Introduction

This present article aims to make a analysis within the context of VAR models, allowing inferences and forecasts about the variables worked on. In this case, we will deal with the Brazilian variables of GDP, real exchange rate and trade balance.

Understanding how the real exchange rate impacts the trade balance, affecting the growth of the GDP, allows one to see what the economic scenario would be like with different decisions regarding economic policy. Besides emphasizing that greater integration in international trade is associated with higher levels of growth and social well-being.

The sample period of the monthly series is restricted from January 1995 to December 2013 (228 observations).

Descriptive Data Analysis

The GDP series used is measured in millions of dollars (source: Banco Central do Brasil). An upward trend is visible during much of the sample period (**Appendix 1**). In the period from 2004 to 2008 there was a stronger growth of the GDP followed by a fall in economic activity, derived from the international economic crisis (2008-2009). In 2010 a strong recovery in economic activity is already obtained, through the anti-cyclical policy adopted.

The analysis of the ACF correlogram (**Appendix 2**) helps to realize the characteristics of a series with unit root, in addition to possible seasonality. To prove such existence of unit root, the augmented Dickey Fuller or ADF test will be performed (**Appendix 3**). The formalization of the test applied to the already transformed (log) GDP series can be described as follows:

$$\Delta y_{1,t} = \beta_0 + \beta_1 t + \Phi y_{1,t-1} + \varepsilon_t$$

Where $\boldsymbol{\beta_0}$ represents the inclusion of the constant in the test statistic, $\boldsymbol{\beta_1 t}$ represents the deterministic trend, as it is reasonable to imagine that for GDP series there is such a component. Not including this deterministic component, in case it is needed, causes a potency problem in the ADF test. We seek to test the null hypothesis: H_0: $\boldsymbol{\phi} = 0$ against the alternative hyphotesis H_1: $\boldsymbol{\phi} < 0$. the test performed must use the critical values tabulated by Dickey and Fuller, since the usual asymptotic properties do not hold. Since we do not reject H_0, we have statistical evidence of unit root existence. On **appendix 4**, **5** e **6** we could see: the series after applied the first differences, after logging; the new correlograms after the transformations; the result of ADF Test, which, there isn't evidence of the unit root(the null hyphotesis is rejected).

VAR Estimation

The choice of the VAR model order respects the following process:

- Analysis of the cross-correlograms of the differentiated series (**Appendix 5**)
- Choosing the order of the VAR model based on the selection criteria, after running a series of models of different orders.

With the signs of possible existence of seasonality and relevance of some lags in these correlograms, the estimation of VAR with seasonality 12 (11 time dummies) up to order 8 (lag max = 8) was performed. The equation can be described in (1), where: v represents the intercepts; A1 are the 1st order parameters; A2 are the 2nd order parameters; Dsazonal is the set of time dummies and ut represents the model errors (white noise).

$$Y_t = v + A_1 Y_{t-1} + A_2 Y_{t-2} + \varphi Dsazonal + u_t \quad u_t \sim \text{RB } (0, \textstyle\sum_u) \quad (1)$$

In **appendix 7**, we can see the different information criteria for the estimated models. Having forecasting as the main objective of the analysis, we opted to choose the AIC criterion for model selection, since it does not penalize an eventual over-parameterization so much. In addition, the addition of an order to be estimated compared to the BIC and HQ criteria gives us a greater margin to avoid possible problems with autocorrelation of errors. It is worth mentioning that our sample is relatively large, so we could consider the BIC or HQ criteria, which have desirable properties in large samples.

The model estimations chosen for the equations for GDP, exchange rate and trade balance can be seen in **Appendix 8, 9 and 10**, respectively.

Once the choice of model (VAR 2) has been defined, an attempt is made to validate the model with the diagnostic tests performed. The common procedure observed in the literature for the diagnostic tests is to use the regression residuals in place of the unobserved errors. If the autocorrelation test fails, one can go back to the previous step and test another model:

- Tests for Error Normality
- test for autocorrelation in the erros
- Test for heteroscedasticity in erros

The normality test performed is the Jarque-Bera, in which the kurtosis and asymmetry coefficients are used in its statistics to show this normality.

. We test the null hypothesis that the data are normally distributed, against the alternative that they are not. **Appendix 11** gives us statistical evidence that the residuals do not follow normality. This does not create a big problem, since the sample is large enough, relative to the estimated coefficients. The inference can be based on the asymptotic properties.

