Analysis of Irreversible Phenomena via τ-Manifold: A Formal τ-Analysis Approach

Author: Vitor Sbizera Simões

Section 0 – Introduction:

Irreversibility is a fundamental topic in both pure and applied sciences, including mathematics, mathematical physics, and complex systems. Despite extensive efforts by leading researchers, traditional methods often rely on approximations or computationally intensive algorithms. Consequently, many irreversible phenomena — such as entropy evolution or complex economic fluctuations — remain only partially understood.
In this preprint I will introduce a novel formalism for treating irreversibility not merely as a unavoidable loss of information, but as a structured domain – a τ manifold, systematically designed to contain and control information dissipation.

To formalize an irreversible process, we introduce a unit τ satisfying:

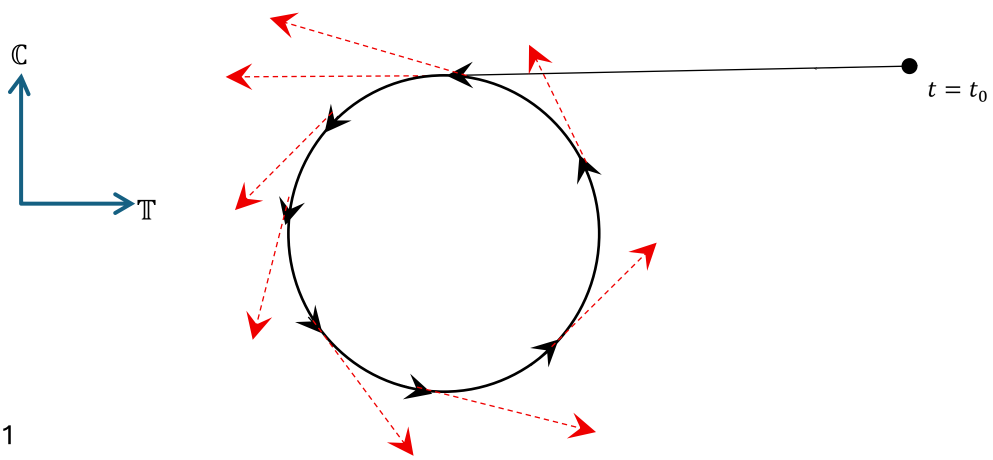
$$\sqrt{\tau} = -1$$

Squaring both sides yields $\tau = 1$, however the original negative sign is lost in the squaring process, indicating an intrinsic loss of information. This property exemplifies the irreversible nature encoded in τ and motivates its use as a foundational unit within the τ-manifold.

Section 1 – The τ-Sphere :

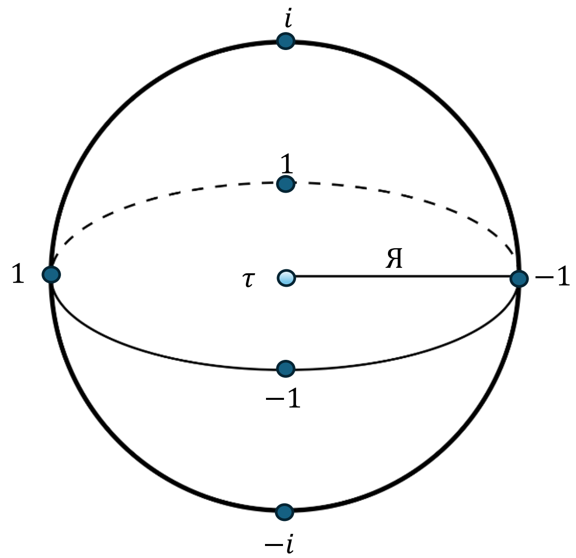
To follow the chronological sequence of the theory, for an advance on mathematical abstraction I shall introduce the idea of time-current. The time is one of the most logical and direct ideals to comprehend irreversibility, in the sight that, a change in some parameter in some point in a given time position, may affect future events chaotically.

The interpretation of time-current is directly connected with the definition of circulation - described as follows:



Once a perturbation at t_0 is done, the time current takes the path towards a point t; the path walked by the current passes along a circumference, which represents geometrically the τ space. According to the degree of reversibility of the analyzed system, the current scapes by an angle ξ compared with the original route, and such an angle describes the degree of irreversibility of the system. By convention, because the degree of irreversibility is not periodic, the angle ξ is denoted to be hyperbolic, $-\infty < \xi < \infty$, such that, when $\xi = 0$, there is absence of deviation, and when $\xi \to \infty$ the complete irreversibility happens (in hyperbolic coordinates it's similar to affirm orthogonality).

