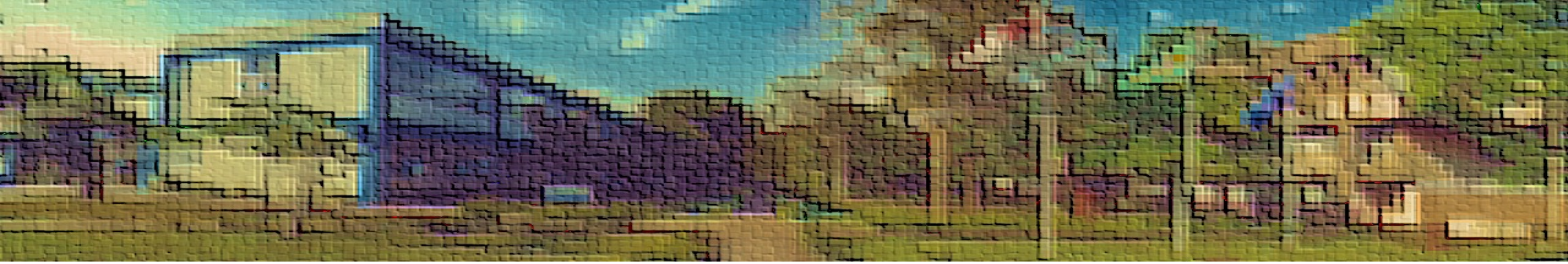

MAC317
Introdução ao Processamento de Sinais Digitais

Prof. Marcel P. Jackowski

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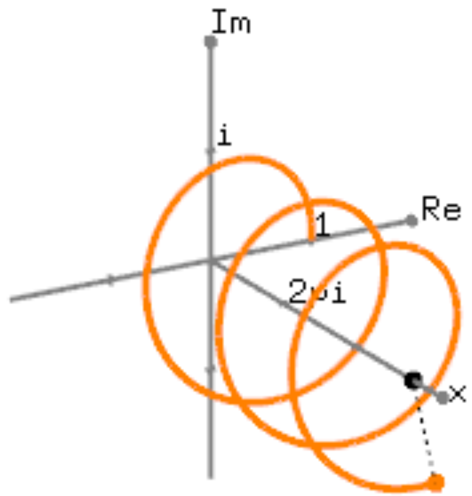
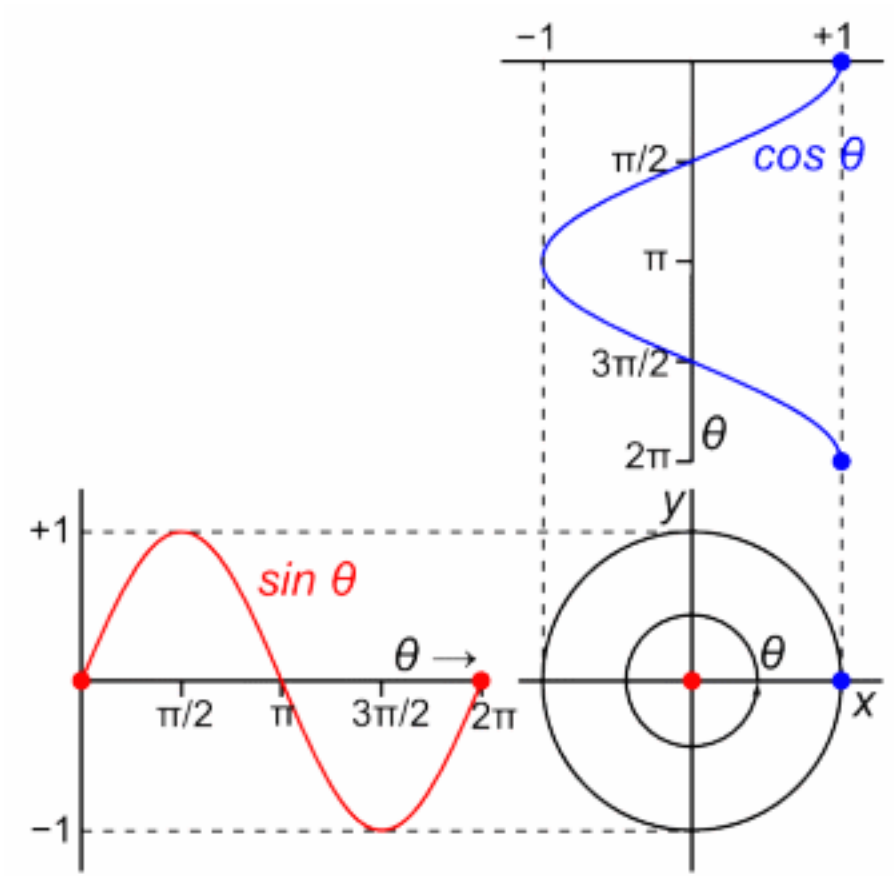
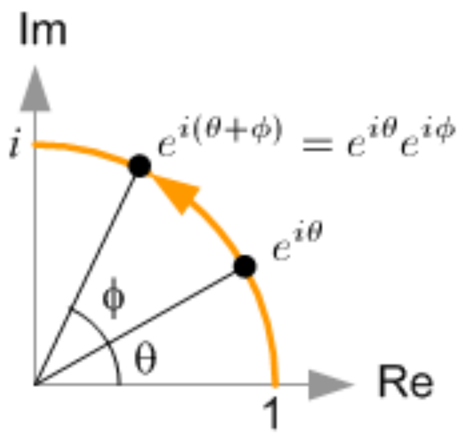
Aula #4: Amostragem e aliasing

