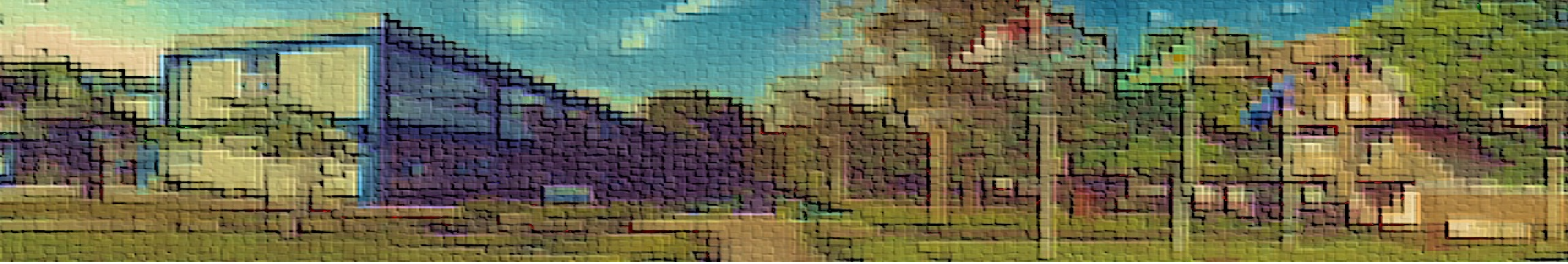

MAC317
Introdução ao Processamento de Sinais Digitais

Prof. Marcel P. Jackowski

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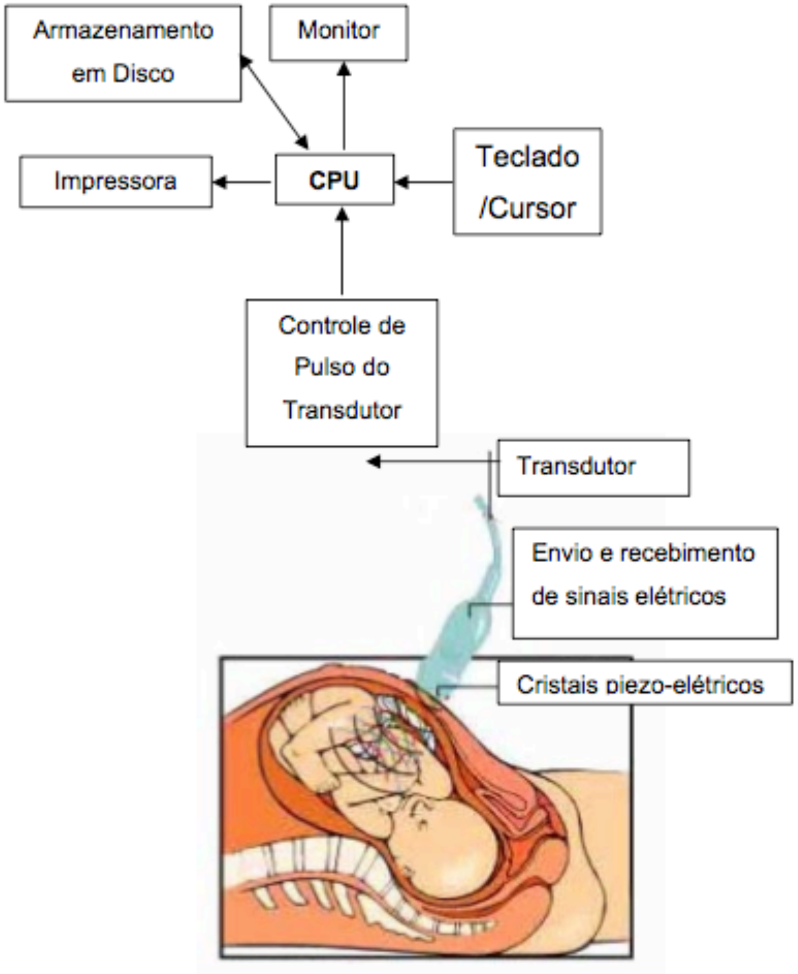
Aula #2: Sinais

Aplicação: ultrassonografia

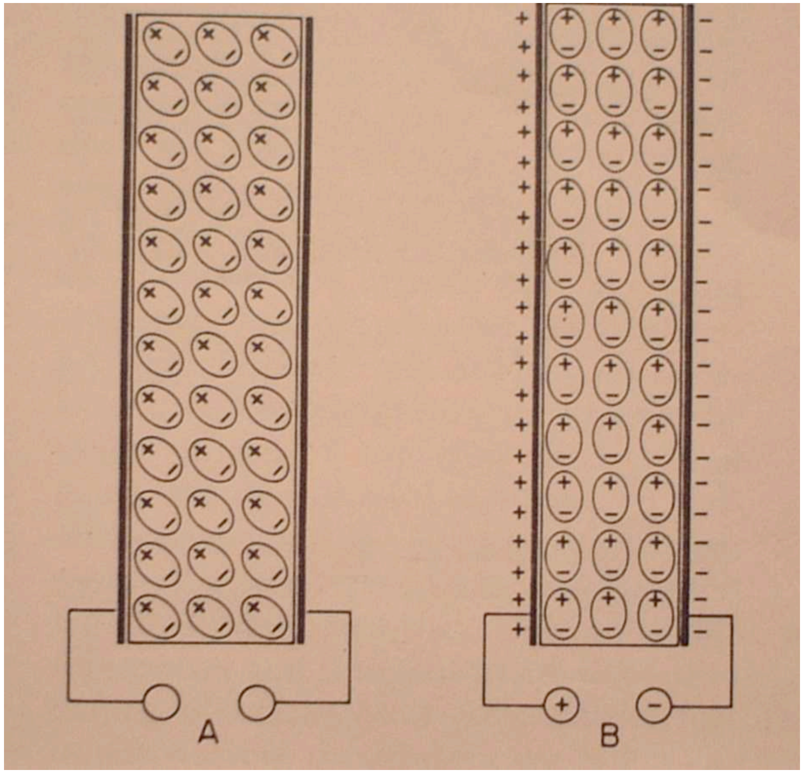


Geração de ondas acústicas

- As ondas de ultrassom são geradas devido ao efeito piezoelétrico
  - Consiste na variação das dimensões físicas de certos materiais quando sujeitos a campos elétricos
  - O contrário também ocorre, ou seja, a aplicação de pressões
- Por exemplo, pressões acústicas que causam variações nas dimensões de materiais piezoelétricos provocam o aparecimento de campos elétricos

Efeito piezoelétrico

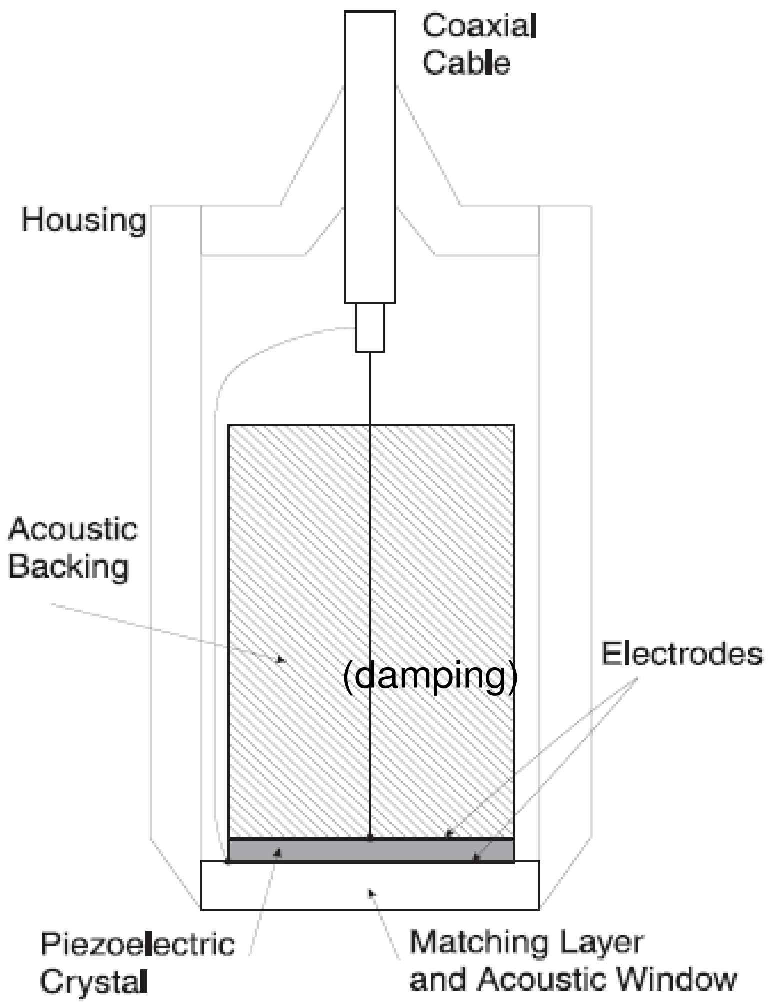
- O efeito piezoelétrico foi descoberto por Pierre e Jacques Curie em 1880 e consiste na variação das dimensões físicas de certos materiais sujeitos a campos elétricos
- O quartzo e a turmalina, cristais naturais, são piezoelétricos
- Tensão alternada produz oscilações nas dimensões do cristal.