The autocorrelation test used is the Breusch Godfrey, LM-type, where one regresses the residuals obtained in the estimation of the original model against the variables (regressors) used originally and the lags of the residuals. These repeated variables in the auxiliary regression serve to whiten the residuals and make the characteristics of the errors more visible. The test can be formalized as follows, starting with the lag of order 3, since the other two are present in the model (VAR 2), and therefore do not apply:

$$\hat{u}_t = \nu + A_1\, Y_{t-1} + A_2\, Y_{t-2} + \varphi Dsazonal + D_3 \hat{u}_{t-3} + \cdots + D_{10} \hat{u}_{t-10} + \varepsilon_t$$

The null hyphotesis: H_0: $D_3 = D_4 = \ldots = D_{10} = 0$ against the alternative hyphotesis H_1: $\exists\ D_i \neq 0$, i = 3,..., 10. There is no evidences to reject H_0 (**Appendix 12)**, therefore there is no symptoms pelo of autocorrelation of errors and we can stay with the chosen model.

The heteroscedasticity test (ARCH) seeks to assess whether the variance of the errors varies across time series. The test can be formalized as follows, starting with the lag of order 3, since the other two are present in the model (VAR 2), and therefore do not apply.

$$\hat{u}_t^2 = \alpha_0 + \alpha_3 \hat{u}_{t-3}^2 + \alpha_4 \hat{u}_{t-4}^2 + \cdots + \alpha_{10} \hat{u}_{t-10}^2 + \varepsilon_t$$

The null hyphotesis: H_0: $\alpha_3 = \alpha_4 = \ldots = \alpha_{10} = 0$ against the alternative hyphotesis H_1: $\exists\ \alpha_i \neq 0$, i = 3,..., 10. There is no statistical evidence to reject H0 (**Appendix** 12), so it seems to be an adequate model

Forecasting

The VAR model chosen for this work, based on the information criteria, allows us to make forecasts for the series of the vector worked on. Given the sample size (19 years), it is reasonable to establish a forecast for the period subsequent to the two-year sample. That is, to make the forecast for the months in the years 2014 and 2015. **Appendix 13** shows the graph of the forecast for the series vector. **Appendix 14, 15 and 16** show the forecast values for the series (GdP, Exchange Rate and Trade Balance) and their respective confidence intervals

The forecasts for each series at one step ahead and their respective confidence intervals can be described by the expressions below. As already explained above, Yt (1) represents the forecast for the series or vector 1 step ahead; the element within the square root represents the estimate for the variance of the respective series 1 step ahead (element k of the main diagonal of the matrix ∑y (1)

$$I.C_{95\%}\ (Y_{1,t+1}) = \widehat{Y_{1,t}}(1) \pm 1{,}96\sqrt{\hat{\sigma}_{11}(1)}$$
$$I.C_{95\%}\ (Y_{2,t+1}) = \widehat{Y_{2,t}}(1) \pm 1{,}96\sqrt{\hat{\sigma}_{22}(1)}$$
$$I.C_{95\%}\ (Y_{3,t+1}) = \widehat{Y_{3,t}}(1) \pm 1{,}96\sqrt{\hat{\sigma}_{33}(1)}$$

To assess the quality of the forecast model, which is the core of this work, we can compare the forecast for the transformed GDP series, based on the information set of this variable and the others present in the VAR model with an alternative model. This alternative model - which only uses historical information of this variable was chosen based on the correlograms (**Appendix 17**) and also on a loop generated to find the model with the lowest BIC - is the SARIMA (0,1,0)(0,1,1)12 model. For such a comparison, the total sample is divided into training and test samples, where the test period corresponds to the last 24 observations of the original sample. That is, training sample comprises the period from January 1995 to December 2011, while test sample corresponds to the period from January 2012 to December 2013. The comparison criterion to verify which best model is used is the mean square error. The forecast of the VAR model can be seen in **Appendix 18**, while that of the chosen SARIMA can be seen in **appendix 19**.

In **appendix 20** the predictions for both models can be seen, as well as the mean square errors of prediction. It should be noted that, due to different commands used in R, the actual observed and forecast values in the table are different. That said, the key information in this table is the mean square error for both prediction models. It is evident that the use of other series in the VAR model contributes to a more accurate forecast of the GDP series.