Fórmula de Euler

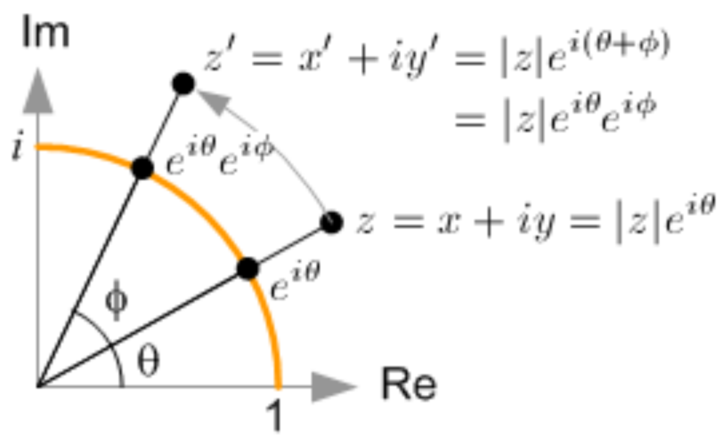




Rotações em 2D

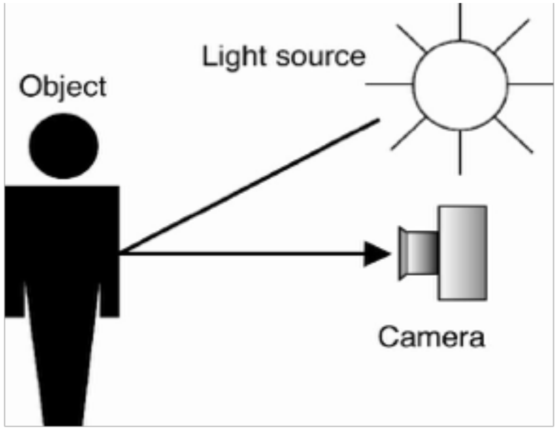


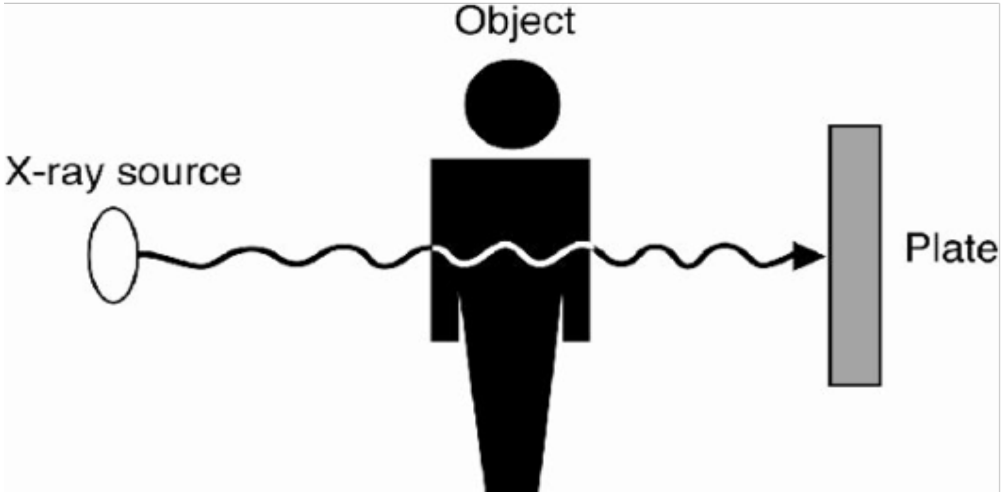
$$
\begin{aligned}
z' &= |z'|e^{i(\theta+\phi)} \\
&= |z|e^{i(\theta+\phi)} \qquad (\because |z'| = |z|) \\
&= |z|e^{i\theta}e^{i\phi} \\
&= (x+iy)e^{i\phi} \qquad (\because z = |z|e^{i\theta} = x+iy) \\
&= (x+iy)(\cos\phi + i\sin\phi) \\
&= \cos\phi \cdot x - \sin\phi \cdot y + i(\sin\phi \cdot x + \cos\phi \cdot y)
\end{aligned}
$$