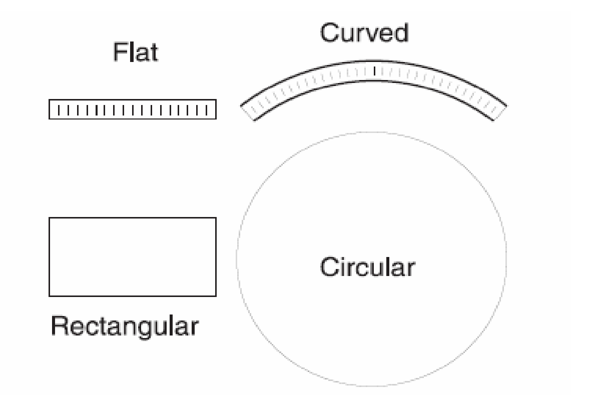


Transdutores

- Pulsos de alta frequência são produzidos por um gerador de pulso e enviados ao transdutor por um transmissor
- O sinal elétrico faz com que o cristal momentaneamente mude de tamanho, aumentando e diminuindo a pressão na frente do transdutor, produzindo as ondas de ultrasom
- Os ecos ultrassônicos que retornam são convertidos novamente em sinais elétricos utilizando o mesmo ou outro transdutor

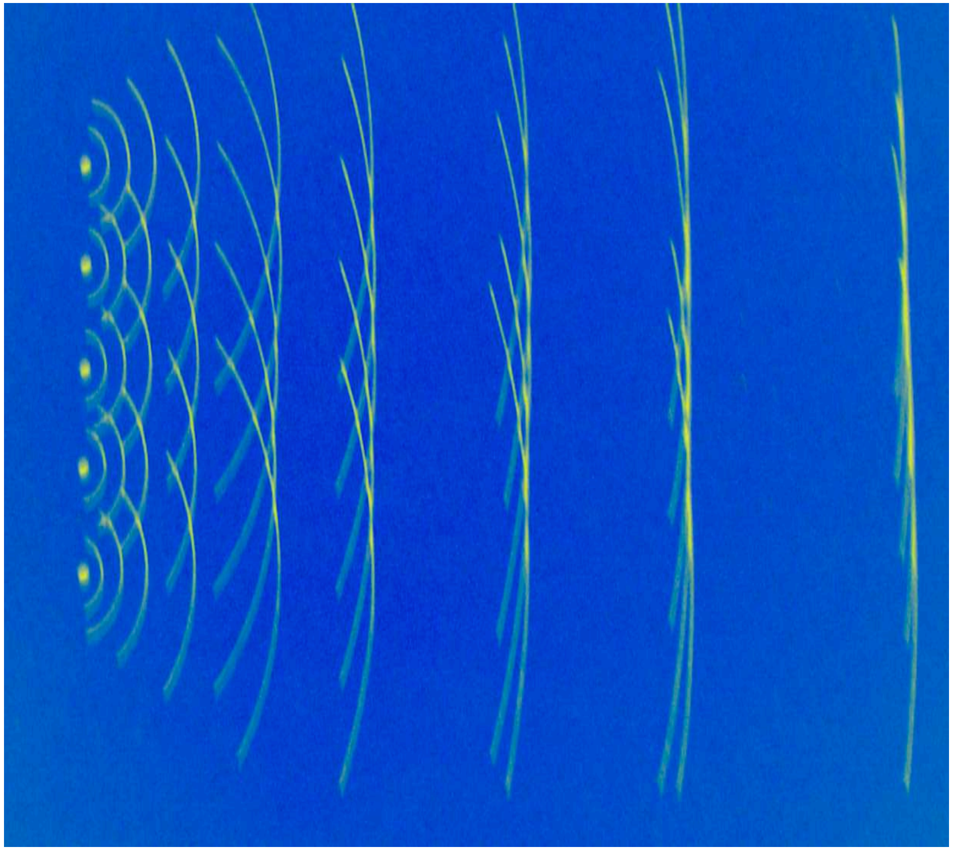
Transdutor de cristal único





Características de um feixe de ultrassom

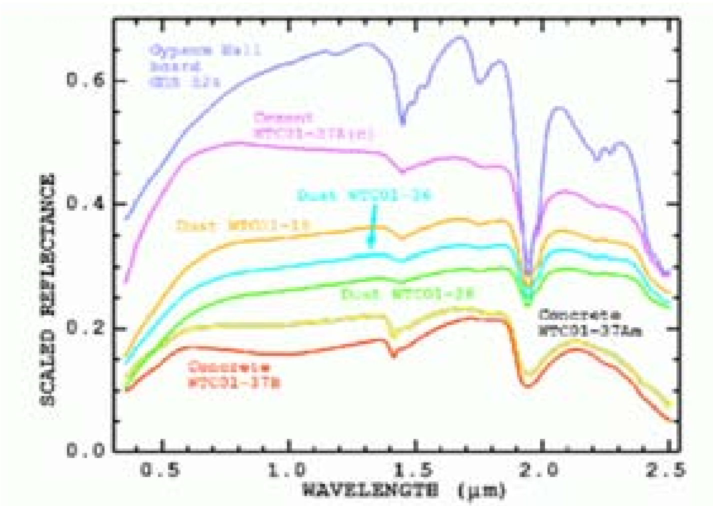
- Exemplo:
  - Transdutor de 5 pontos
- Frentes da onda são paralelas à superfície do transdutor
- A frente navega por uma certa distância até se dispersar ou ser refletida



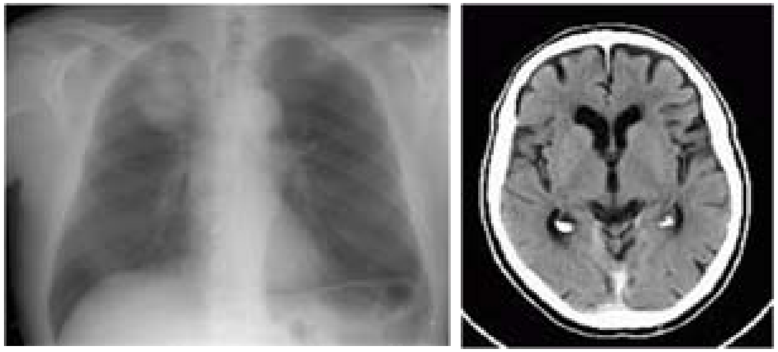
Sinais e sistemas

- Sinais são funções matemáticas de uma ou mais variáveis independentes, capazes de representar uma variedade de processos físicos
- Sistemas são entidades que respondem aos sinais e por sua vez, produzem novos sinais
  - Exemplo: instrumentos médicos realizam mensurações físicas (i.e. adquirem sinais) e os transformam em imagens (i.e. geram novos sinais).

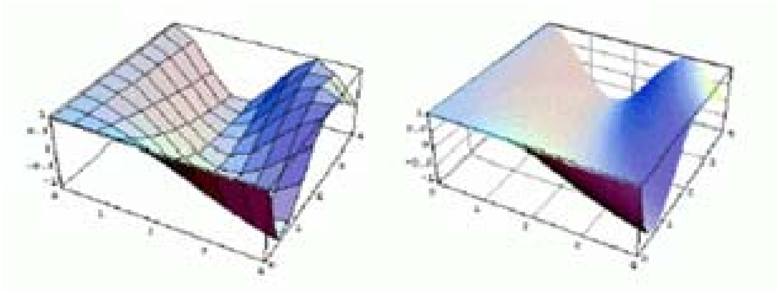
Dimensionalidade



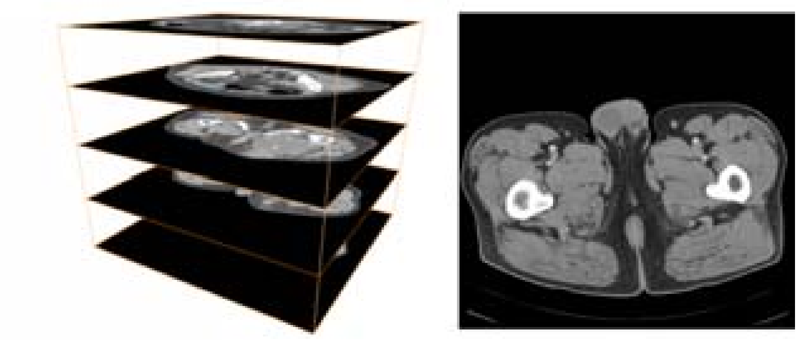
1D signal $f(x)$



2D signal $f(x, y)$



2D signal, shown as height field



3D signal $f(x, y, z)$

4D signal $f(x, y, z, t\text{=time})$
example: 3D heart in motion

Representação

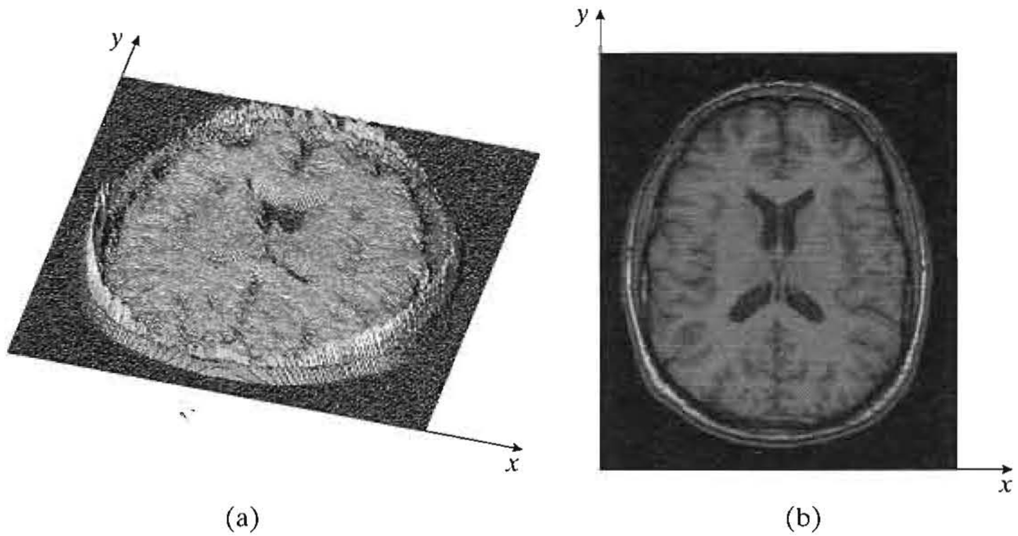


Figure 2.1
Two alternative signal visualizations: (a) a functional plot and (b) an image display.

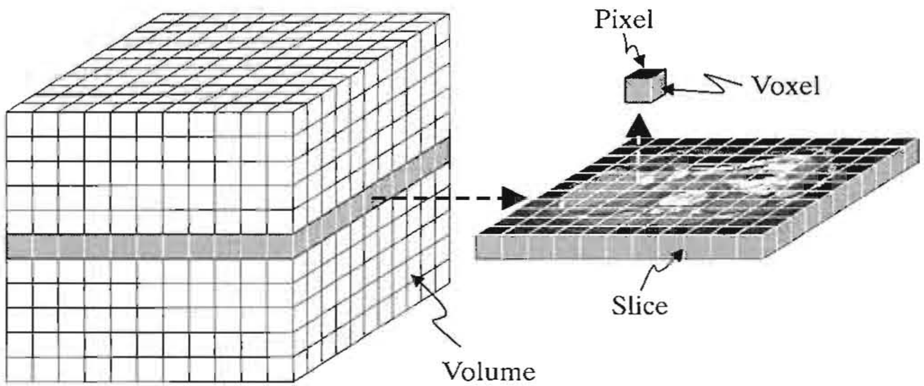


Figure 2.2
The representation of a 3-D object as a 2-D image. The 3-D object is represented by a collection of voxels; the 2-D image is comprised of pixels. In this example, the image is a slice through the 3-D object.

