CS 132 – Spring 2020, Assignment 3

Answer 1.

First lets build up the matrix from these values:

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix}$$

If we solve matrix, we are left with:

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 1 & \frac{4}{5} & \frac{1}{5} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We can see that this system is inconsistent, as such, there is no linear combination of a_1, a_2 and a_3 that leads to b.

Answer 2.

Let's arrange this in a matrix:

$$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{bmatrix}$$

If we solve this we get:

$$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 0 & -3+12h \end{bmatrix}$$

For the system to be consistent, $-3 + 12h = 0$, so, h must equal $\frac{1}{4}$.

Answer 3.

A - $x_1 \times 27.6 \times 10^6 + x_2 \times 30.2 \times 10^6$

B -

Let a be the values for A and v_2 the values for B:

$$a = \begin{bmatrix} 27.6 \\ 3100 \\ 250 \end{bmatrix}$$

$$b = \begin{bmatrix} 30.2 \\ 6400 \\ 360 \end{bmatrix}$$

With that in mind, the linear equation would be: $x_1 \times a + x_2 \times b$:

$$\begin{bmatrix} 27.6 \times x_1 + 30.2 \times x_2 \\ 3100 \times x_1 + 6400 \times x_2 \\ 250 \times x_1 + 360 \times x_2 \end{bmatrix}$$

C -

$$\begin{bmatrix} 27.6 & 30.2 & 162 \\ 3100 & 6400 & 23610 \\ 250 & 360 & 1623 \end{bmatrix}$$

Solving this we get:

$$\begin{bmatrix} 1 & 0 & 3.9 \\ 0 & 1 & 1.8 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $x_1 = 3.9$ and $x_2 = 1.8$

```
A = np.array([[27.6, 30.2, 162], [3100, 6400, 23610], [250, 360, 1623]])
AEchelon = forwardElimination(A)
if (not inconsistentSystem(AEchelon)):
    AReducedEchelon = backsubstitution(AEchelon)


print(AReducedEchelon)
```

```
[[1.  0.  3.9]
 [0.  1.  1.8]
 [0.  0.  0. ]]
```

Answer 4.

This computation is impossible, as we have an extra weight when compared to the number of values available.

Answer 5.

$$\begin{bmatrix} 8 & 3 & -4 \\ 5 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8+3-4 \\ 5+1+2 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

Answer 6.

$$-2 \times \begin{bmatrix} 7 \\ 2 \\ 9 \\ -3 \end{bmatrix} - 5 \times \begin{bmatrix} -3 \\ 1 \\ -6 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$$

Answer 7.

$$\begin{bmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{bmatrix} \times \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$$

Answer 8.

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & \frac{1}{5} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

So: $-\frac{2}{5} \times x_1 + \frac{1}{5} \times x_2$

Answer 9.

If u is a subset of R^3 spanned by the columns of A then the following matrix must have a solution.

$$\begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{bmatrix}$$

If we try solving this matrix, we get:

$$\begin{bmatrix} 1 & \frac{8}{5} & \frac{7}{5} & \frac{2}{5} \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & \frac{29}{7} \end{bmatrix}$$

This is an inconsistent matrix, hence u cannot be a subset of R^3 spanned by the columns of A.

Answer 10.

The matrix must have a pivot value in each row when reduced so it can span R^4 so if we solve B we get:

$$\begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

As the third row doesn't have a pivot value, B does not span R^4, furthermore, if we add random values a, b, c and d to give us an augmented matrix, we will reach the same conclusion, as there is only one value that c can take up for it to be true, hence $Bx = y$ is not true for every y.

Answer 11.

We've already seen that the columns of be don't span R^4, as shown above.
With that said, if each vector in R^4 can be represented by the columns of B, then:

$$\begin{bmatrix} 1 & 3 & -2 & 2 & 1 \\ 0 & 1 & 1 & -5 & 1 \\ 1 & 2 & -3 & 7 & 1 \\ -2 & -8 & 2 & -1 & 1 \end{bmatrix}$$

If we solve this, then we get that $x_1 = 0.2$, $x_2 = -0.4$, $x_3 = 0$, $x_4 = -0.5$, meaning that we can get every unit vector in R^4 with those weights, thus we can represent any other vector in that space.

Answer 12.

Applying the same logic as before, but this time using python, if we plug the matrix into the code:

$$\begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 5 & -12 \end{bmatrix}$$

Since each of these rows has a pivot value, for which we can operate, we can further reduce them thus getting them to span R^4.

```
#####################


A = np.array([[8, 11, -6, -7, 13], [-7, -8, 5, 6, -9], [11, 7, -7, -9, -6], [-3, 4, 1, 8, 7]])
AEchelon = forwardElimination(A)
#if (not inconsistentSystem(AEchelon)):
#   AReducedEchelon = backsubstitution(AEchelon)


print(AEchelon)
```

```
[[  8  11  -6  -7  13]
 [  0   1   0   0   2]
 [  0   0   1   0  -7]
 [  0   0   0   5 -12]]
```