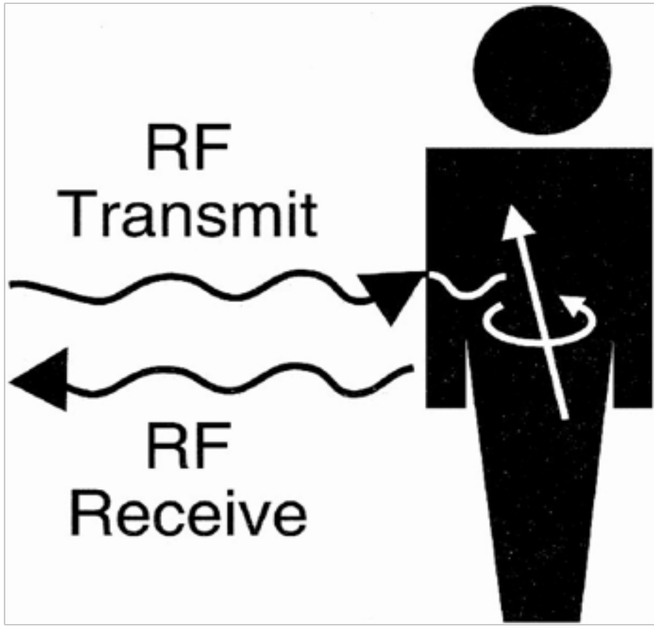


$$
\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}
$$

Ressonância magnética



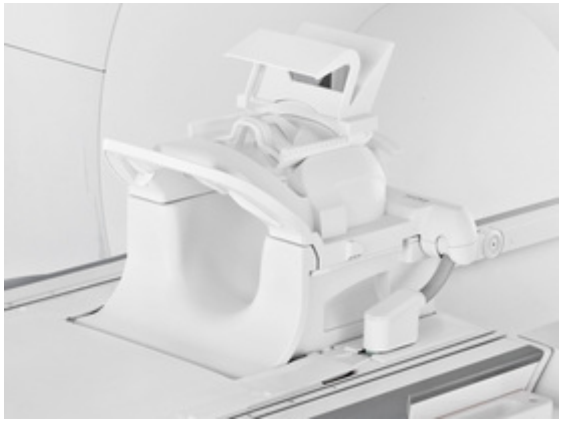




Philips 3T

Philips 3T

Diferentes tipos de bobinas



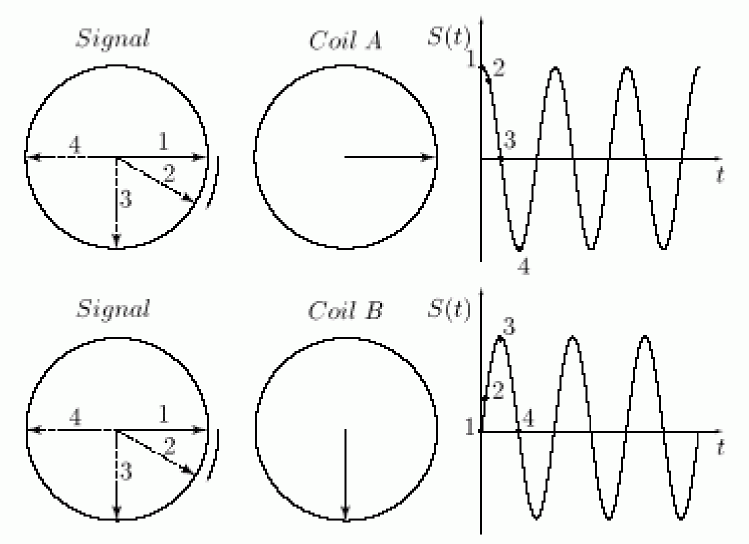
Representação do sinal

To improve SNR, we use two coils, one aligned with the *x*-axis and one aligned with the *y*-axis (*quadrature* scheme)

- the detected signal can then be represented as follows:

$$s_x(t) = Ae^{-\frac{t}{T_2}} \cos(-\omega_0 t)$$

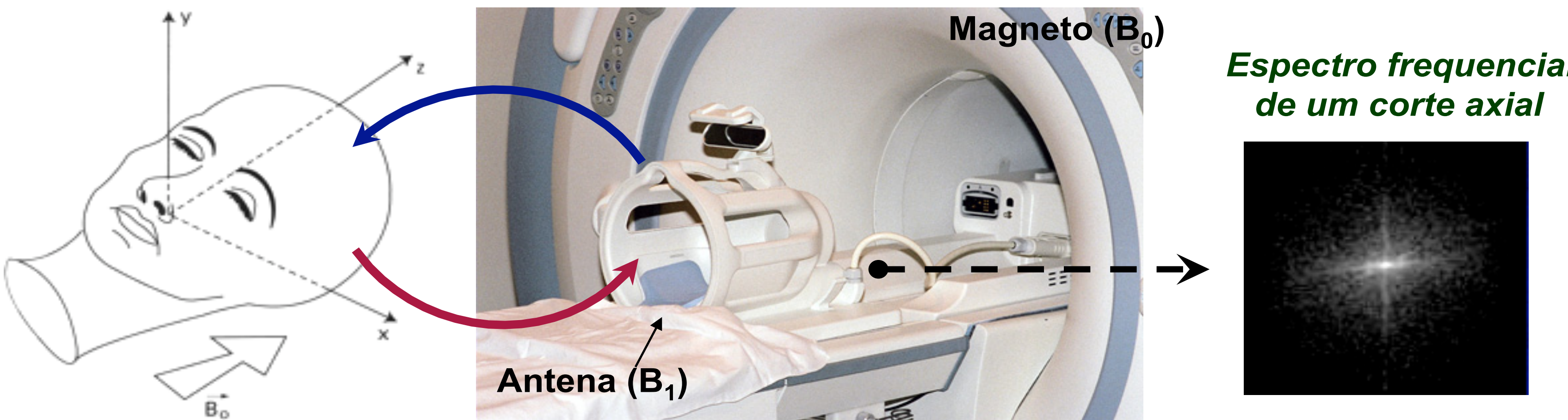
$$s_y(t) = Ae^{-\frac{t}{T_2}} \sin(-\omega_0 t)$$



- thus, coil x gives the real part and coil y the imaginary part of a complex-valued signal:

$$s(t) = Ae^{-\frac{t}{T_2}} e^{-i\omega_0 t}$$

Aquisição de imagens



Aquisição Axial

XY: Plano Axial

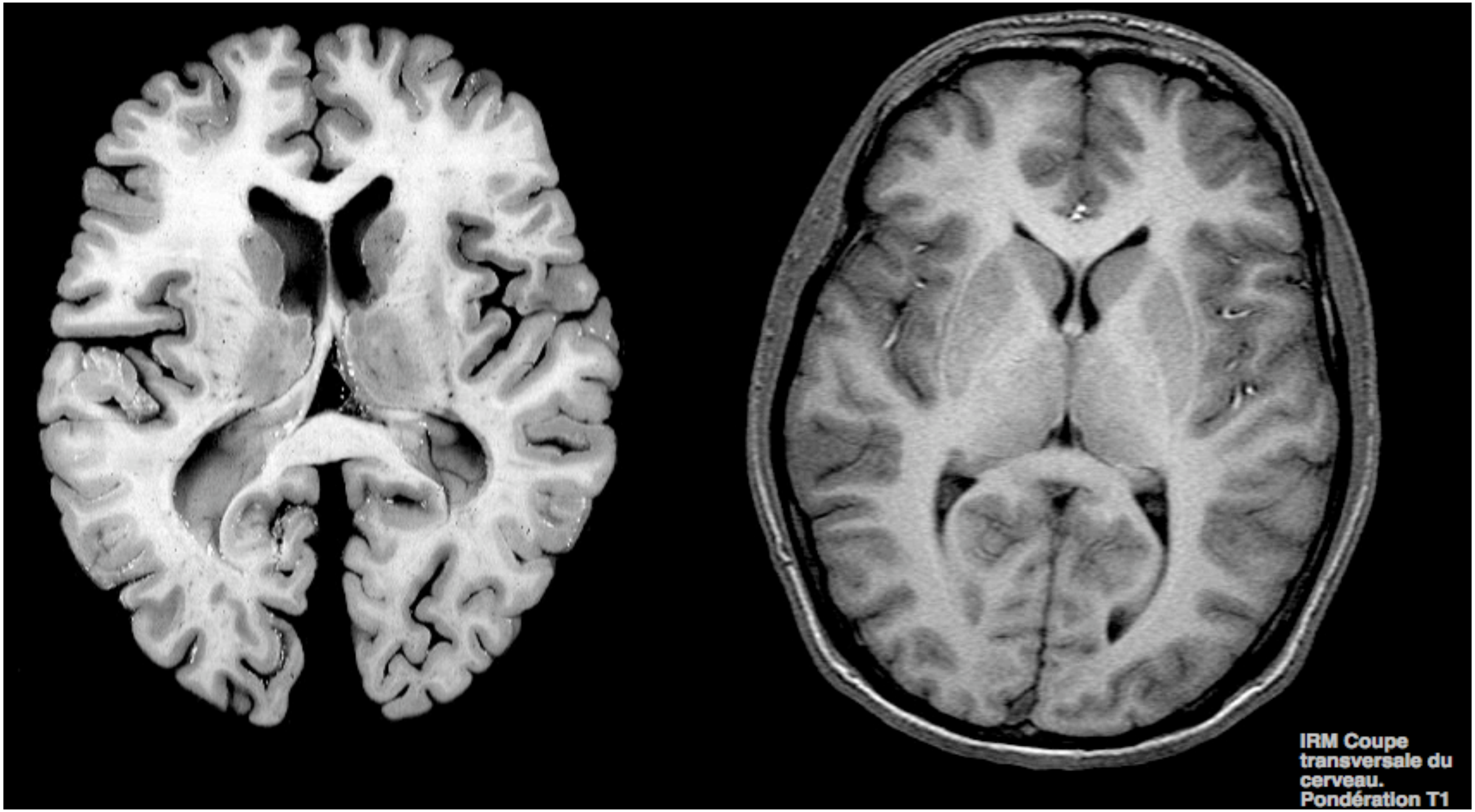
YZ: Plano Sagittal

XZ: Plano Coronal