Analógico e digital

- Signals can be analog or digital
- <u>Analog signals</u>
  - Represent data that is continuous
  - Analog signals can have an infinite number of values in a range
- <u>Digital signals</u>
  - Represent data that can take only discrete values
  - Digital signals have a limited number of values

Sinal analógico

- An analog signal may be modeled as a real-valued function of a real independent variable *t*, which is usually time.
  - Suppose that at each time *t* within some interval $a<t<b$ we perform a measurement, and this measurement yields a real number
  - In this case our measurements are naturally represented by a real-valued function $x(t)$ with domain $a<t<b$.
  - We will refer to $x(t)$ as an *analog signal.*

Sinal analógico

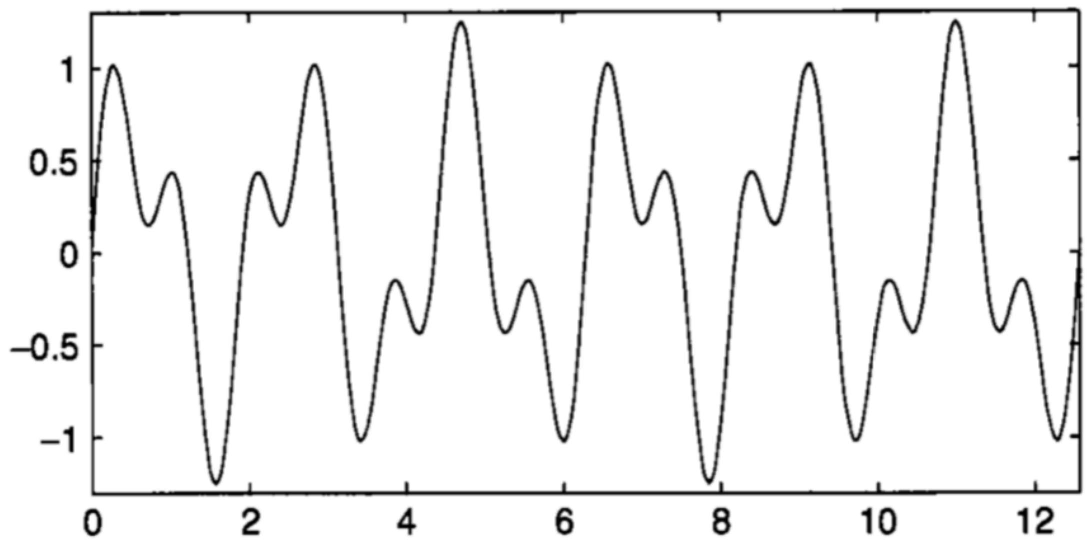


FIGURE 1.1 Analog or continuous model $x(t) = 0.75 \sin(3t) + 0.5 \sin(7t)$.

- The function *x(t)* might represent the intensity of sound at a given location (an audio signal), the current through a wire, the speed of an object, and so on.

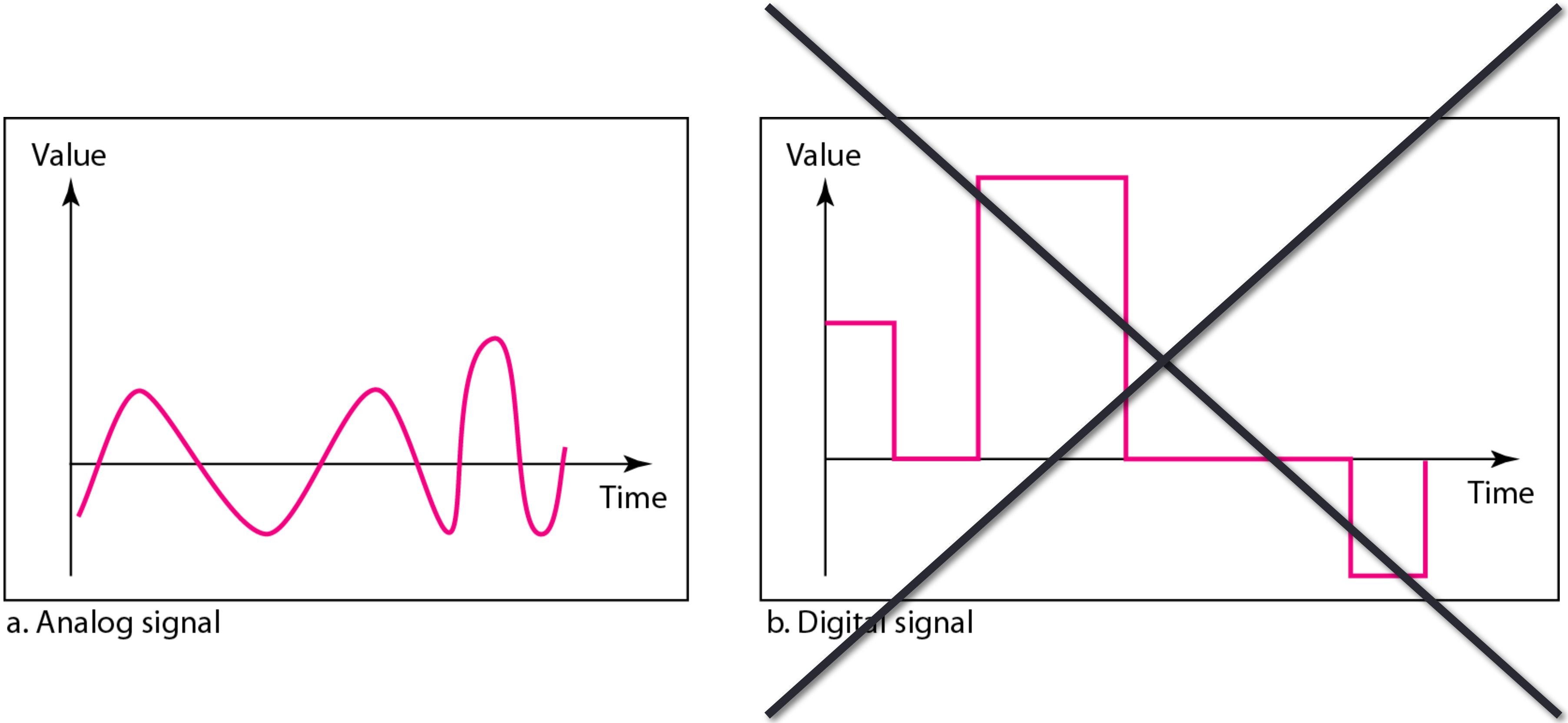
Sinais analógicos

- Many physical processes are naturally modeled by analog signals
- Analog models also have the advantage of being amenable to analysis using methods from calculus and differential equations
- In general, in signal processing we are faced by a few persistent annoyances:
  - We almost never have an explicit formula for $x(t)$
  - Most signals are very complex
  - Most signals have noise.

Sinais digitais

- Represent signals by a sequence of numbers
  - Sampling or analog-to-digital conversions
- Perform processing on these numbers with a digital processor
  - Digital signal processing (DSP)
  - Reconstruct analog signal from processed numbers
  - Reconstruction or digital-to-analog conversion

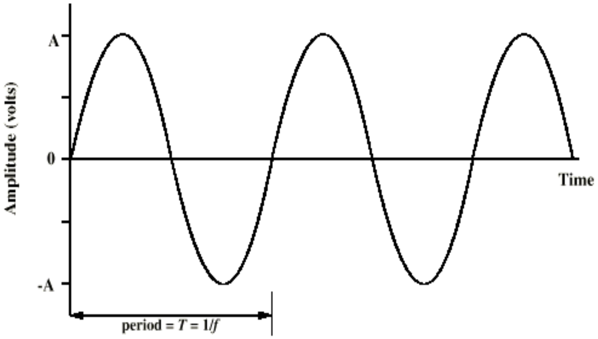
Sinais analógicos vs. digitais



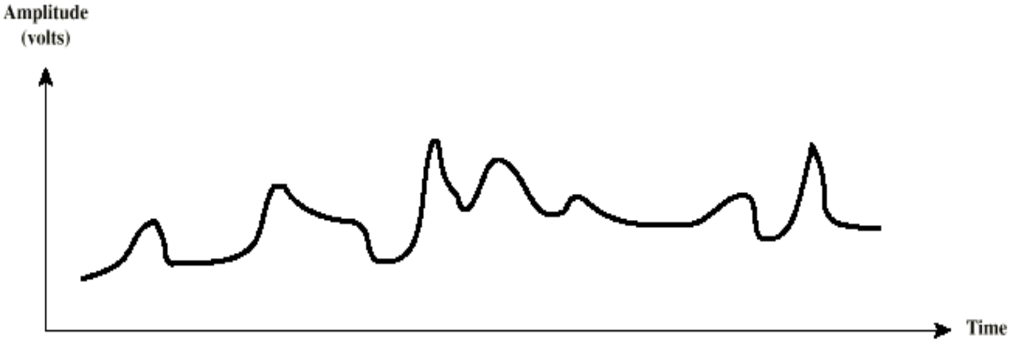
a. Analog signal

b. Digital signal

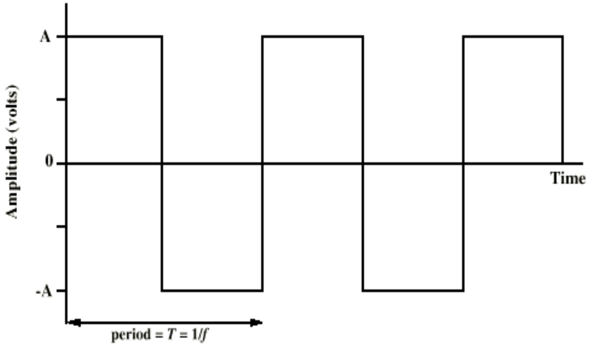
Periodicidade de um sinal



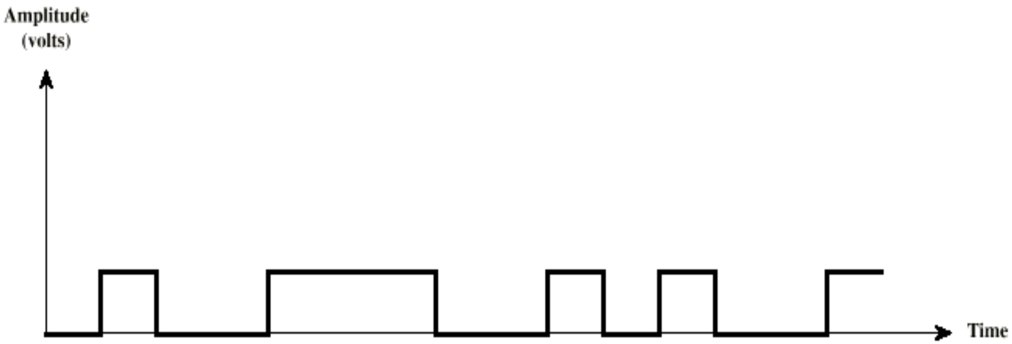
(a) Sine wave



(a) Continuous



(b) Square wave



(b) Discrete

Sinais periódicos

A signal is periodic if f(x)=f(x+X)=f(x)