Conclusions

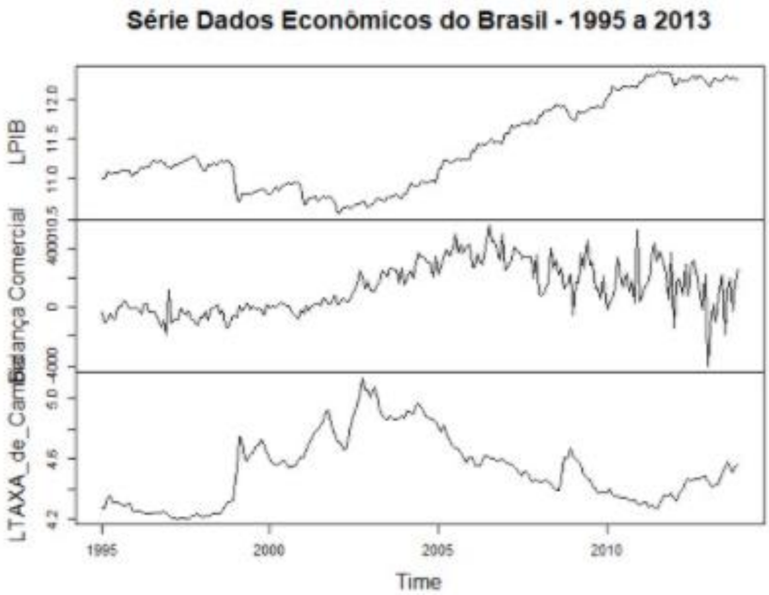
The objective of the present work was to perform an analysis in the context of VAR models, making inferences and forecasts about the Brazilian economic variables: GDP, real exchange rate and trade balance.

It was verified that these series presented seasonality, and only the series referring to GDP presented an upward trend. All series presented unit roots, as verified through the ADF test, which made it necessary to use some data transformations in order to make them stationary.

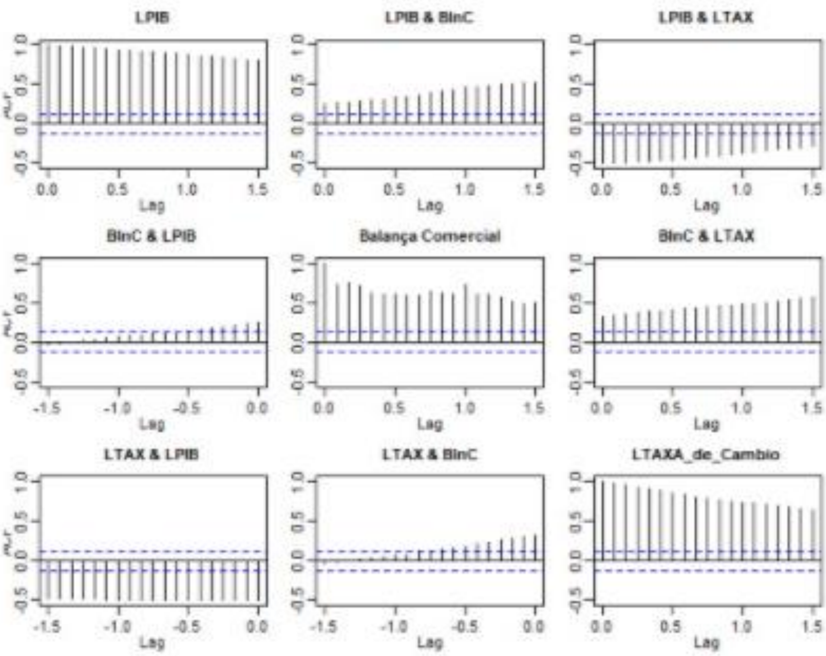
The estimated VAR model was of second order, and we believe that the model fitted the data well, since the results of the diagnostic tests presented a satisfactory result. Even the model presented a good predictive ability, considering the comparison with the competing SARIMA model.

