CS 332 – Spring 2021, Assignment 4

Colaborators: None

I assumed that every TM tape is infinite in both directions, and that slot -1, -2, ... as well as the slots after the string are just $\sqcup$. *Since I did this for basically every exercise I thought I would note it here.*

Answer 1

(a)

(i) $q_1 010\#111$
$\text{x}q_2 10\#111$
$\text{x}1q_2 0\#111$
$\text{x}10q_2\#111$
$\text{x}10\#q_4 111$
$\text{x}10q_6\#\text{x}11$
$\text{x}1q_7 0\#\text{x}11$
$\text{x}q_7 10\#\text{x}11$
$q_7\text{x}10\#\text{x}11$
$\text{x}q_1 10\#\text{x}11$
$\text{xx}q_3 0\#\text{x}11$
$\text{xx}0q_3\#\text{x}11$
$\text{xx}0\#q_5\text{x}11$
$\text{xx}0\#\text{x}q_5 11$
$\text{xx}0\#q_6\text{xx}1$
$\text{xx}0q_6\#\text{xx}1$
$\text{xx}q_7 0\#\text{xx}1$
$\text{x}q_7\text{x}0\#\text{xx}1$
$\text{xx}q_1 0\#\text{xx}1$
$\text{xxx}q_2\#\text{xx}1$
$\text{xxx}\#q_4\text{xx}1$
$\text{xxx}\#\text{x}q_4\text{x}1$
$\text{xxx}\#\text{xx}q_4 1$
$\text{xxx}\#\text{x}q_6\text{xx}$
$\text{xxx}\#q_6\text{xxx}$
$\text{xxx}q_6\#\text{xxx}$
$\text{xx}q_7\text{x}\#\text{xxx}$
$\text{xxx}q_1\#\text{xxx}$
$\text{xxx}\#q_8\text{xxx}$
$\text{xxx}\#\text{x}q_8\text{xx}$
$\text{xxx}\#\text{xx}q_8\text{x}$
$\text{xxx}\#\text{xxx}q_8$
$\text{xxx}\#\text{xxx}\sqcup q_{accept}$

(ii) $q_1 101\#110$
$xq_3 01\#110$
$x0q_3 1\#110$
$x01q_3\#110$
$x01\#q_5 110$
$x01q_6\#x10$
$x0q_7 1\#x10$
$xq_7 01\#x10$
$q_7 x01\#x10$
$xq_1 01\#x10$
$xxq_2 1\#x10$
$xx1q_2\#x10$
$xx1\#q_4 x10$
$xx1\#xq_4 10$
$xx1\#q_6 xx0$
$xx1q_6\#xx0$
$xxq_7 1\#xx0$
$xq_7 x1\#xx0$
$xxq_1 1\#xx0$
$xxxq_3\#xx0$
$xxx\#q_5 xx0$
$xxx\#xq_5 x0$
$xxx\#xxq_5 0$
$xxx\#xx0q_{reject}$

(iii) $q_1 0\#\#0$
$xq_2\#\#0$
$x\#q_4\#0$
$x\#\#q_{reject}0$

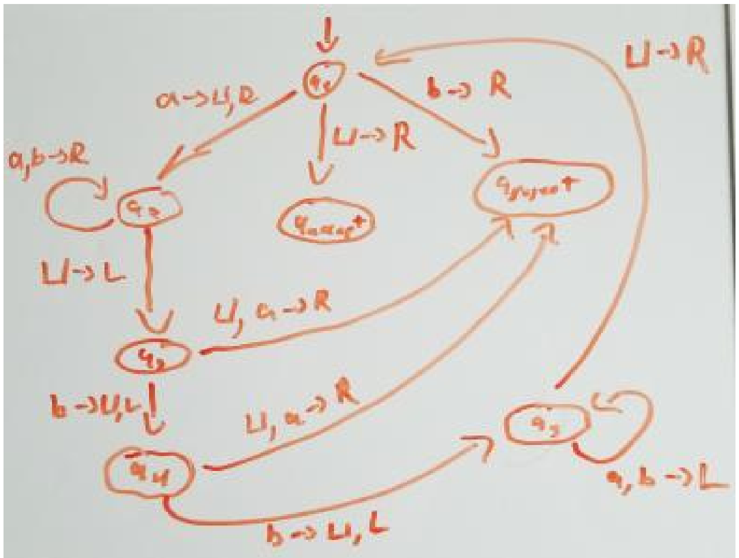
b) This TM gets a string, and checks whether it is a part of the language $L = \{wy \| (w \in \{0,1\}^* \wedge y \in \{1\}^*) \wedge \|y\| = \|w\|\}$, it starts by checking the first symbol of the string, and regardless of whether it is a 0 or a 1, it replaces it with an x and moves to the right until it finds the central $\#$, then it replaces the first non-x symbol for an x and it begins moving left until it finds the central $\#$ again. It then proceeds to move left until it finds an x, then moving right and restarting the process.