- we call X the period of the signal
- if there is no such X then the signal is aperiodic

Sinusoids are periodic functions

- sinosoids will play an important role in this course

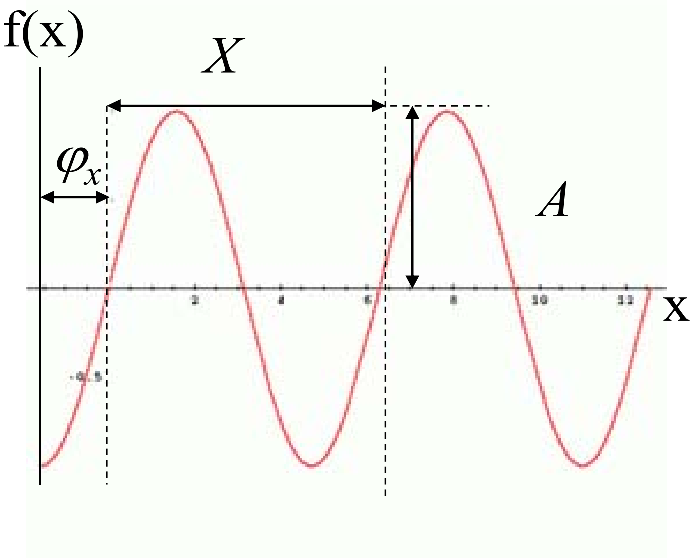
Write as:

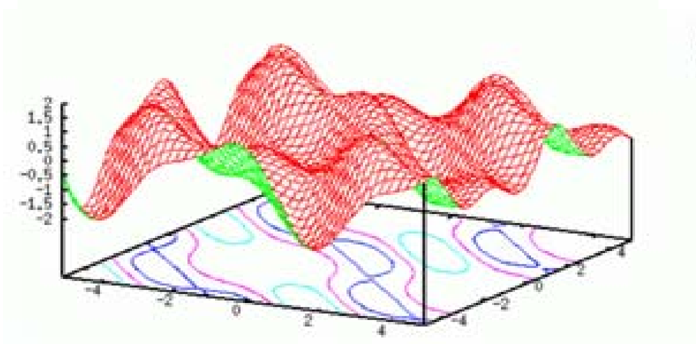
$$A\sin(\frac{2\pi x}{X}+\varphi_x)$$

- where φ_x is the phase shift and A is the amplitude

Sinusoids can combine

- they can also occur in higher dimensions:





Pros and Cons of Digital Signal Processing

- Advantages
  - Accuracy can be controlled by choosing word length
  - Dynamic range can be controlled using floating point numbers
  - Flexibility can be achieved with software implementations
  - Digital storage is cheap
  - Digital information can be encrypted for security
  - Price/performance and reduced time-to-market
- Disadvantages
  - Sampling causes loss of information
  - A/D and D/A requires mixed-signal hardware
  - Limited speed of processors
  - Quantization and round-off errors

Amostragem

- To store a signal in a computer we must first *digitize* the signal.
- The first step in digitization consists of measuring the signal's instantaneous value at specific times over a finite interval of interest. This process is called *sampling*.
- The process of sampling the signal converts it from an analog form to a finite list of real numbers, and is usually carried out by a piece of hardware known as an *analog-to-digital* ("A-to-D") converter.

Amostragem

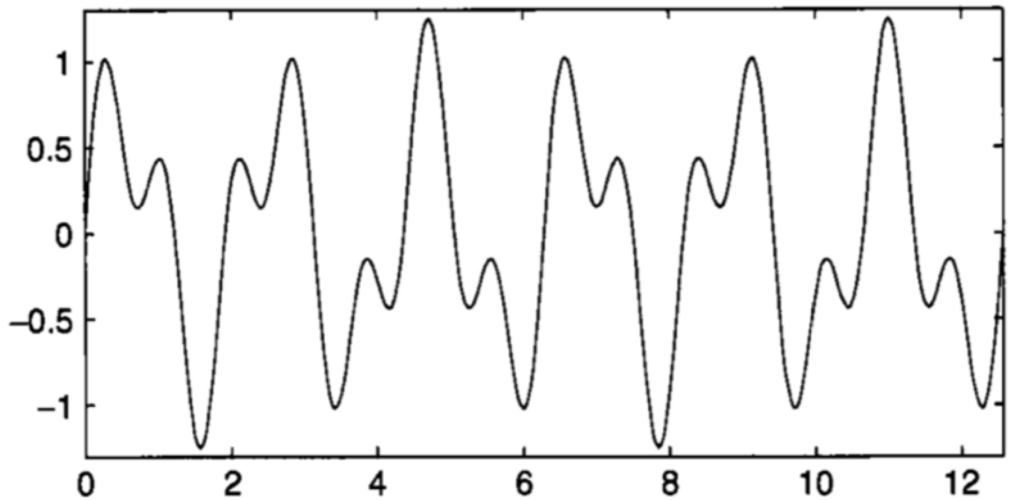


FIGURE 1.1 Analog or continuous model $x(t) = 0.75\ \sin(3t) + 0.5\ \sin(7t)$.

More explicitly, suppose that the signal $x(t)$ is defined on the time interval $a \leq t \leq b$. Choose an integer $N \geq 1$ and define the *sampling interval* $\Delta t = (b - a)/N$. We then measure $x(t)$ at times $t = a, a + \Delta t, a + 2\Delta t, \ldots$, to obtain samples

$$x_n = x(a + n\Delta t), \qquad n = 0, 1, \ldots, N.$$

Define

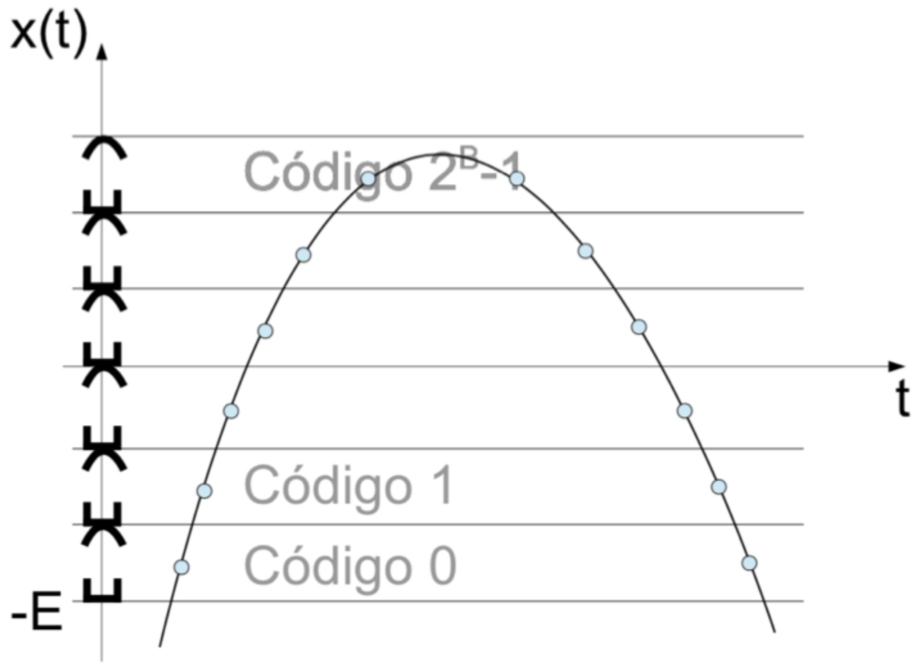
$$\mathbf{x} = (x_0, x_1, \ldots, x_N) \in \mathbb{R}^{N+1}.$$

Amostragem

- O sinal tem que ser representado em uma quantidade finita de bits, assim é necessário fixar um certo nível de discretização para $f(t)$
  - Exemplo: $2^8 = 256$
- Converter cada valor $x_k \in R$ para uma coleção finita de códigos/valores
  - Não importa se vamos pensar em números em ponto flutuante, ASCII, etc
- É muito comum utilizar valores inteiros $\{0, 1, \ldots, 2^{B-1}\}$ onde B é o número de bits utilizados para representar cada x_k.

Amostragem

Podemos mapear o intervalo $[-E, +E] \subseteq \mathbb{R}$ em 2^B códigos distintos subdividindo o intervalo em segmentos de tamanho $\frac{2E}{2^B}$ e associando cada intervalo da forma $\left[-E + k\frac{2E}{2^B}, -E + (k+1)\frac{2E}{2^B}\right)$ ao código k.



Amostragem e quantização

- The combination of sampling and quantization allows us to *digitize* a signal, and thereby convert it into a form suitable for computer storage and processing
- Unfortunately, quantization is a process that corrupts the algebraic structure afforded by the vector space model
- In addition quantization introduces irreversible, though usually acceptable, loss of information.

Ruído de amostragem

- If the noiseless samples are given by x_n, the noisy sample values y_n might be modeled as $y_n=x_n+e_n$ where e_n represents the noise in the nth measurement.
- The errors e are usually assumed to be distributed according to some probability distribution, known or unknown.