Excitação Eletromagnética B_1

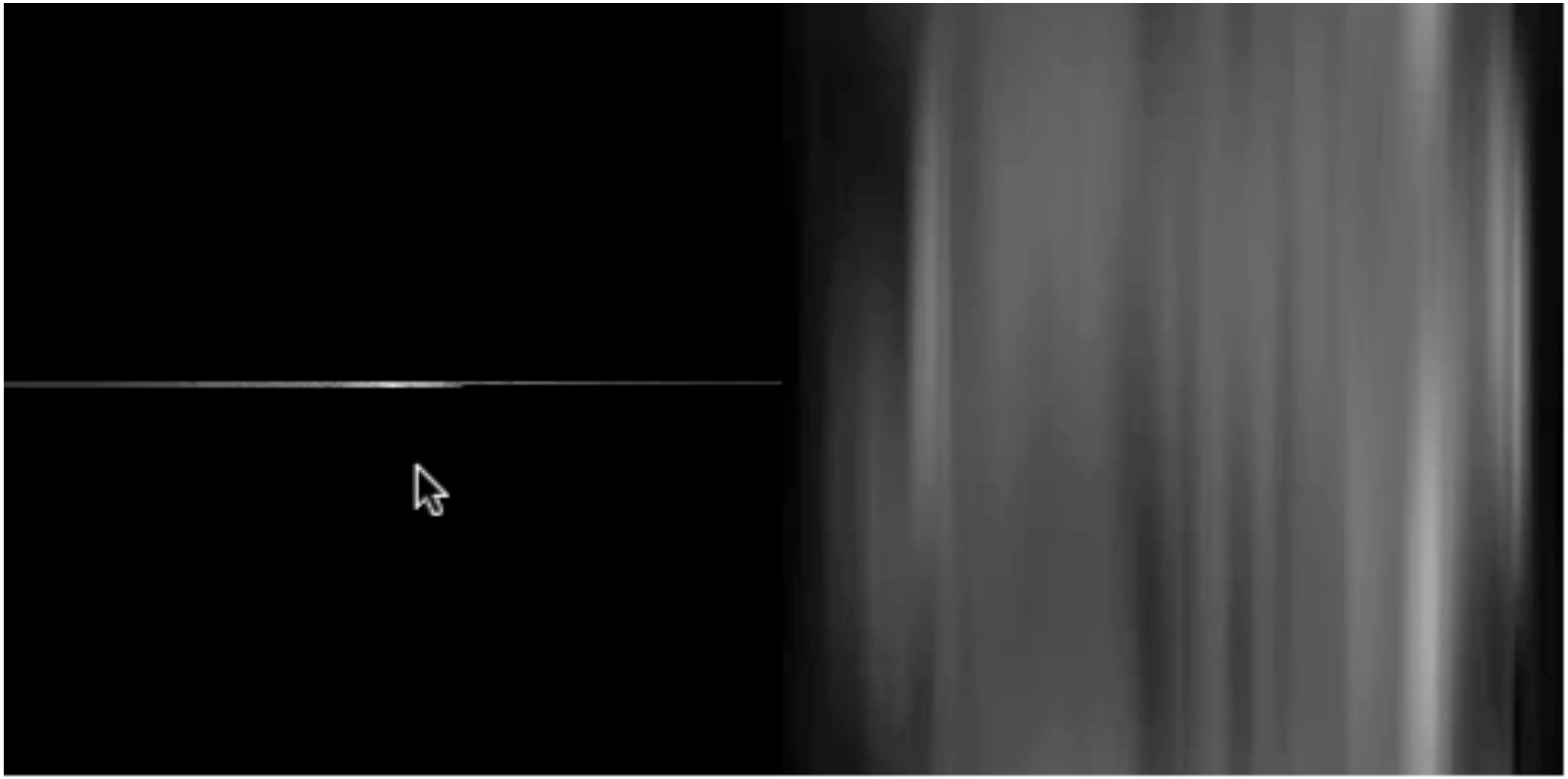
Recepção do Sinal

Exemplo

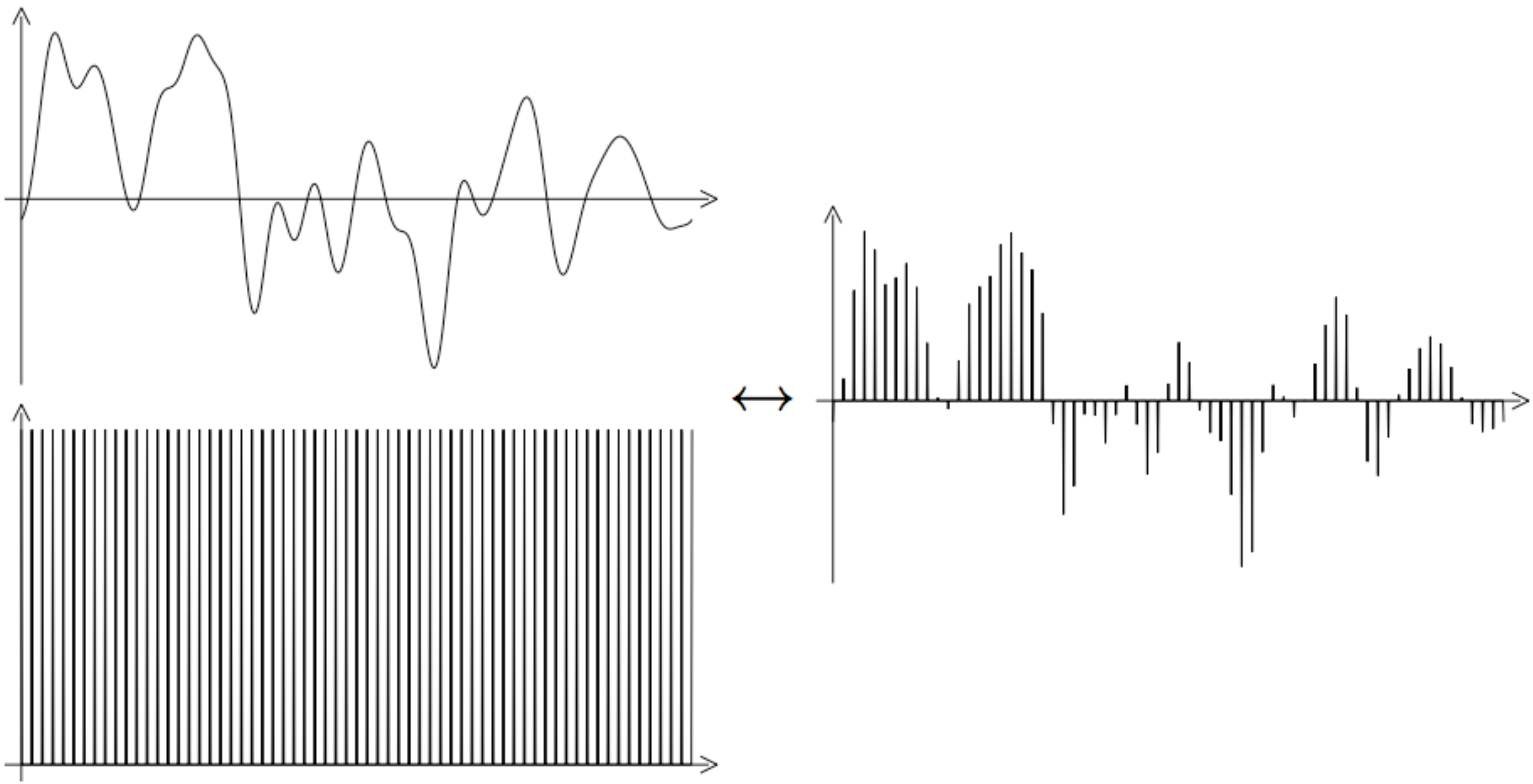


Reconstrução

Reconstrução (ii)

