Answer 1

(a) If $p_1 = \{1, 6, 5\}$ and $p_2 = \{5, 3, -7\}$, then $\overrightarrow{v_2} = p_2 - p_1 = \{4, -3, -9\}$.

(b) If $p_1 = \{1, 6, 5\}$ and $p_3 = \{1, 6, 4\}$, then $\overrightarrow{v_3} = p_3 - p_1 = \{0, 0, -1\}$.

(c) $\|\overrightarrow{v_2}\| = \sqrt{4^2 + (-3)^2 + (-9)^2} = 10.2956$ and $\|\overrightarrow{v_3}\| = \sqrt{0^2 + 0^2 + (-1)^2} = 1$.

(d) $u_2 = v_2 / \|\overrightarrow{v_2}\| = \{4/10.1956, -3/10.2956, -9/10.2956\} = \{0.3885, -0.2914, -0.8742\}$ and $u_3 = v_3 / \|\overrightarrow{v_3}\| = \{0/1, 0/1, -1/1\} = \{0, 0, -1\}$.

Answer 2

(a) Using the following formula:

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$$

Then we have that:

$$\overrightarrow{v_2} \times \overrightarrow{v_3} = \begin{bmatrix} (-3 \times -1) - (-9 \times 0) \\ (-9 \times 0) - (4 \times -1) \\ (4 \times 0) - (-3 \times 0) \end{bmatrix} = \{3, -3, 0\}.$$

(b) Using the same formula as above, we have that:

$$\overrightarrow{v_3} \times \overrightarrow{v_2} = \begin{bmatrix} (0 \times -9) - (-1 \times -3) \\ (-1 \times 4) - (0 \times -9) \\ (0 \times -3) - (0 \times 4) \end{bmatrix} = \{-3, 3, 0\}.$$

(c) Using the following formula:

$$\overrightarrow{a} \cdot \overrightarrow{b} = a_1b_1 + a_2b_2 + a_3b_3$$

Then we have that:

$$\overrightarrow{v_2} \cdot \overrightarrow{v_3} = (4 \times 0) + (-3 \times 0) + (-9 \times -1) = 9$$

Answer 3

The dot product of two orthogonal vectors is 0.

Answer 4

Considering that a Unit Vector is a vector with magnitude of 1, then both (b) and (c) are Unit Vectors.

Answer 5

(a) Using the following formula:

$cos\theta = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{\|\overrightarrow{a}\| \cdot \|\overrightarrow{b}\|}$

We have that:

$cos\theta = \frac{\overrightarrow{v_2} \cdot \overrightarrow{v_3}}{\|\overrightarrow{v_2}\| \cdot \|\overrightarrow{v_3}\|} = \frac{9}{10.2956 \cdot 1\|} = cos\theta = 0.87416.$

(b) Using the following formula:

$sin\theta = \frac{\|\overrightarrow{a}\overrightarrow{b}\|}{\|\overrightarrow{a}\| \times \|\overrightarrow{b}\|}$

We have that:

$sin\theta = \frac{\|v_2 v_3\|}{\|v_2\| \times \|v_3\|} = \frac{\sqrt{3^2+(-3)^2+0^3}}{10.2956 \times 1} = \frac{3}{10.2956} = 0.291387.$

(c) Using the formula for the cross product:

$\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c}$ where $\overrightarrow{c} \cdot \overrightarrow{a} = 0$ and $\overrightarrow{c} \cdot \overrightarrow{b} = 0$, then $\overrightarrow{c}$ is perpendicular to both a and b.

Then we already have a previously calculated answer to $\overrightarrow{v_2} \times \overrightarrow{v_3} = \overrightarrow{v_p} = \{3, -3, 0\}$.

Answer 6

Given that Q, R, and S are square matrices, then, in general, both (c) and (d) are true.

Answer 7

(a) If the columns of A form an orthonormal basis, that means they are perpendicular amongst themselves. With that in mind, the dot product of any two columns will always be 0.

(b) The inverse of a matrix A (A^{-1}) whoose columns form an orthonormal basis is equal to A^T.