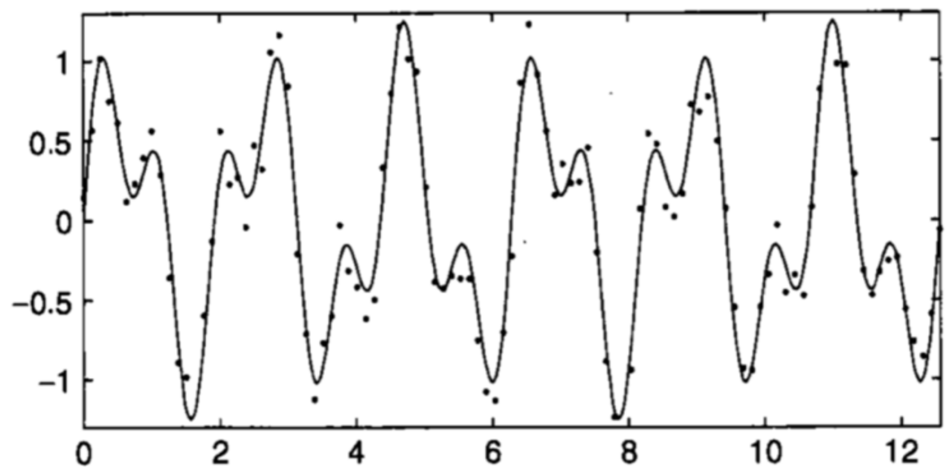


FIGURE 1.3 Analog model and discrete model with noise, $x(t) = 0.75\ \sin(3t) + 0.5\ \sin(7t)$.

Sinais bidimensionais

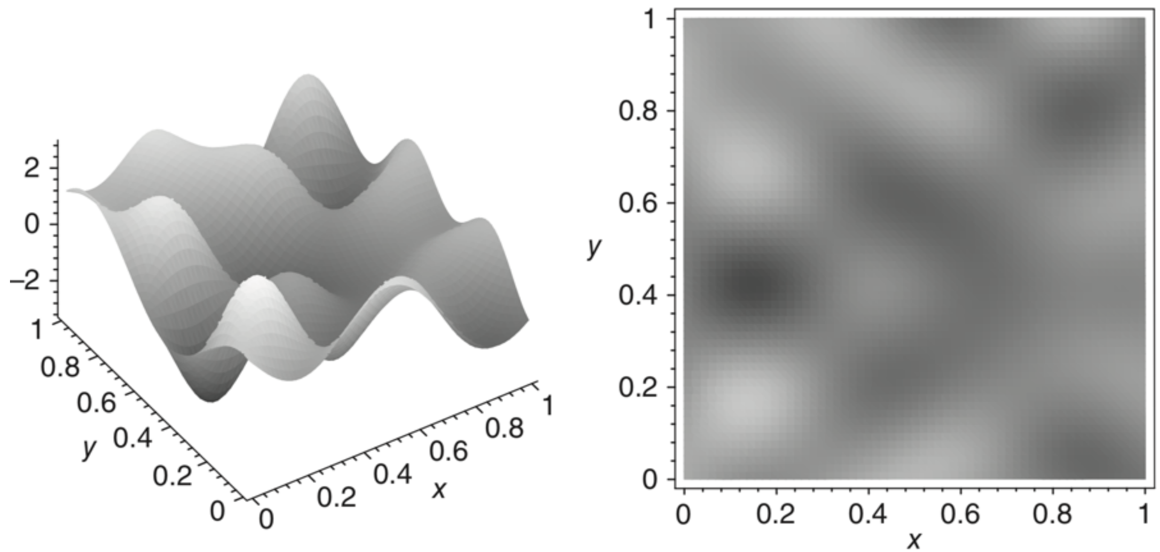


FIGURE 1.4 Grayscale image from two perspectives.

For natural images $f(x, y)$ would never be a simple function. Nonetheless, to illustrate let us consider the image defined by the function

$$f(x, y) = 1.5\ \cos(2x)\cos(7y) + 0.75\ \cos(5x)\sin(3x)$$
$$-1.3\ \sin(9x)\cos(15y) + 1.1\ \sin(13x)\sin(11y)$$

Trabalharemos com $f : \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$(ou **C**) ou $f : [a,b] \times [c,d] \rightarrow \mathbf{R}$(ou **C**)

Amostragem em 2 dimensões

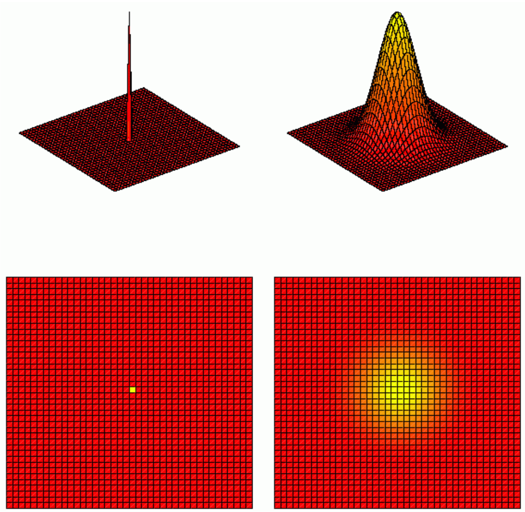
- Obtenção de valores $f_{m,n}$, para m = 1,...,M e n = 1,...,N onde $f_{m,n}=f(x_m, y_n)$,
- Os valores $x_1, x_2, \ldots$ são equiespaçados com intervalo Δx
  - ou frequência de amostragem $1/\Delta x$
- Os valores $y_1, y_2 \ldots$ são equiespaçados com intervalo Δy ou Δx
  - ou frequência de amostragem $1/\Delta y$
- Podemos utilizar algum tipo de função integral ou média próxima ao ponto (x_r, y_s)

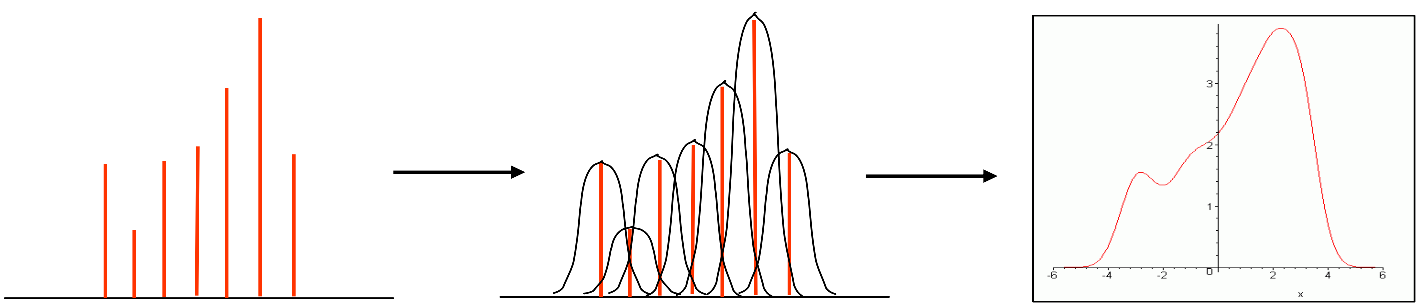
Função de espalhamento

Each pixel is not a sharp spike, but represented by a point spread function (PSF)

The PSFs overlap and form a continuous function (for the eye)

Smaller PSFs give sharper images



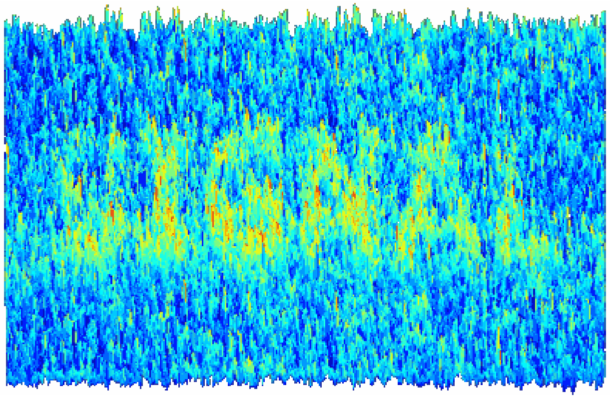


Razão sinal-ruído (SNR)

Signal-to-Noise ratio (SNR) = S_{RMS} / N_{RMS}

- RMS: root mean square

high SNR



low SNR

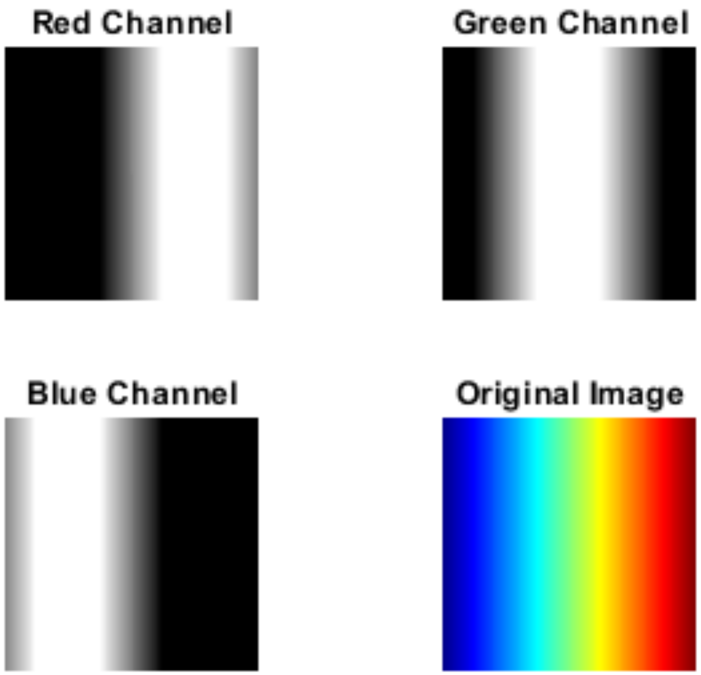
$$x_{\mathrm{rms}} = \sqrt{\frac{1}{n}\left(x_1^2 + x_2^2 + \cdots + x_n^2\right)}.$$

The corresponding formula for a continuous function (or waveform) $f(t)$ defined over the interval $T_1 \le t \le T_2$ is

$$f_{\mathrm{rms}} = \sqrt{\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} [f(t)]^2 \, dt},$$

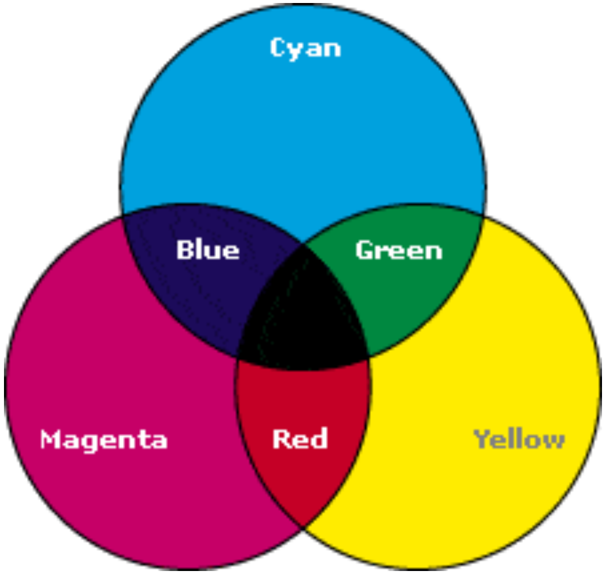
Imagens coloridas

- One of the simplest is the "RGB" (red, green, blue) model in which a color image is described using three functions *r(x, y)*, *g(x, y)*, and *b(x, y)*, appropriately scaled, that correspond to the intensities of these three additive primary colors at the point *(x, y)* in the image domain.