If the machine encounters any 0's in the second half, it halts and rejects, while it only accepts after finally reaching the middle $\#$ when every other character is an x.

c) $L = \{wy \| (w \in \{0,1\}^* \wedge y \in \{1\}^*) \wedge \|y\| = \|w\|\}$

Answer 2

(a)



(b)

(i) $q_1aabbbb$
q_2abbbb
aq_2bbbb
abq_2bbb
$abbq_2bb$
$abbbq_2b$
$abbbbq_2$
$abbbq_3b$
$abbq_4b$
abq_5b
aq_5bb
q_5abb
$q_5 \sqcup abb$
q_1abb
q_2bb
bq_2b
bbq_2
bq_3b
q_4b
q_5
q_1
q_{accept}

(ii) q_1aabb

q_2abb

aq_2bb

abq_2b

$abbq_2$

abq_3b

aq_4b

q_5a

$q_5 \sqcup a$

q_1a

q_2

q_3

q_{reject}

c) $L = \{wy \| w \in \{a\}^n \wedge y \in \{b\}^{2n}\}$

Answer 3

(a) My BUID is U55891002, so $u = 55891002$, $s = 30$ and $x = 0011110$

(b)

(i) This Turing machine will have 2 states, plus an accept and a reject state. The starting state q_0, will represent a string with an even number of 1's, while q_1 will represent an odd number of ones.

Once a string enters q_0, if it finds a 0, it'll move right, and replace that 0 with $\sqcup$ while staying in the same state. If it finds $\sqcup$ it rejects, and if it finds a 1, it erases that 1, moves to the right, and goes to q_1. In q_1, if it finds a 0 it'll erase it, move right, and stay in the same state, if it finds a 1, it erases it, moves right, and goes to q_0, and finally if it finds $\sqcup$ it accepts the string.

(ii)

```
; State 0 - Even number of 1's
0 0 _ r 0
0 1 _ r 1
0 _ _ * halt-reject

; State 1 - Odd number of 1's
1 0 _ r 1
1 1 _ r 0
1 _ _ * halt-accept
```

(iii)

(1) $q_0 00$
$q_0 0$
q_0
q_{reject}

(2) $q_0 00111$
$q_0 0111$
$q_0 111$
$q_1 11$
$q_0 1$
q_1
q_{accept}

(3) $q_0 0011110$
$q_0 011110$
$q_0 11110$
$q_1 1110$
$q_0 110$
$q_1 10$
$q_0 0$
q_0
q_{reject}

(c)

(i) This Turing machine will have 6 states, plus an accept and a reject state. The starting state will be q_0.

Once in q_0, if the machine will loop through the string from left to right until it finds a 0, or until it finds a $\sqcup$. If it finds a 0, it replaces it with an x, moves right, and goes to q_1. If it manages to reach the end of the string without finding a 0, it halts and accepts the input.

While in q_1 the TM will loop through from left to right until it finds a 1. If it doesn't find any, it'll halt and reject the input. Otherwise once it finds a 1, it'll go to q_2, replacing the 1 with an x and move right. Once in q_2 it'll start moving left without changing the string until it finds a 0, when that happens it'll replace it with an x, move right, and go to q_5. If it reaches $\sqcup$ then it will move left, and go to q_3.

Once in q_3, it'll begin iterating through the entire string from left to right again, until it either reaches a 0, or $\sqcup$. If it reaches a 0, it'll move left, replace the 0 with an x and go to q_5. If it finds a $\sqcup$ it'll move left, and go to q_4. In q_4, the TM will preform a final iteration of the String, replacing every x and 1 with $\sqcup$, moving left, once it finds a $\sqcup$ it'll move right, halt and accept.