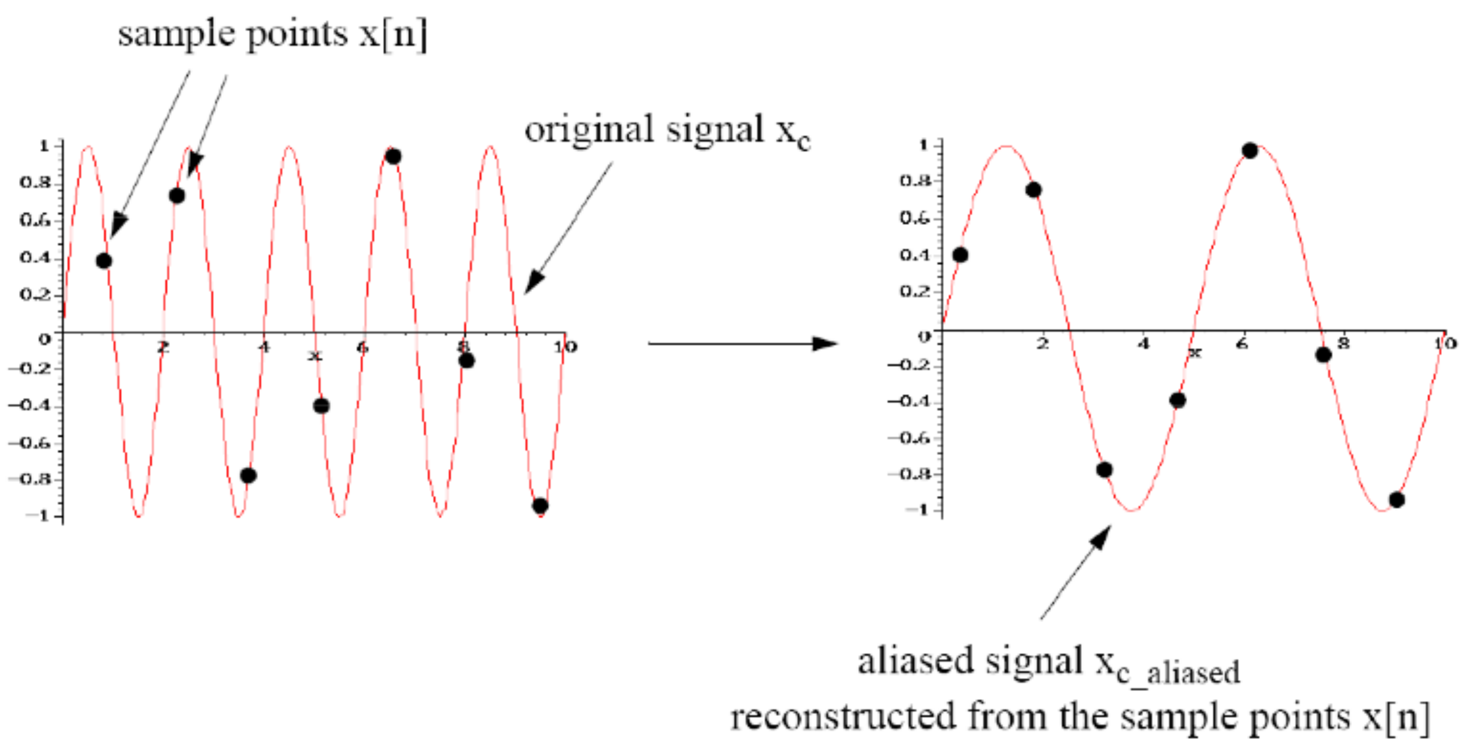


Amostragem



Exemplo de amostragem

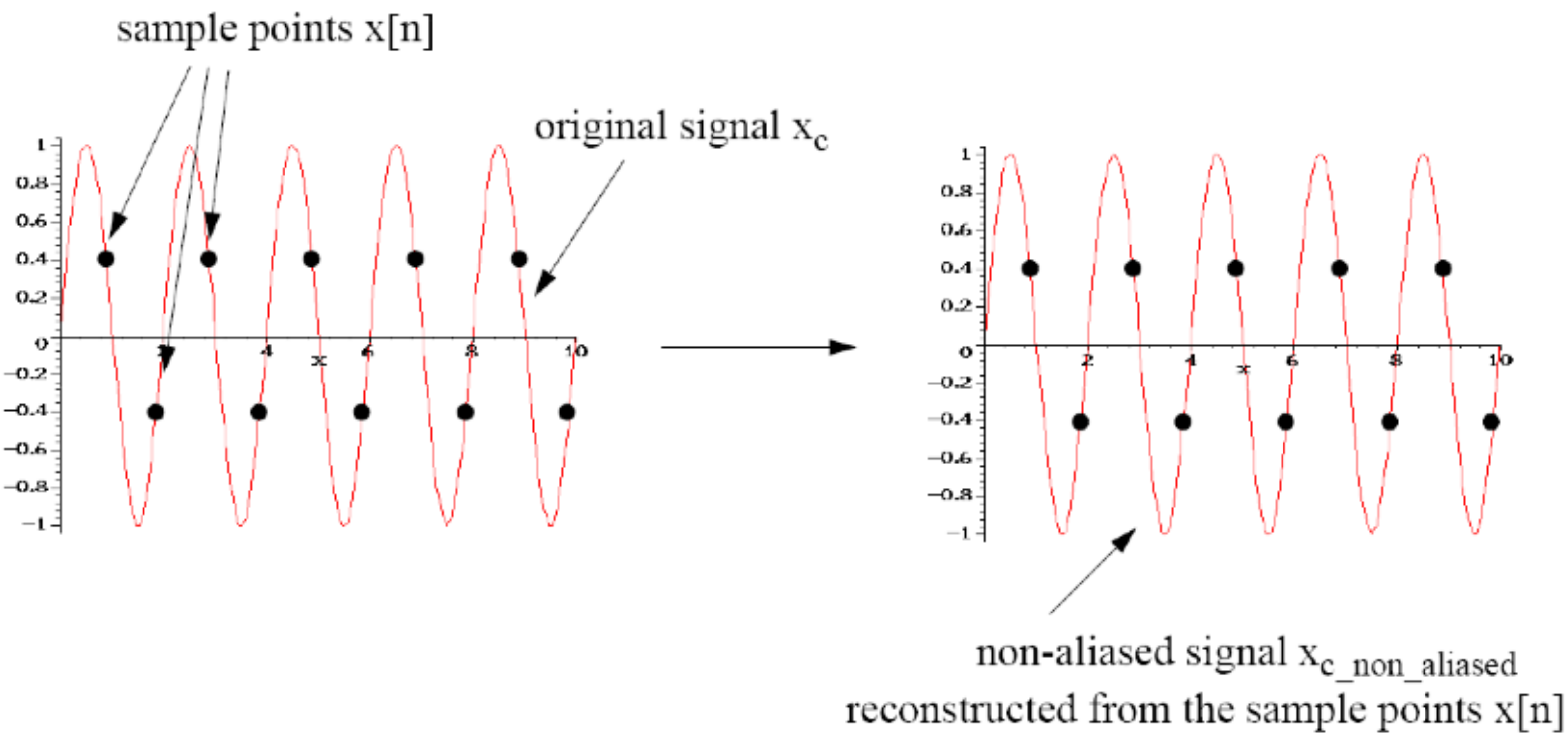
- Frequency of original signal: 0.5 (oscillations per time unit)
- Sampling frequency: 0.7 (samples per time unit)
- Looking at the sample points x[n], they appear to originate from a sine wave $x_{c\_aliased}$ of much lower frequency → again, the original sine wave is lost and can not be recovered



Exemplo de aliasing

Exemplo

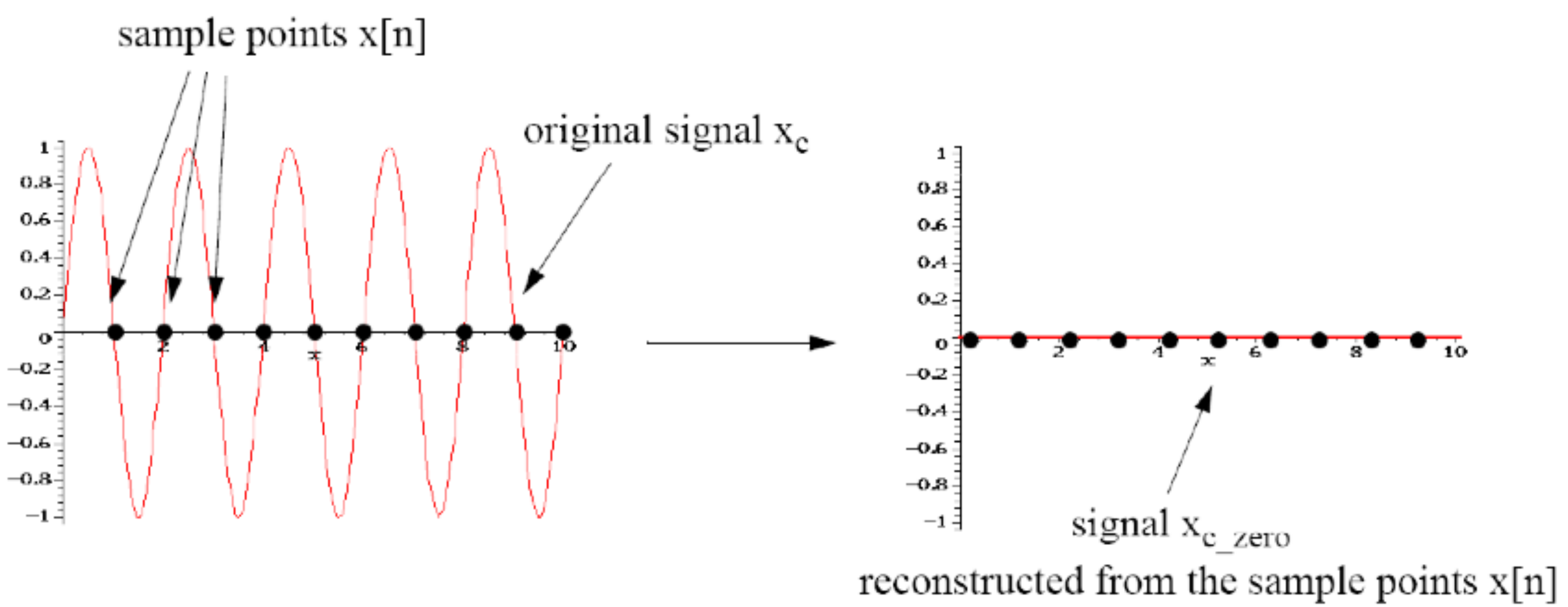
- Frequency of original signal: 0.5 (oscillations per time unit)
- Sampling frequency: 1.0 (sample per time unit) → original signal can be recovered
- We learn that we need to sample each oscillation period twice for good reconstruction