Imagens coloridas

- It is worth noting that the actual JPEG compression standard specifies color images with a slightly different scheme, the luminance-chrominance or "YCbCr" scheme.
- An example where the CMY model needs to be considered is in color laser printers that use cyan, magenta, and yellow toner.



Ruído de amostragem e quantização

- Just as for one-dimensional signals, quantization error is introduced when an image is digitized.
- Like one-dimensional signals, images may contain sampling noise.

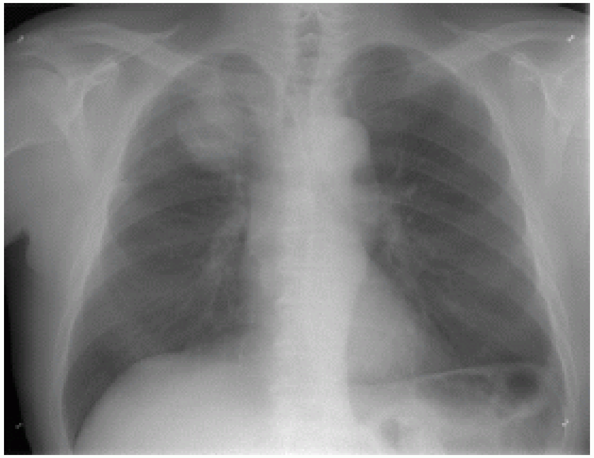
(a) (b)

Figure 1.5 Image without (a) and with additive noise (b).

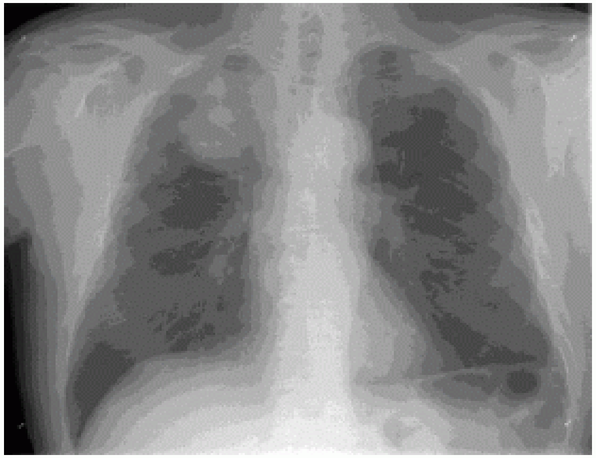
Exemplo: ruído de quantização

Minimal number of bits = 6 (64 greylevels or 4 levels for R,G,B)

- most medical digital images have 12 bits (4096 grey levels)



8 bits



4 bits

- not enough bits leads to quantization artifacts and loss of resolution

Espaços vetoriais

The vector space $\mathbb{R}^N$ consists of vectors $\mathbf{x}$ of the form

$$\mathbf{x} = (x_1, x_2, \ldots, x_N), \tag{1.6}$$

where the x_k are all real numbers. Vector addition and scalar multiplication are defined component by component as

$$\mathbf{x} + \mathbf{y} = (x_1 + y_1, x_2 + y_2, \ldots, x_N + y_N), \quad c\mathbf{x} = (cx_1, cx_2, \ldots, cx_N),$$

where $\mathbf{y} = (y_1, y_2, \ldots, y_N)$ and $c \in \mathbb{R}$. The space $\mathbb{R}^N$ is appropriate when we work with sampled audio or other one-dimensional signals. If we allow the x_k in (1.6) and scalar c to be complex numbers, then we obtain the vector space $\mathbb{C}^N$. That $\mathbb{R}^N$ or $\mathbb{C}^N$ satisfy the properties of a vector space (with addition and scalar multiplication as defined) follows easily, with zero vector $\mathbf{0} = (0, 0, \ldots, 0)$ and additive inverse $(-x_1, -x_2, \ldots, -x_n)$ for any vector $\mathbf{x}$.

$$\mathbb{R}^n = \{(x_1, x_2, \ldots, x_n) | x_i \in \mathbb{R}, i = 1, 2, \ldots, n\}$$

$$\mathbb{C}^n = \{(x_1, x_2, \ldots, x_n) | x_i \in \mathbb{C}, i = 1, 2, \ldots, n\}$$

Espaço vetorial

Um espaço vetorial sobre $\mathbb{R}$ (ou $\mathbb{C}$) é um conjunto V munido das operações de soma (entre elementos de V) e multiplicação de elementos de V por escalares (em $\mathbb{R}$ ou em $\mathbb{C}$) com as seguintes propriedades:

1. Fecho por adição: $\forall u, v \in V$ a soma $u + v$ está bem definida e pertence a V;

2. Fecho por multiplicação por escalar: $\forall u \in V$ e $\forall \alpha \in \mathbb{R}$ (ou $\forall \alpha \in \mathbb{C}$) temos αu bem definido e pertence a V;

Espaço vetorial

3. A soma e o produto por escalar satisfazem as propriedades algébricas abaixo $\forall a, b \in \mathbb{R}$ (ou $\mathbb{C}$) e $\forall u, v \in V$:

   (a) $u + v = v + u$ (comutatividade)

   (b) $(u + v) + w = u + (v + w)$ (associatividade)

   (c) $\exists$ um vetor $\mathbf{0}$ t.q. $u + \mathbf{0} = \mathbf{0} + u = u$ (elemento neutro da soma)

   (d) $\forall u \in V$ $\exists$ um vetor w t.q. $u + w = \mathbf{0}$ (elemento inverso da soma)

   (e) $(ab)u = a(bu)$

   (f) $(a + b)u = au + bu$

   (g) $a(u + v) = au + av$

   (h) $1u = u$

Espaço vetorial

$$M_{m,n}(\mathbb{R}) = \mathbb{R}^{m\times n} = \left\{ \begin{pmatrix} a_1 1 & \dots & a_1 n \\ a_2 1 & \dots & a_2 n \\ \vdots & \ddots & \\ a_n 1 & \dots & a_n n \end{pmatrix} a_{i,j} \in \mathbb{R}, i = 1, \dots, m, j = 1, \dots, n \right\}$$

$$M_{m,n}(\mathbb{C}) = \mathbb{C}^{m\times n} = \left\{ \begin{pmatrix} a_1 1 & \dots & a_1 n \\ a_2 1 & \dots & a_2 n \\ \vdots & \ddots & \\ a_n 1 & \dots & a_n n \end{pmatrix} a_{i,j} \in \mathbb{C}, i = 1, \dots, m, j = 1, \dots, n \right\}$$

É um espaço vetorial com as operações:

$$\begin{pmatrix} \ddots & \vdots & \\ \dots & a_{i,j} & \dots \\ & \vdots & \ddots \end{pmatrix} + \begin{pmatrix} \ddots & \vdots & \\ \dots & b_{i,j} & \dots \\ & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} \ddots & \vdots & \\ \dots & a_{i,j} + b_{i,j} & \dots \\ & \vdots & \ddots \end{pmatrix}$$

$$\alpha \begin{pmatrix} \ddots & \vdots & \\ \dots & a_{i,j} & \dots \\ & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} \ddots & \vdots & \\ \dots & \alpha a_{i,j} & \dots \\ & \vdots & \ddots \end{pmatrix}$$

Espaços de interesse

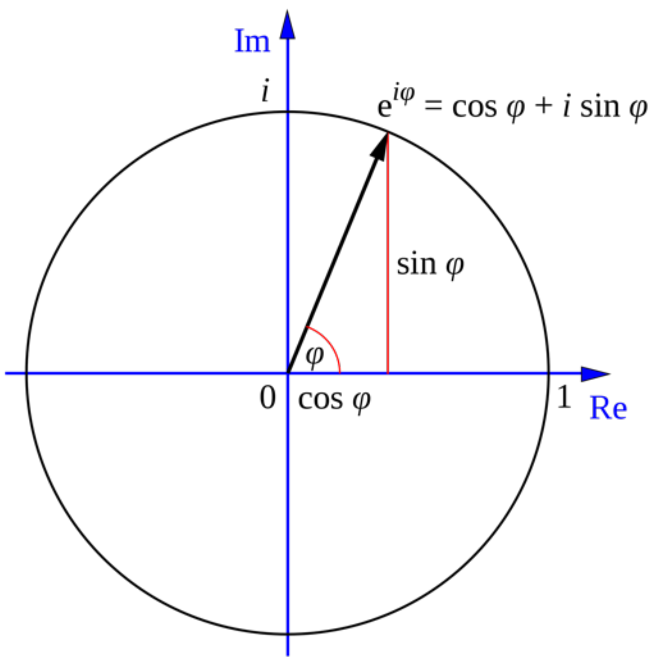
TABLE 1.1 Discrete Signal Models and Uses