The τ-sphere - S_τ, is a remarkable way to interpret the τ-space: It consists of six poles representing the supremum of S_τ $(\sup(S_\tau))$: (-1,1); (-i, i); (-1,1). The previously seen circulation consists in a sub circumference of S_τ. Inside the sphere the pure τ domain is evident; the center of the sphere is the τ-unit, far from a distance Я of the vertexes (that changes according to the chosen direction).



- ***Theorem I:*** *The τ-unit $\{\tau\}$ is a basis for S_τ and $\mathbb{C}$ concomitantly, but not for $\mathbb{R}$.*

Let $\sqrt{\tau} = -1$, then:
1) $\sqrt[4]{\tau} \equiv i \in \mathbb{C}$

2) $\tau^2 \neq 1$, otherwise, information dispersion occurs.

Therefore, rational powers of the τ-unit are represented as complex numbers, but the same is not true for real numbers.

Section 2 – Mathematical Framework:

This section will be entirely dedicated to analyzing what these simple properties of S_τ imply in a more systematic mathematical framework.

The current situation requires an introduction of operators; their function in τ-analysis is extremely important and provides a direct and clear interpretation and application using the concepts explored hitherto.

Let $\Upsilon: \mathbb{C} \to \mathbb{T}$ be the called τ-inversion operator, which diverse meanings depending the analyzed situations – I shall present some of them in detail further.

For every operator, there must be a function in which the same can be applied:

Remark: We will be using Dirac notation for inner products; bra's ⟨ | and ket's | ⟩, due to many similarities our approach has with Quantum Mechanics.

Let $\boldsymbol{q = z + \tau w}$; $z, w \in \mathbb{C}$ be a τ-variable – this is, a well defined variable inside S_τ. Then, let $\boldsymbol{f_\tau(q)}$ be a function $f_\tau: \mathbb{T} \to \mathbb{T}$. The following process will be, with caution, to adopt a similar approach to what is done in Complex Analysis and derive precisely the two kinds of τ-functions:

1) **$\boldsymbol{\tau}$-trivial functions**
2) **$\boldsymbol{\tau}$-analytic functions.**

<u>The $\boldsymbol{\tau}$-integration:</u>

- For a well behaved τ-function $f_\tau: \mathbb{T} \to \mathbb{T}$ in which is integrable; differentiable by each one of its components (z, w); continuous under the period of integration we define a τ-integral of f_τ as follows:

$$\oint_{S_\tau} f_\tau(q)dq$$

Where f_τ is identified as a **field**, and S_τ as a **path** in which the field acts.

1) <u>Definition of $\boldsymbol{\tau}$-trivial functions:</u>

- Given $f_\tau(q) = \text{Ո}(\mathrm{z}, \mathrm{w}) + \tau\text{Ᵽ}(\mathrm{z}, \mathrm{w})$:

$$\oint_{S_\tau} f_\tau(q)dq = \oiint \langle \text{Ո}(\mathrm{z}, \mathrm{w}) + \tau\text{Ᵽ}(\mathrm{z}, \mathrm{w}) | dz + \tau \mathrm{dw} \rangle$$

- By Stokes Theorem we get:

$$= \oiint \langle \text{Ո}(\mathrm{z}, \mathrm{w}) + \tau\text{Ᵽ}(\mathrm{z}, \mathrm{w}) | dz + \tau \mathrm{dw} \rangle = \oiint (\nabla_q \times f_\tau) dzdw$$

$$= \oiint \frac{\partial \text{Ᵽ}(\mathrm{z,w})}{\partial \mathrm{z}} - \frac{\partial \text{Ո}(\mathrm{z,w})}{\partial w} dzdw$$

- τ-trivial functions are those in which $\frac{\partial \text{Ᵽ}(\mathrm{z,w})}{\partial \mathrm{z}} = \frac{\partial \text{Ո}(\mathrm{z,w})}{\partial w}$, so then, the τ-integral is zero.

τ-trivial functions led to instantaneous understanding about the rotation the field f_τ enforces – the rotation is zero, therefore the angle ξ has an absence of deviation, then, τ-trivial functions represent the called irreversibility static functions.

