(z, w) + \tau \text{Ᵽ}(z, w) | \boldsymbol{dz} + \tau dw \rangle$$

- By Stokes Theorem we get:

$$= \oiint \langle \text{Ꞃ}(z, w) + \tau \text{Ᵽ}(z, w) | \boldsymbol{dz} + \tau dw \rangle = \oiint (\nabla_q \times f_\tau) \boldsymbol{dz} dw$$

$$= \oiint \frac{\partial \text{Ᵽ}(z,w)}{\partial z} - \frac{\partial \text{Ꞃ}(z,w)}{\partial w} dz dw$$

- τ-trivial functions are those in which $\frac{\partial \text{Ᵽ}(z,w)}{\partial z} = \frac{\partial \text{Ꞃ}(z,w)}{\partial w}$, so then, the τ-integral is zero.

τ-trivial functions led to instantaneous understanding about the rotation the field f_τ enforces – the rotation is zero, therefore the angle ξ has an absence of deviation, then, τ-trivial functions represent the called irreversibility static functions.

1) Definition of τ-analytic functions:

- *Theorem II (Structural reversibility condition): Given* $\Upsilon : \mathbb{C}^1 \rightarrow \mathbb{T}^1$ *is a linear map from* $\mathbb{C}^1$ ***onto*** $\mathbb{T}^1$*, prove that* Υ *is an anti-homomorphism from* $\mathbb{C}$ *to* $\mathbb{T}$ *,and since the eigenvalue* $ю = 1$*, prove that* $\mathbb{C}^1 \cong \mathbb{T}^1$.

; $\Upsilon|\Phi \circledast \varsigma\rangle = \Upsilon|\Phi\rangle \circledast \Upsilon|\varsigma\rangle$, for all $\Phi, \varsigma \in \mathbb{C}$, and some operation $\circledast$ relating them.

- ***Remark: The notation*** $A \cong B$ ***for two sets denotes "A is isomorphic to B"***