Notation	Vector Space Description
$\mathbb{R}^N$	$\{\mathbf{x} = (x_1, \ldots, x_N) : x_i \in \mathbb{R}\}$, finite sampled signals
$\mathbb{C}^N$	$\{\mathbf{x} = (x_1, \ldots, x_N) : x_i \in \mathbb{C}\}$, analysis of sampled signals
$L^\infty(\mathbb{N})$ or ℓ^∞	$\{\mathbf{x} = (x_0, x_1, \ldots) : x_i \in \mathbb{R},\ \lvert x_i\rvert \leq M$ for all $i \geq 0\}$ bounded, sampled signals, infinite time
$L^2(\mathbb{N})$ or ℓ^2	$\{\mathbf{x} = (x_0, x_1, \ldots) : x_i \in \mathbb{R}$ or $x_i \in \mathbb{C}$, $\sum_k \lvert x_k\rvert^2 < \infty\}$ sampled signals, finite energy, infinite time
$L^2(\mathbb{Z})$	$\{\mathbf{x} = (\ldots, x_{-1}, x_0, x_1, \ldots) : x_i \in \mathbb{R}$ or $x_i \in \mathbb{C}$, $\sum_k \lvert x_k\rvert^2 < \infty\}$ sampled signals, finite energy, bi-infinite time
$M_{m,n}(\mathbb{R})$	Real $m \times n$ matrices, sampled rectangular images
$M_{m,n}(\mathbb{C})$	Complex $m \times n$ matrices, analysis of images

Decomposição

- To analyze signals and images, it can be extremely useful to decompose them into a sum of more elementary pieces or patterns, and then operate on the decomposed version, piece by piece
- They serve as the essential building blocks for signals and images
  - In the context of Fourier analysis for analog signals these basic waveforms are simply <u>sines</u> and <u>cosines</u>, or equivalently, <u>complex exponentials</u>
- Formas ”senoidais” como sen(ωt), cos(ωt), sen($\omega t+\phi$) correspondem a projeções de movimentos circulares uniformes

Fórmula de Euler

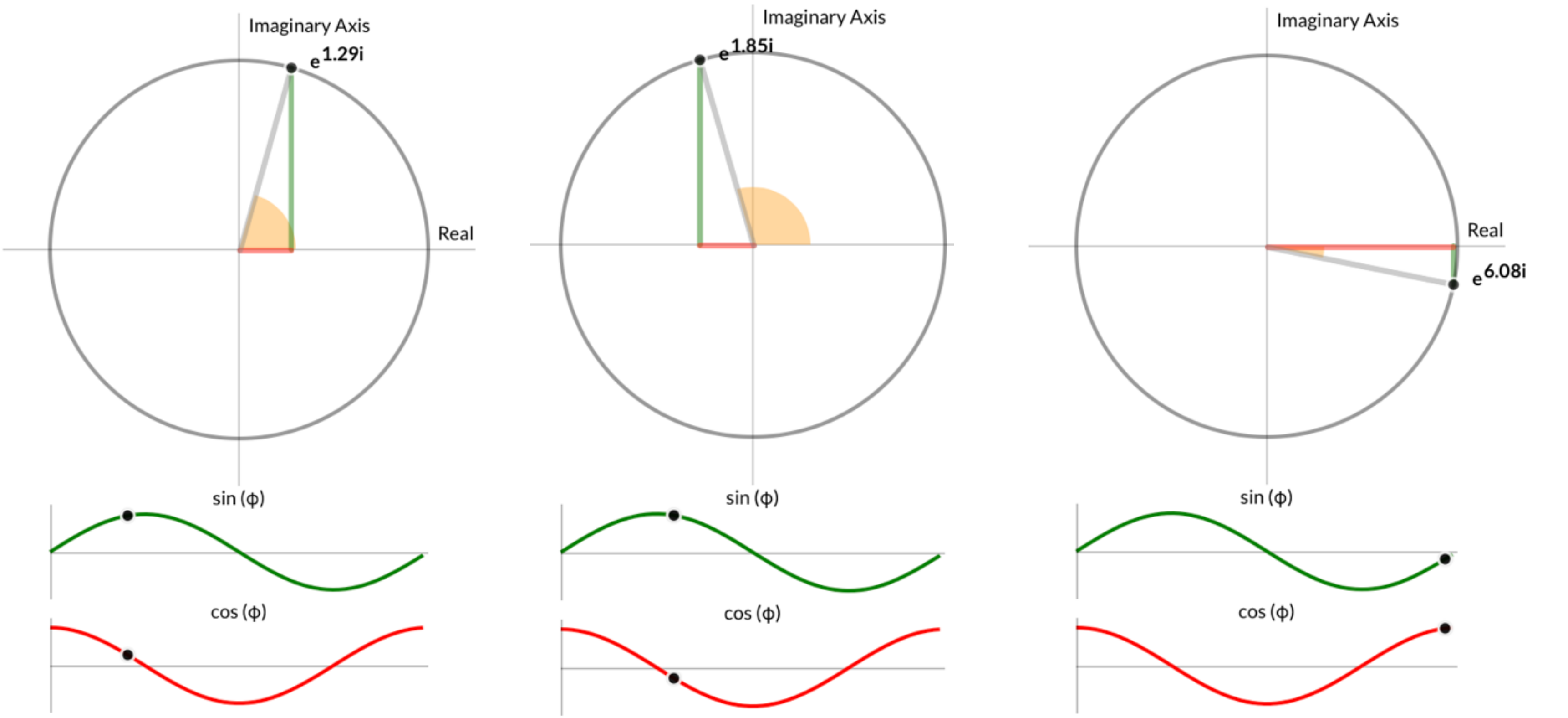


$$z(t) = e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$
$$= e^{ix} = \cos x + i\sin x$$

In the real-valued sine/cosine case we only need to work with $\omega > 0$, since $\cos(-\omega t) = \cos(\omega t)$ and $\sin(-\omega t) = -\sin(\omega t)$.

Any function that can be constructed as a sum using negative values of ω has an equivalent expression with positive ω.

Fórmula de Euler



Relação de Euler

$$e^{i\theta} = \cos(\theta) + i\,\sin(\theta).$$

From this we have (with $\theta = \omega t$ and $\theta = -\omega t$)

$$\begin{aligned} e^{i\omega t} &= \cos(\omega t) + i\ \sin(\omega t), \\ e^{-i\omega t} &= \cos(\omega t) - i\ \sin(\omega t), \end{aligned} \tag{1.12}$$

which can be solved for $\cos(\omega t)$ and $\sin(\omega t)$ as

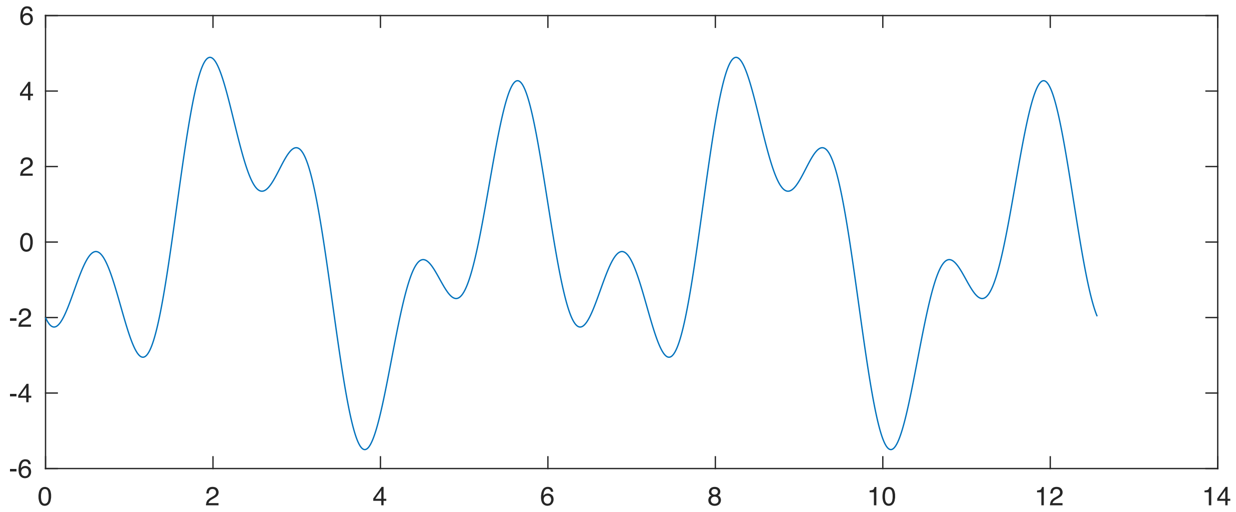
$$\begin{aligned} \cos(\omega t) &= \frac{e^{i\omega t} + e^{-i\omega t}}{2}, \\ \sin(\omega t) &= \frac{e^{i\omega t} - e^{-i\omega t}}{2i}. \end{aligned} \tag{1.13}$$

Formas senoidais

- If we can decompose a given signal *x(t)* into a linear combination of waveforms cos(ωt) and sin(ωt), these equations make it clear that we can also decompose *x(t)* into a linear combination of appropriate complex exponentials.
- Similarly equations can be used to convert any complex exponential decomposition into sines and cosines.
- We thus also consider the complex exponential functions $e^{i\omega t}$ as basic waveforms.