1) <u>Definition of $\boldsymbol{\tau}$-analytic functions:</u>

- Let $q = z + \tau w$; $f_\tau(q) = f_\tau(q(z, w))$, then:

$$\oint_{S_\tau} f_\tau(q)dq \quad = \quad \oint_{S_\tau} f_\tau(q(z, w)dq$$

- Analyzing a specific case – when $w = 0 \Rightarrow q = z \in \mathbb{C}$ – we are led to the Corollary:

- ***Corollary I***: *The case $w = 0$ implies $q = z$, then $f_\tau(q(z,w)) = f_\tau\big(q(z,0)\big) = f_\tau(q(z))$, however, $q = z$, therefore, $\boldsymbol{f_\tau(q(z,0)) \mapsto f(z)}$ – implying the **absence of the influence of $\boldsymbol{S_\tau}$**. Such an absence implies a remarkable simplification to the standard τ-integral:*

$$\oint_{S_\tau} f_\tau(q(z,0))dq \;\mapsto\; \oint_{\Gamma\in\mathbb{C}} f(z)dz$$

- Therefore, such an integral is on the complex domain – which is clearly reversible.
- This concludes that the complex space $\mathbb{C}$ **is a proper subset of the $\boldsymbol{\tau}$-space** $\mathbb{T}$: Given $\mathrm{q} \in \mathbb{T};\ \mathrm{q} = \mathrm{z} + \tau\mathrm{w},\ \mathrm{w} = 0$ implies $q = z$; z is a variable in the complex space.
  $\therefore \mathrm{z} \in \mathbb{C};\ \mathrm{z} = \mathrm{q}(\mathrm{z}, \mathrm{w} = 0) \in \mathbb{T} \Rightarrow \mathbb{C} \subset \mathbb{T} \Rightarrow \mathbb{C}$ is a proper subset of $\mathbb{T}$**.**

Now, the context requires the introduction of the **reversibility operator (ϒ)**: Such an operator takes a function in the complex manifold $f(z)\colon \mathbb{C} \to \mathbb{C}$ and maps the latter in the τ-manifold - $\Upsilon\colon \mathbb{C} \to \mathbb{T}$. Symmetrically, Υ^{-1} is defined as being a mapping of $\mathbb{T}$ **onto** $\mathbb{C}$ – $\Upsilon^{-1}\colon \mathbb{T} \to \mathbb{C}$.

The junction of ϒ and Υ^{-1} forms the operator $Ѫ = \Upsilon\Upsilon^{-1}$. Applying the new definitions to the latter analyzed case – $w = 0$: $\Upsilon\Upsilon^{-1}|f_\tau(q(z,0))\rangle = Ѫ|f_\tau(q(z,0))\rangle = \mathcal{I}|f_\tau(q(z,0))\rangle$; $\mathcal{I}$ is called the identity eigenvalue. In such a case the irreversible component of the τ-variable, w, is zero, so then, $f_\tau(q(z,0)) \mapsto f(z)$, and behaves statically with respect to dissipation of information.

For generalize the operator approach for an arbitrary well-behaved endofunction f_τ is crucial the addition of the variable w – which carries the irreversibility. Then, the influence of the reversibility operator under a potentially irreversible τ-function requires a **scale of measurement of the dissipation** – an eigenvalue labeled ю, such that:

- $Ѫ|f_\tau(q(z,w))\rangle = \text{ю}|f_\tau(q(z,w))\rangle$

Such a relation is well known in linear algebra – the eigenvalue relation. τ-functions that respect the eigenvalue relation (those which are eigenfunctions of ю) are said to be **$\boldsymbol{\tau}$-analytic**.

- The reversibility operators can be recognized as homomorphisms betwixt $\mathbb{C}$ and $\mathbb{T}$; and this relation, assuming that ϒ consists in a linear map is explored in Theorem *II:*

- ***Theorem II (Structural reversibility condition)***: *Let* $\Upsilon: \mathbb{C}^1 \to \mathbb{T}^1$ *a mapping from* $\mathbb{C}^1$ **onto** $\mathbb{T}^1$ *- then* Υ *is an anti-homomorphism from* $\mathbb{C}$ *to* $\mathbb{T}$ *and* Υ^{-1} *is a homomorphism from* $\mathbb{T}$ *to* $\mathbb{C}$*, and, since the eigenvalue* $\text{ю} = 1$*, then* $\mathbb{C}^1 \cong \mathbb{T}^1$.
    - ***Remark: The notation*** $A \cong B$ ***for two sets denotes "A is isomorphic to B"***