Appendix

Appendix 1



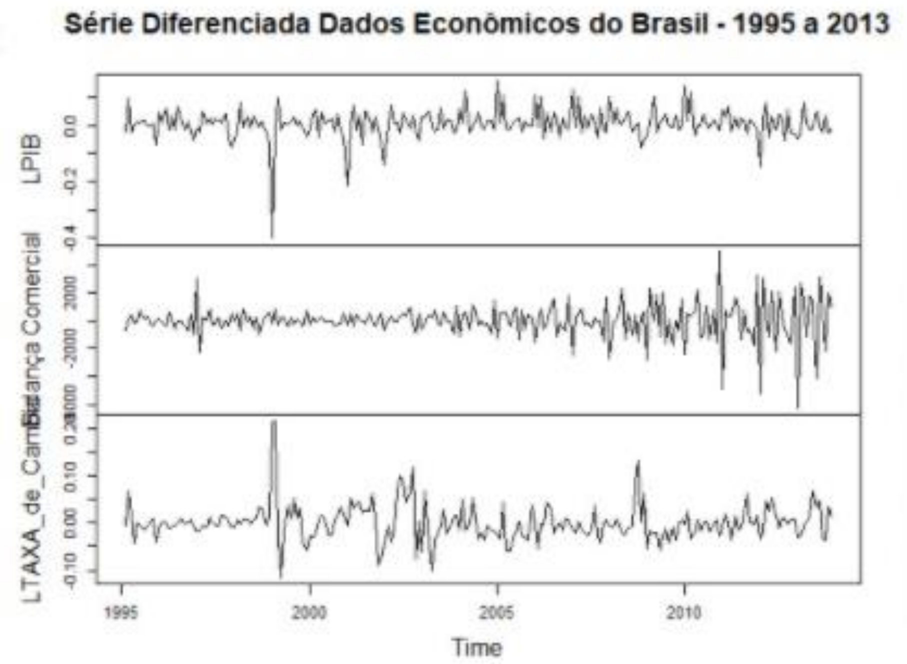
Appendix 2



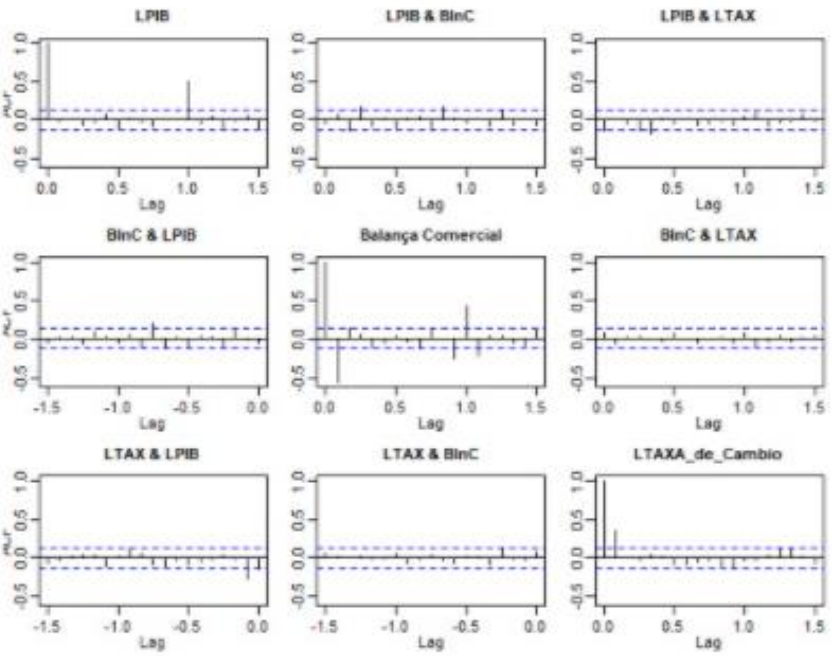
Appendix 3

Séries Originais		
Série	Dickey-Fuller	p-value
PIB	-1.4635	0.7451
Balança Comercial	-2.589	0.3285
Taxa de Cambio	-1.8093	0.656

Appendix 4



Appendix 5



Appendix 6

Séries Diferenciadas		
Série	Dickey-Fuller	p-value
PIB	-6.7773	0.01
Balança Comercial	-8.9999	0.01
Taxa de Cambio	-6.2356	0.01

Appendix 7

Critérios de Seleção				
Ordem	AIC	BIC	FPE	HQ
1	0.97	1.67	2.65	1.25
2	0.93	1.77	2.55	1.27
3	0.99	1.96	2.69	1.38
4	1.01	2.13	2.77	1.46
5	1.03	2.28	2.81	1.53
6	1.07	2.47	2.95	1.64
7	1.11	2.64	3.07	1.73
8	1.12	2.79	3.09	1.79