Exemplo #1

Consider the signal $x(t) = \sin(t) + 3\ \sin(-2t) - 2\ \cos(-5t)$.



$$x(t) = \sin(t) - 3\ \sin(2t) - 2\ \cos(5t)$$

$$x(t) = \frac{1}{2i}e^{it} - \frac{1}{2i}e^{-it} - \frac{3}{2i}e^{2it} + \frac{3}{2i}e^{-2it} - e^{5it} - e^{-5it},$$

Exemplo #2

$$\cos(\omega t) = \mathrm{Re}(e^{i\omega t}),$$

$$\sin(\omega t) = \mathrm{Im}(e^{i\omega t}).$$

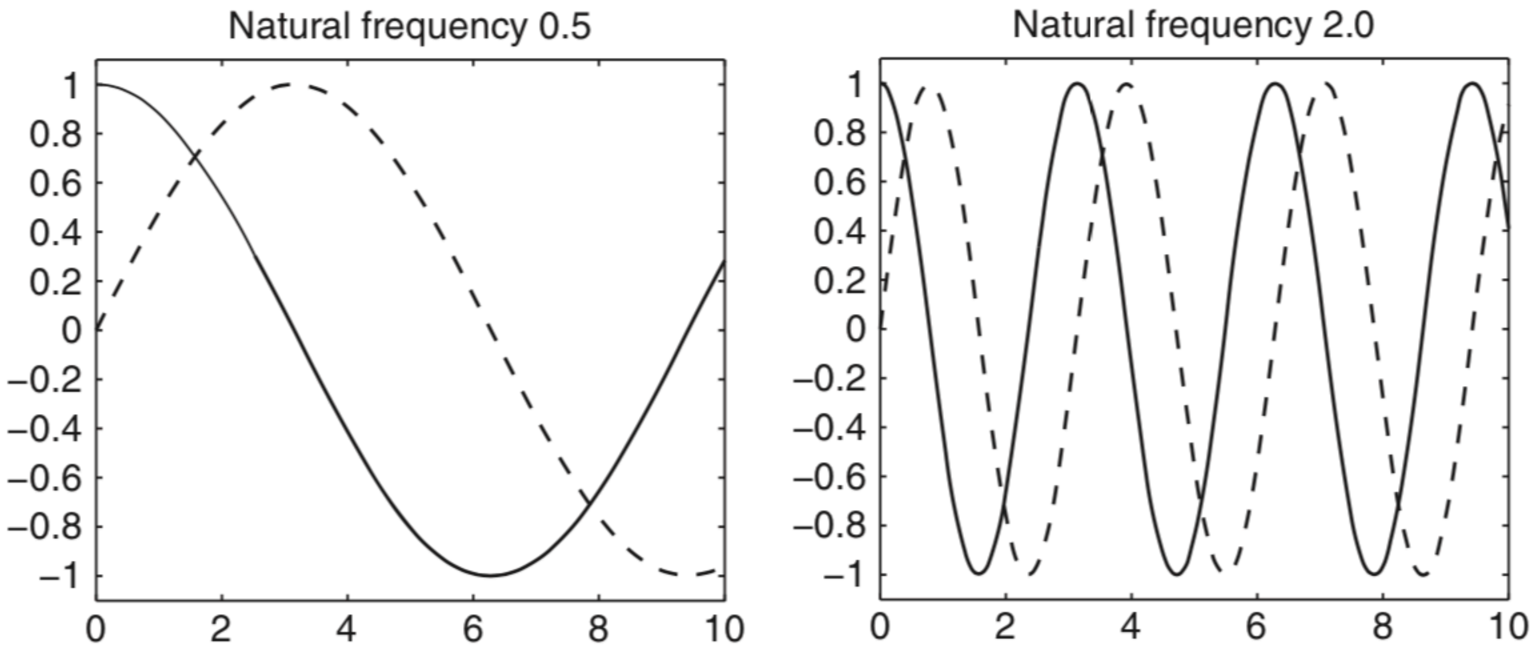


Figure 1.6 Real (*solid*) and imaginary (*dashed*) parts of complex exponentials.

Periodicidade de $e^{i\omega t}$

O período (comprimento de onda) λ é o menor valor positivo que verifica a seguinte propriedade:

$$\lambda = \text{ período de } e^{i\omega t} \Leftrightarrow e^{i\omega(t+\lambda)} = e^{i\omega t}, \forall t \in \mathbb{R}$$

$$\begin{aligned}
e^{i\omega t}e^{i\omega\lambda} &= e^{i\omega(t+\lambda)} = e^{i\omega t}, \forall t \in \mathbb{R} \\
\Rightarrow e^{i\omega\lambda} &= 1 \\
\Rightarrow |\omega|\lambda &= 2\pi \\
\Rightarrow \lambda &= \frac{2\pi}{|\omega|}
\end{aligned}$$

Frequência

A frequência é dada por:

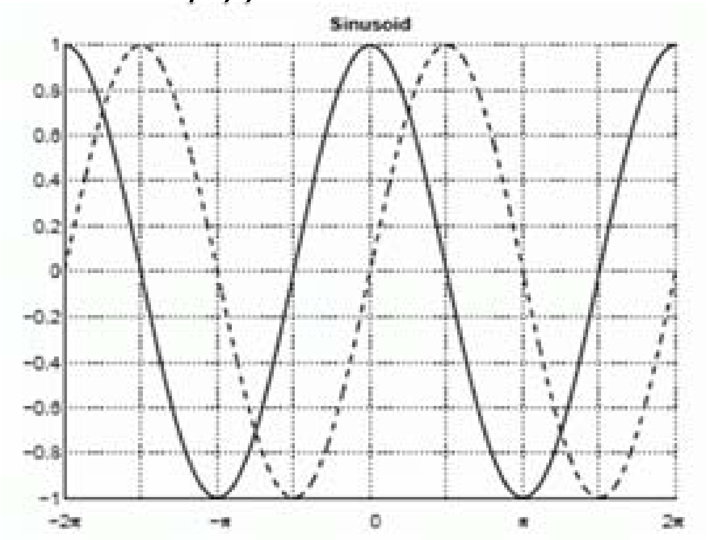
$$q = \frac{1}{\lambda}$$

que é o número de oscilações completas por unidade da variável t. Se t é tempo, q é medido em Hz. Se t é espaço, q é medido em ciclos/unidade espacial.

Note que $q = \frac{1}{\lambda} = \frac{|\omega|}{2\pi}$. A função básica $e^{i\omega t}$ pode ser expressada equivalentemente como $e^{2\pi \cdot i \cdot q \cdot t}$ (admitindo que q poderia ser negativo também).

Representação geral

$$Ae^{i(2\pi kx+\phi)} = A(\cos(2\pi kx+\phi)+i\sin(2\pi kx+\phi))$$



As before

- the *cos* term is the signal's real part
- the *sin* term is the signal's imaginary part
- *A* is the amplitude, φ the phase shift, *k* determines the frequency

Vamos mostrar (mais adiante) que se $f : [0,T] \to \mathbb{R}$ (ou $\mathbb{C}$) é limitada e tiver uma quantidade finita de descontinuidades, existem constantes $\{c_k\}_{k\in\mathbb{Z}} \subseteq \mathbb{C}$ tais que $f(t) = \sum_{k=-\infty}^{\infty} c_k e^{2\pi i k(\frac{1}{T})t}$

Sinais bidimensionais

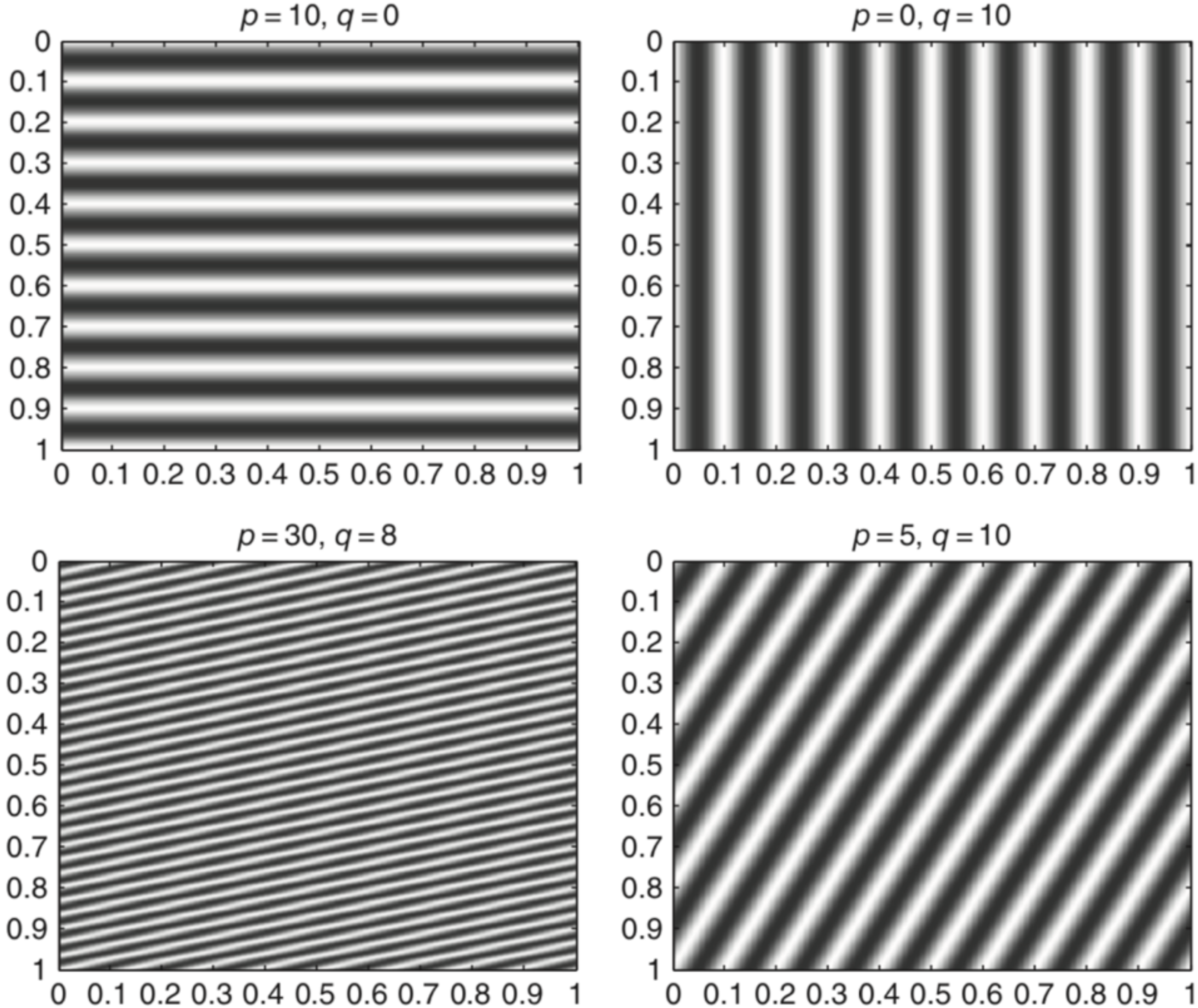


Figure 1.7 Grayscale image of $\cos(2\pi(px + qy)) = \mathrm{Re}(e^{2\pi i(px+qy)})$ for various p and q.

Tarefa de casa

- Leitura capítulo 1 até seção 1.5.1
- Seção ”Matlab project”, exercício 2 (em Python)