Exemplo

- In practice, it is best to use more than 2 samples per oscillation period

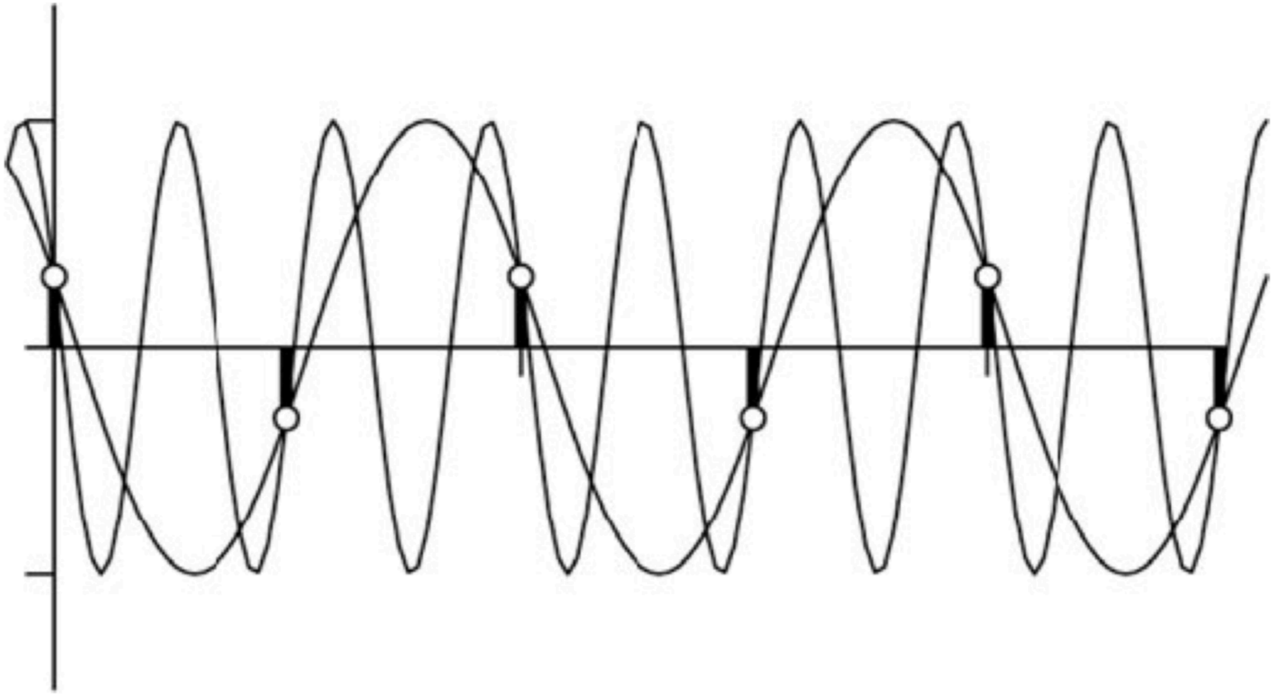
    - else one may get wrong reconstructions for some special sample alignments



- Thus, to be on the safe side:

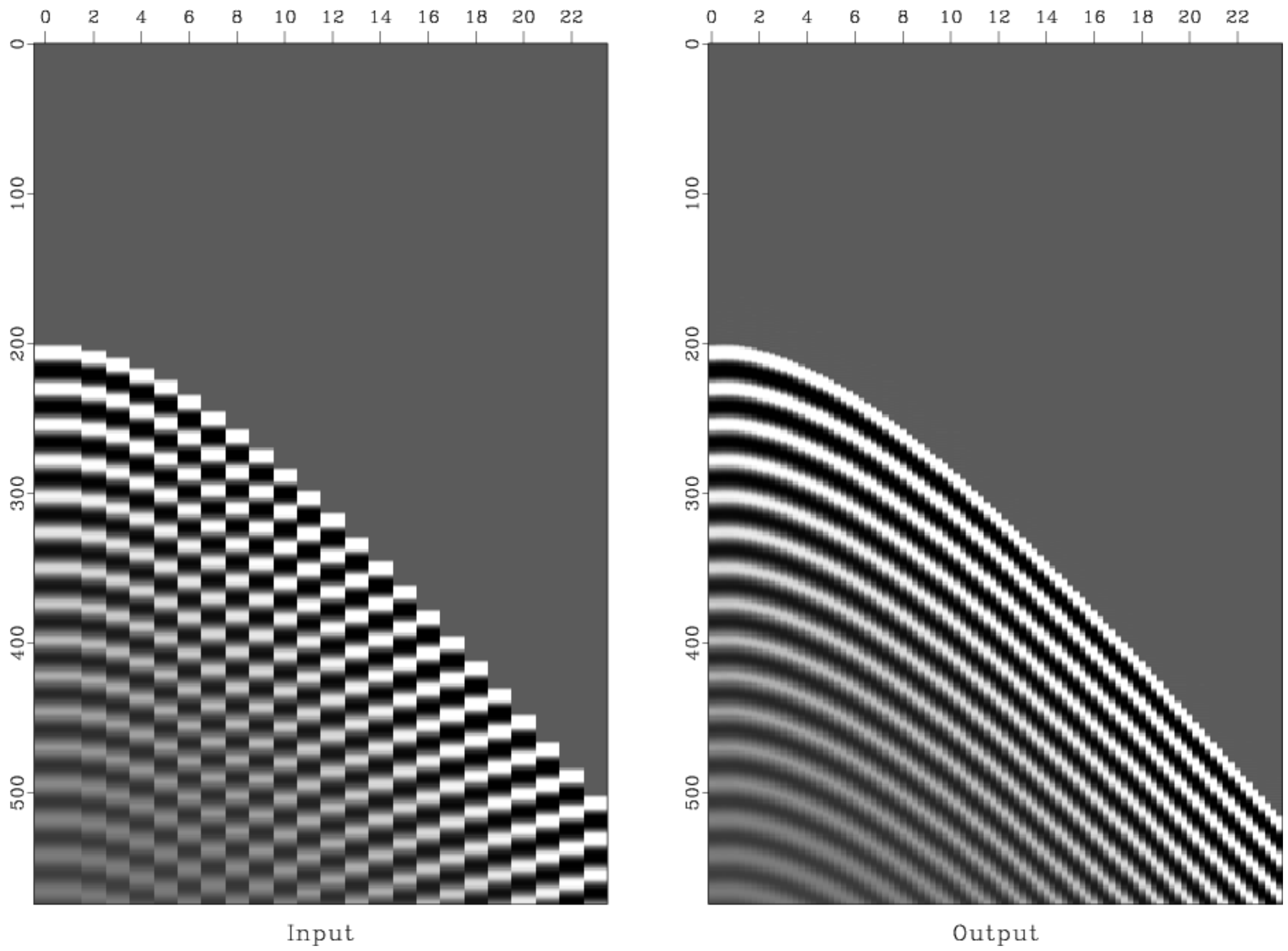
    - sample each oscillation period more than twice

Aliases



These two sine waves are indistinguishable
Indistinguishable frequencies are called "aliases"

Aliasing


0 2 4 6 8 10 12 14 16 18 20 22
0
100
200
300
400
500
Input
0 2 4 6 8 10 12 14 16 18 20 22
0
100
200
300
400
500
Output

Critério de Nyquist

Definition: The Nyquist frequency is ½ the sampling frequency (1/Ts)

Frequencies above the Nyquist frequency appear as aliases

No aliases appear if the function being sampled has no frequencies above the Nyquist frequency

Antialiasing

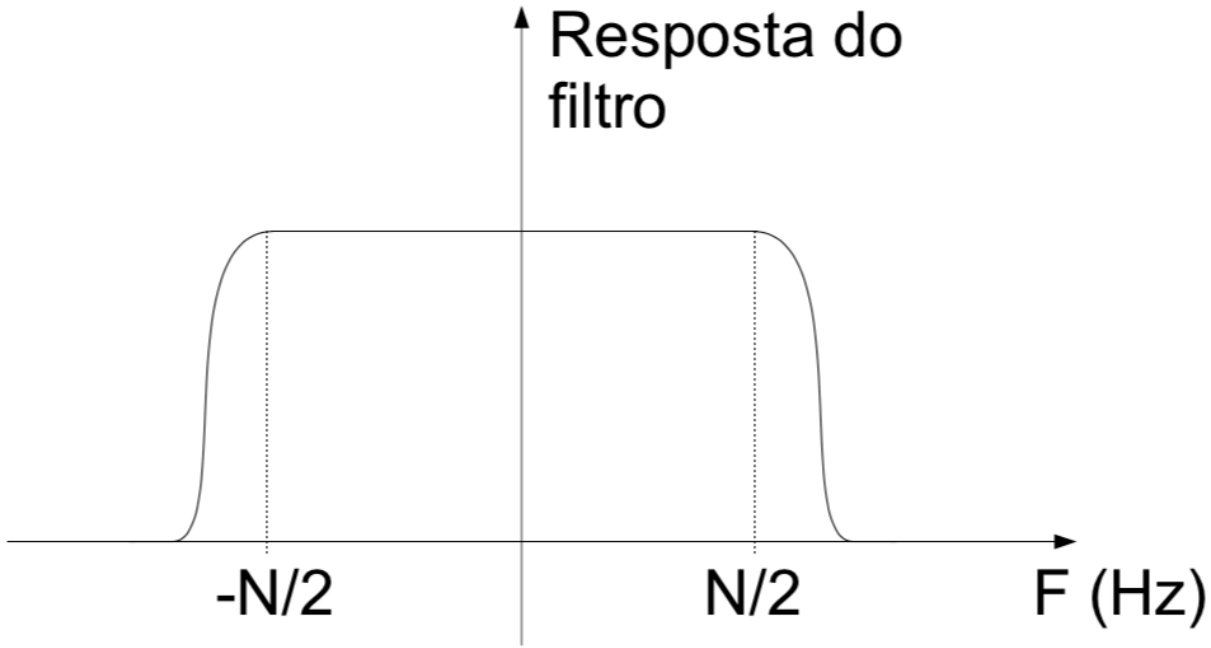
Simple idea:

Remove frequencies above the Nyquist frequency before sampling

How? Filtering before sampling

Filtragem

Na conversão analógico → digital é necessário garantir que a entrada contenha apenas frequências representáveis, o que é feito por um filtro passa-baixa com frequência de corte $\frac{N}{2}$ Hz:



Tarefa em sala

- Dado o sinal analógico:
  - $f(x, y) = 256 \sin(2\pi(50x + 70y))$
- Quais são as frequências utilizadas ?
- Representar na forma de Euler
- Visualize o sinal usando scipy (matplotlib)
- Agora faça a amostragem com:
  - $60{\times}60$, $100{\times}100$, $300{\times}300$ e $1000{\times}1000$ amostras
  - Mostre o resultado com imagens em tons de cinza
- Quantas amostras são necessárias para reconstruir o sinal analógico ?

Tarefa de casa

- Leitura do livro texto até a seção 1.7