Appendix 8

	Estimate	Std. Error	t value	Pr(>|t|)
LPIB.l1	0.0636	0.0710	0.8952	0.3717
Balança_Comercial.l1	0.0000	0.0000	0.5428	0.5879
LTAXA_de_Cambio.l1	-0.0157	0.0869	-0.1801	0.8572
LPIB.l2	0.0340	0.0733	0.4643	0.6429
Balança_Comercial.l2	0.0000	0.0000	-0.5255	0.5998
LTAXA_de_Cambio.l2	-0.0071	0.0838	-0.0843	0.9329
const	0.0048	0.0030	1.5871	0.1140
sd1	0.0254	0.0170	1.4979	0.1357
sd2	0.0926	0.0162	5.7312	0.0000
sd3	-0.0088	0.0173	-0.5087	0.6115
sd4	0.0267	0.0166	1.6043	0.1102
sd5	0.0062	0.0154	0.4032	0.6872
sd6	0.0520	0.0155	3.3541	0.0009
sd7	0.0239	0.0161	1.4841	0.1393
sd8	-0.0055	0.0159	-0.3439	0.7312
sd9	0.0411	0.0155	2.6541	0.0086
sd10	-0.0044	0.0162	-0.2695	0.7878
sd11	-0.0219	0.0159	-1.3770	0.1700

Appendix 9

	Estimate	Std. Error	t value	Pr(>|t|)
LPIB.l1	-0.1806	0.0566	-3.1902	0.0016
Balança_Comercial.l1	0.0000	0.0000	-0.8854	0.3770
LTAXA_de_Cambio.l1	0.3974	0.0693	5.7353	0.0000
LPIB.l2	0.0515	0.0585	0.8801	0.3798
Balança_Comercial.l2	0.0000	0.0000	-1.2115	0.2271
LTAXA_de_Cambio.l2	-0.1360	0.0668	-2.0365	0.0430
const	0.0013	0.0024	0.5448	0.5865
sd1	0.0282	0.0135	2.0810	0.0387
sd2	0.0006	0.0129	0.0455	0.9638
sd3	0.0168	0.0138	1.2170	0.2250
sd4	0.0175	0.0133	1.3228	0.1873
sd5	0.0224	0.0122	1.8322	0.0684
sd6	0.0111	0.0124	0.8996	0.3694
sd7	0.0311	0.0128	2.4233	0.0162
sd8	0.0190	0.0127	1.4995	0.1353
sd9	0.0176	0.0124	1.4282	0.1547
sd10	0.0083	0.0129	0.6388	0.5237
sd11	0.0093	0.0127	0.7340	0.4638

Appendix 10

	Estimate	Std. Error	t value	Pr(>|t|)
LPIB.l1	-2952.7835	1419.5009	-2.0802	0.0387
Balança_Comercial.l1	-0.7297	0.0670	-10.8923	0.0000
LTAXA_de_Cambio.l1	-2.0972	1737.7127	-0.0012	0.9990
LPIB.l2	-258.7635	1466.0486	-0.1765	0.8601
Balança_Comercial.l2	-0.2764	0.0673	-4.1048	0.0001
LTAXA_de_Cambio.l2	-143.3009	1674.3380	-0.0856	0.9319
const	37.1739	60.7083	0.6123	0.5410
sd1	707.8941	339.5999	2.0845	0.0383
sd2	1260.7622	322.9083	3.9044	0.0001
sd3	1831.4292	346.4314	5.2866	0.0000
sd4	1831.8137	332.3784	5.5112	0.0000
sd5	1629.7174	307.0415	5.3078	0.0000
sd6	1045.0202	310.2071	3.3688	0.0009
sd7	1245.9772	321.5283	3.8752	0.0001
sd8	979.0566	318.3560	3.0754	0.0024
sd9	470.9500	309.7959	1.5202	0.1300
sd10	309.1872	323.9006	0.9546	0.3409
sd11	1751.8880	318.2013	5.5056	0.0000

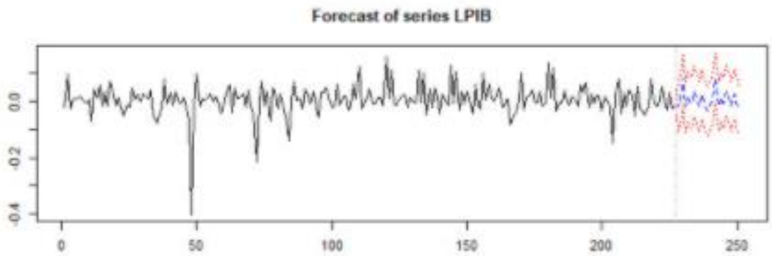
Appendix 11

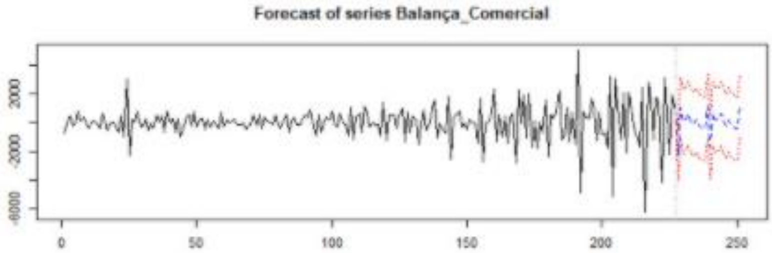
Testes Normalidade		
Teste	Valor	P-value
JB	9728.1	0.0000
Skewness	368.77	0.0000
Kurtosis	9359.3	0.0000