In q_5, the TM will backtrack to the start of the string (leftmost symbol) without altering anything, and once it finds a $\sqcup$ it'll move right, and go back to q_1.

In hindsight, the final q_4 state wouldn't be needed, as by then we have no zeros, so nothing to deal with, I only realized this after I'd done the walk through for every single example, anb since the machine works regardless I just decided to let it be.

(ii)

```
; State q0
0 x x r 0
0 1 1 r 0
0 0 x r 1
0 _ _ r halt-accept

; State q1
1 x x r 1
1 0 0 r 1
1 1 x r 2
1 _ _ r halt-reject

; State q2
2 1 1 l 2
2 x x l 2
2 0 x r 5
2 _ _ r 3

; State q3
3 1 1 r 3
3 x x r 3
3 0 x l 5
3 _ _ r 4

; State q4
4 1 _ l 4
4 x _ l 4
4 _ _ r halt-accept

; State q5
5 x x l 5
5 1 1 l 5
5 0 0 l 5
5 _ _ r 1
```

(iii)

(1) q_000
q_10
$0q_1$
q_{reject}

(2) q_000111
q_10111
$0q_1111$
$0xq_211$
$0q_2x11$
q_20x11
xq_5x11
q_5xx11
$q_5\sqcup xx11$
q_1xx11
xq_1x11
xxq_111
$xxxq_21$
xxq_2x1
xq_2xx1
q_2xxx1
$q_2\sqcup xxx1$
q_3xxx1
xq_3xx1
xxq_3x1
$xxxq_31$
$xxx1q_3$
$xxxq_41$
xxq_4x
xq_4x
q_4x
q_4
q_{accept}

(3) $q_00011110$
$q_1011110$
$0q_111110$
$0xq_21110$
$0q_2x1110$
$q_20x1110$
xq_5x1110
$q_5xx1110$
$q_5\sqcup xx1110$
$q_1xx1110$
xq_1x1110
xxq_11110
$xxxq_2110$
xxq_2x110
xq_2xx110

$q_2xxx110$
$q_2 \sqcup xxx110$
$q_3xxx110$
xq_3xx110
xxq_3x110
$xxxq_3110$
$xxx1q_310$
$xxx11q_30$
$xxx1q_51x$
$xxxq_511x$
xxq_5x11x
xq_5xx11x
$q_5xxx11x$
$q_5 \sqcup xxx11x$
$q_1xxx11x$
xq_1xx11x
xxq_1x11x
$xxxq_111x$
$xxxxq_21x$
$xxxq_2x1x$
xxq_2xx1x
xq_2xxx1x
$q_2xxxx1x$
$q_2 \sqcup xxxx1x$
$q_3xxxx1x$
xq_3xxx1x
xxq_3xx1x
$xxxq_3x1x$
$xxxxq_31x$
$xxxx1q_3x$
$xxxx1xq_3$
$xxxx1q_4x$
$xxxxq_41$
$xxxq_4x$
xxq_4x
xq_4x
q_4x
q_4
q_{accept}

Answer 4

a) Since we already know that the set of decidable languages is closed under itself, and we know that, to invert a decidable language TM (ie: a TM that halts on every input, either accepting or rejecting), all we have to do is turn the accept state in the original TM to a reject state, and vice versa with the reject state, and thus we have the complement of the original TM.

Since no alterations to transitions were made, we know that this TM is still a decidable TM, thus, the class of decidable TMs is closed under complement.

b) Unlike decidable languages, recognizable languages are all languages that for some TM M, all input strings are either halt-accepted, halt-rejected, or run forever in the TM. This last part, makes it harder to complement a language, since we cannot, as we did above, just reverse the accept and reject states, rather we need to find a way for all the inputs that would run forever to be accepted. Thus my construction above is not enough to prove that all recognizable languages are closed under complement.

Since if a string runs forever in the TM, it isn't apart of the language, then if we complement the language these strings must be apart of it, thus we need to find a way to get the TM to accept them, but switching accept/reject states isn't enough.