Let $f_\tau(\mathrm{q}): \mathbb{C}^1 \to \mathbb{C}^1; g_\tau: \mathbb{C}^1 \to \mathbb{C}^1 \;\Rightarrow\; \Upsilon^{-1}|f_\tau(\mathrm{q}(\mathrm{z},\mathrm{w}))\rangle = \mathrm{f}(\mathrm{z});\; \Upsilon^{-1}|g_\tau(\mathrm{q}(\mathrm{z},\mathrm{w}))\rangle = g(\mathrm{z}) \Rightarrow$

$Ѫ\left|\begin{matrix} f_\tau \\ g_\tau \end{matrix}\right\rangle = \text{ю}\left|\begin{matrix} f_\tau \\ g_\tau \end{matrix}\right\rangle$. The bijectivity of the operator is defined exclusively when $\text{ю} = 1 \;\Rightarrow\; Ѫ\left|\begin{matrix} f_\tau \\ g_\tau \end{matrix}\right\rangle = \left|\begin{matrix} f_\tau \\ g_\tau \end{matrix}\right\rangle \Rightarrow Ѫ = \hat{\mathrm{I}}$, otherwise:
$Ѫ = \Upsilon\Upsilon^{-1}: \mathbb{T}^1 \to \mathbb{C}^1 \to \mathbb{T}^{*1}$, i.e., $Ѫ: \mathbb{T}^1 \to \mathbb{T}^{*1}$, unless $\text{ю} = 1$ – the unique case in which $\mathbb{T}^1 \cong \mathbb{T}^{*1}$, that proves the statement two.

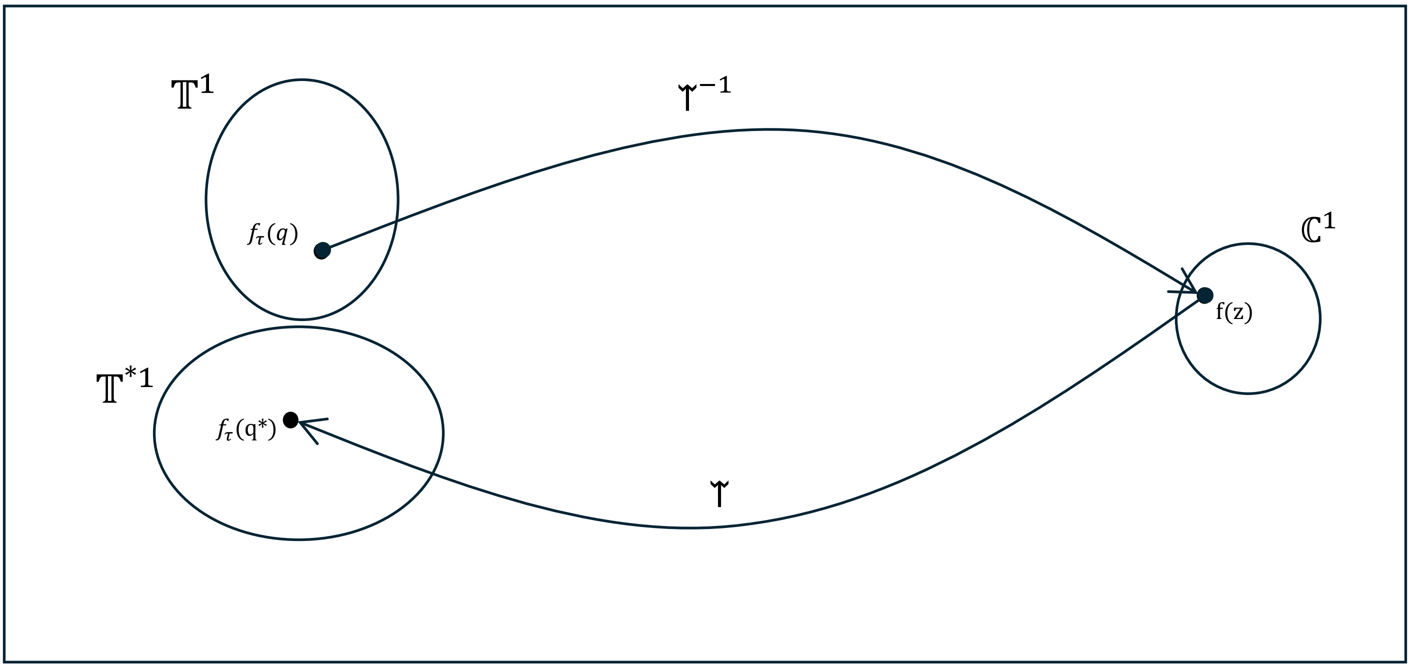
; Since Υ returns to a different set of the which Υ^{-1} has departed; and as *Corollary I* affirms that $\mathbb{C}$ is a proper subset of $\mathbb{T}$, then:

1) $\Upsilon^{-1}: \mathbb{T}^1 \overset{\Upsilon^{-1}}{\mapsto\!\!\!\!\twoheadrightarrow} \mathbb{C}^1$ – (Υ^{-1} is a homomorphism from $\mathbb{T}^1$ to $\mathbb{C}^1$)

2) $\Upsilon: \mathbb{C}^1 \overset{\Upsilon}{\mapsto\!\!\!\!\twoheadrightarrow} \mathbb{T}^{*1}$ – (Υ is an anti-homomorphism from $\mathbb{C}^1$ to $\mathbb{T}^{*1}$)

Which proves statement one, and completes the proof ■

- Below is a schematic representation of *Theorem II*:



- ***Remark:*** $\boldsymbol{f_\tau(q)}$ ***is an endofunction in*** $\mathbb{T}^1$***, while*** $\boldsymbol{f_\tau(q\,*)}$ ***is an endofunction in*** $\mathbb{T}^{*1}$.

Section 3 – Degrees of Freedom of Irreversibility & Multiple τ-Dimensions:

A diversity of observable phenomena – the majority of them, have a complex system of dependencies of variables – and therefore a more entangled chain of irreversible factors; that when blended, transforms a predictable system into pseudo-chaotic or even chaotic systems.

The main goal of this section is to develop a higher understanding of pseudo-chaotic systems, and how the latter can be broken out in isolated and simplest systems, that yet connect with each other.

Pseudo-chaotic systems are likewise called due to the false judgement and assumption of chaoticity, whereas in reality, they are predictable. Such a phenomena is observed when an anti-homomorphism is applied successive times in a initially well-behaved pattern.

The anti-homomorphism and the codification of irreversibility:

Definition I: Let Ͳ; $Ͳ^{-1}$ represent an anti-homomorphism and homomorphism, respectively (as proved in *Theorem II*); $Ͳ: \mathbb{C}^1 \mapsto\!\!\!\!\!\twoheadrightarrow \mathbb{T}^1; Ͳ^{-1}: \mathbb{T}^1 \mapsto\!\!\!\!\!\twoheadrightarrow \mathbb{C}^1$; $Ѫ = ͲͲ^{-1}$ and a τ-function $f_\tau(q) = \text{Ⴖ}(z,w) + \tau Ᵽ(\text{z}, \text{w})$. To analyze the nth degree of freedom of irreversibility of $f_\tau(q)$ there is a necessity of extend the -space from $\mathbb{T}^1$ to $\mathbb{T}^{\text{n}}$. Then, each component of f_τ takes the following form:

$$Ᵽ(\text{z}, \text{w}) = \sum_{\sigma=1}^{n} щ_\sigma(z,w) |\, e_\sigma\rangle$$

$$\text{Ⴖ}(z,w) = \sum_{j=1}^{n} \mathfrak{h}_j(z,w) |\, e_j\rangle$$

; the previously defined eigenvalue relation takes the form:

$$\begin{pmatrix} Ѫ_1 & 0 & \cdots & 0 & 0 \\ 0 & Ѫ_2 & \cdots & 0 & 0 \\ 0 & 0 & Ѫ_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & Ѫ_\text{n} \end{pmatrix} \left| \begin{matrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_n \end{matrix} \right\rangle = Ю \left| \begin{matrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_n \end{matrix} \right\rangle$$

Such a definition led us to the comprehension of why the *Theorem II* limited the degree of freedom of Ͳ to $n = 1$ – for $n > 1$ is necessary to analyze each eigenvalue component by itself, with respect to its respective $Ѫ_\text{n}$