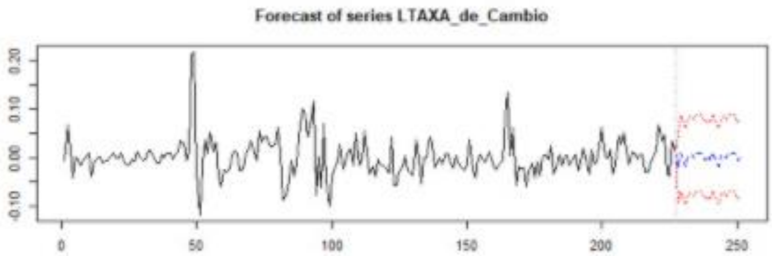
Appendix 12

Testes Diagnósticos				
Testes de autocorrelação nos erros - BG			Teste ARCH	
Lag	Valor	P-Value	Valor	P-Value
3	30.71	0.2831	117.48	0.2508
4	41.17	0.2544	149.02	0.3702
5	45.65	0.4449	178.36	0.5205
6	57.09	0.3608	206.37	0.6694
7	61.7	0.5225	226.57	0.8735
8	73.23	0.4373	255.86	0.9138
9	82.52	0.4318	270.38	0.9864
10	94.06	0.3638	298.18	0.9924

Appendix 13







Appendix 14

$LPIB

	fcst	lower	upper	CI
[1,]	-0.017946147	-0.105540203	0.06964791	0.08759406
[2,]	0.004392004	-0.083451063	0.09223507	0.08784307
[3,]	0.084026350	-0.004044675	0.17209737	0.08807102
[4,]	-0.019314481	-0.107443787	0.06881483	0.08812931
[5,]	0.014818757	-0.073313354	0.10295087	0.08813211
[6,]	-0.006761599	-0.094894387	0.08137119	0.08813279
[7,]	0.036707811	-0.051426098	0.12484172	0.08813391
[8,]	0.011476574	-0.076657659	0.09961081	0.08813423
[9,]	-0.017064617	-0.105198865	0.07106963	0.08813425
[10,]	0.025317215	-0.062817040	0.11345147	0.08813426
[11,]	-0.018065333	-0.106199596	0.07006893	0.08813426
[12,]	-0.036325688	-0.124459954	0.05180858	0.08813427
[13,]	-0.014050028	-0.102184294	0.07408424	0.08813427
[14,]	0.004189354	-0.083944912	0.09232362	0.08813427
[15,]	0.082198055	-0.005936211	0.17033232	0.08813427
[16,]	-0.018248500	-0.106382766	0.06988577	0.08813427
[17,]	0.014469361	-0.073664905	0.10260363	0.08813427
[18,]	-0.006829655	-0.094963921	0.08130461	0.08813427
[19,]	0.036861241	-0.051273025	0.12499551	0.08813427
[20,]	0.011384575	-0.076749691	0.09951884	0.08813427
[21,]	-0.017040927	-0.105175193	0.07109334	0.08813427
[22,]	0.025326271	-0.062807996	0.11346054	0.08813427
[23,]	-0.018079026	-0.106213292	0.07005524	0.08813427
[24,]	-0.036318218	-0.124452484	0.05181605	0.08813427

Appendix 15

$LTAXA_de_Cambio

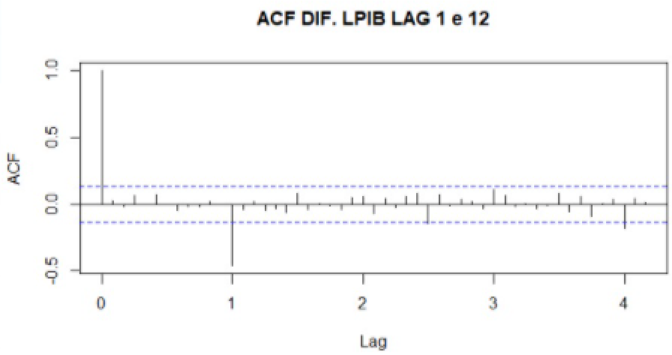
	fcst	lower	upper	CI
[1,]	-0.0215731076	-0.09139923	0.04825301	0.06982612
[2,]	0.0090652842	-0.06902983	0.08716040	0.07809511
[3,]	-0.0037630308	-0.08206325	0.07453719	0.07830022
[4,]	-0.0182977716	-0.09672584	0.06013030	0.07842807
[5,]	0.0041458067	-0.07429989	0.08259151	0.07844570
[6,]	0.0068223704	-0.07162413	0.08526887	0.07844650
[7,]	-0.0007351053	-0.07918244	0.07771223	0.07844734
[8,]	0.0094241745	-0.06902354	0.08787189	0.07844772
[9,]	0.0089191299	-0.06952861	0.08736687	0.07844774
[10,]	0.0095627934	-0.06888495	0.08801053	0.07844774
[11,]	-0.0067126801	-0.08516043	0.07173507	0.07844775
[12,]	-0.0014985197	-0.07994627	0.07694923	0.07844775
[13,]	-0.0092756015	-0.08772335	0.06917215	0.07844775
[14,]	0.0116439177	-0.06680383	0.09009167	0.07844775
[15,]	-0.0048158648	-0.08326361	0.07363188	0.07844775
[16,]	-0.0176639832	-0.09611173	0.06078377	0.07844775
[17,]	0.0036367158	-0.07481103	0.08208446	0.07844775
[18,]	0.0067924540	-0.07165529	0.08524020	0.07844775
[19,]	-0.0006087232	-0.07905647	0.07783903	0.07844775
[20,]	0.0093508183	-0.06909693	0.08779857	0.07844775
[21,]	0.0089468441	-0.06950090	0.08739459	0.07844775
[22,]	0.0095661779	-0.06888157	0.08801393	0.07844775
[23,]	-0.0067235579	-0.08517131	0.07172419	0.07844775
[24,]	-0.0014919205	-0.07993967	0.07695583	0.07844775

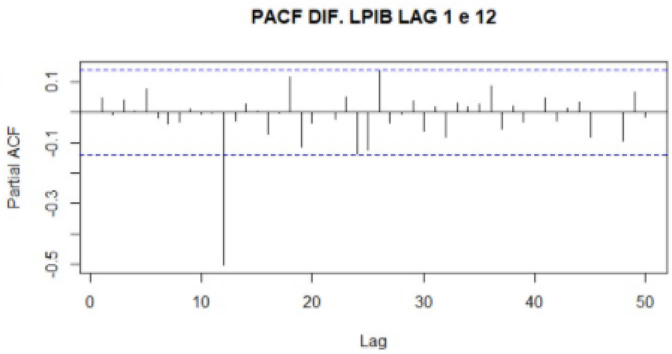
Appendix 16

$Balança_Comercial

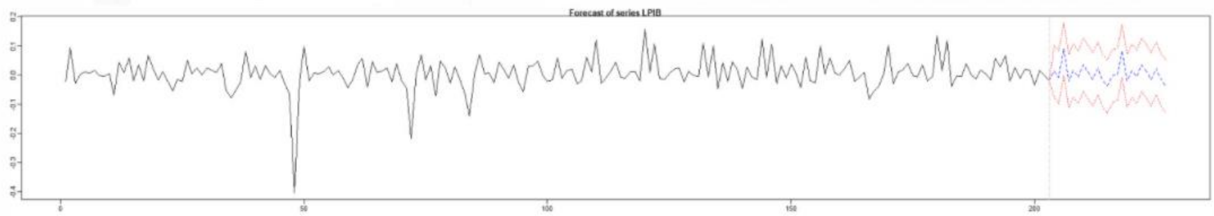
	fcst	lower	upper	CI
[1,]	-2223.686320	-3974.843	-472.5292	1751.157
[2,]	1080.936442	-1097.306	3259.1785	2178.242
[3,]	30.044169	-2196.107	2256.1956	2226.151
[4,]	208.685364	-2018.519	2435.8901	2227.205
[5,]	655.637625	-1577.530	2888.8053	2233.168
[6,]	6.022849	-2229.755	2241.8005	2235.778
[7,]	-176.564356	-2412.593	2059.4642	2236.029
[8,]	214.060183	-2021.984	2450.1047	2236.045
[9,]	-223.093256	-2459.187	2013.0007	2236.094
[10,]	-430.860982	-2666.973	1805.2507	2236.112
[11,]	-437.864738	-2673.978	1798.2482	2236.113
[12,]	1184.456311	-1051.657	3420.5694	2236.113
[13,]	-1681.772639	-3917.886	554.3409	2236.114
[14,]	607.237648	-1628.876	2843.3513	2236.114
[15,]	223.698723	-2012.415	2459.8123	2236.114
[16,]	203.416792	-2032.697	2439.5304	2236.114
[17,]	603.422865	-1632.691	2839.5365	2236.114
[18,]	46.244428	-2189.869	2282.3581	2236.114
[19,]	-191.113758	-2427.227	2044.9999	2236.114
[20,]	213.125998	-2022.988	2449.2396	2236.114
[21,]	-218.175572	-2454.289	2017.9381	2236.114
[22,]	-434.226666	-2670.340	1801.8870	2236.114
[23,]	-436.805229	-2672.919	1799.3084	2236.114
[24,]	1184.651271	-1051.462	3420.7649	2236.114

Appendix 17

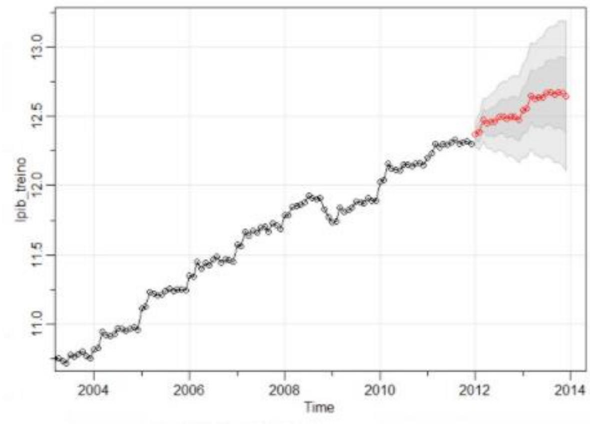




Appendix 18



Appendix 19



Appendix 20

Observação	Previsão Sarima	Valor Real	SE	Previsão VAR	Valor Real	SE
2012-1	0.07340	-0.14769	4.89%	0.01463	-0.14769	2.63%
2012-2	0.01830	0.01710	0.00%	-0.00688	0.01710	0.06%
2012-3	0.09078	0.08023	0.01%	0.08888	0.08023	0.01%
2012-4	-0.02190	-0.04258	0.04%	-0.02108	-0.04258	0.05%
2012-5	0.01738	0.03105	0.02%	0.01614	0.03105	0.02%
2012-6	0.00104	-0.01672	0.03%	-0.00610	-0.01672	0.01%
2012-7	0.03605	0.04078	0.00%	0.03584	0.04078	0.00%
2012-8	0.00992	0.02227	0.02%	0.01143	0.02227	0.01%
2012-9	-0.01956	-0.05227	0.11%	-0.01424	-0.05227	0.14%
2012-10	0.02466	0.05247	0.08%	0.02335	0.05247	0.08%
2012-11	-0.00693	-0.02608	0.04%	-0.01717	-0.02608	0.01%
2012-12	-0.01678	-0.03277	0.03%	-0.03801	-0.03277	0.00%
2013-1	0.07448	-0.04828	1.51%	-0.00226	-0.04828	0.21%
2013-2	0.01939	-0.01768	0.14%	0.00405	-0.01768	0.05%
2013-3	0.09187	0.08078	0.01%	0.08268	0.08078	0.00%
2013-4	-0.02081	0.00919	0.09%	-0.01875	0.00919	0.08%
2013-5	0.01847	-0.01819	0.13%	0.01539	-0.01819	0.11%
2013-6	0.00212	-0.01242	0.02%	-0.00596	-0.01242	0.00%
2013-7	0.03714	0.05042	0.02%	0.03586	0.05042	0.02%
2013-8	0.01101	0.00028	0.01%	0.01140	0.00028	0.01%
2013-9	-0.01847	-0.02998	0.01%	-0.01422	-0.02998	0.02%
2013-10	0.02575	0.03216	0.00%	0.02334	0.03216	0.01%
2013-11	-0.00584	-0.02559	0.04%	-0.01717	-0.02559	0.01%
2013-12	-0.01569	-0.01112	0.00%	-0.03801	-0.01112	0.07%
MSE			0.30%	MSE		0.15%