CS 132 – Spring 2020, Assignment 8

Answer A

For u to be in the subspace generated by A = v1, v2, v3, then there is some x = [x1, x2, x3] where $Ax = b$, hence if we solve this:

$$\boxed{\begin{bmatrix} 1 & 4 & 5 & -4 \\ -2 & -7 & -8 & 10 \\ 4 & 9 & 6 & -7 \\ 3 & 7 & 5 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ 4 & 9 & 6 & -7 \\ 3 & 7 & 5 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

$\boxed{\text{Hence, u is not in the subspace generated by v1, v2, v3.}}$

Answer B

For p to be in Col A then Ax = p must be consistent for some x:

$$\boxed{\begin{bmatrix} -3 & -2 & 0 & 1 \\ 0 & 2 & -6 & 14 \\ 6 & 3 & 3 & -9 \end{bmatrix} \equiv \begin{bmatrix} 1 & \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & 1 & -3 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{cases} x_1 = -\frac{1}{3} - \frac{2}{3}x_2 \\ x_2 = 7 + 3x_3 \\ x_3 - free \end{cases}}$$

$\boxed{\text{Hence, p is in Col A}}$

Answer C

For u to be in Nul A, then Au = 0 must have be consistent:

$$\boxed{\begin{bmatrix} -3 & -2 & 0 \\ 0 & 2 & -6 \\ 6 & 3 & 3 \end{bmatrix} * \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} 6 - 6 + 0 \\ 0 + 6 - 6 \\ -12 + 9 + 3 \end{bmatrix} \equiv \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$\boxed{\text{Hence, u is in Nul A}}$

Answer D

$\boxed{\text{p = 3 q = 4}}$

Answer E

To find a vector u in Nul A, we need to solve Ax = 0:

$$\boxed{\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 7 & 0 \\ -5 & -1 & 0 & 0 \\ 2 & 7 & 11 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & \frac{5}{3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{cases} x_1 = -2x_2 - 3x_3 \\ x_2 = -\frac{5}{3}x_2 \\ x_3 - free \end{cases}}$$

$\boxed{\text{For } x_3 = 3 \text{ than } x_2 = -5 x_1 = 1}$

Answer F

$\boxed{\text{These sets are not bases for } R^2 \text{ as they are linearly dependent, as the first is the second times -2.}}$

Answer G

In order to find if these columns are a basis for R^3 we need to make sure they row reduce into a matrix with 3 pivots:

$$\boxed{\begin{bmatrix} 1 & 3 & -2 & 0 \\ -6 & -4 & 7 & 8 \\ -7 & 7 & 5 & 9 \end{bmatrix} \equiv \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 14 & -5 & 8 \\ 0 & 0 & -9 & -7 \end{bmatrix}}$$

$\boxed{\text{Hence these columns are a subspace of } R^3 \text{ as they span } R^3}$

Answer H

The basis of Col A are the pivot columns of A so:

$$\boxed{\text{The Basis of Col A} = \begin{bmatrix}-3\\2\\3\end{bmatrix}, \begin{bmatrix}-2\\4\\-2\end{bmatrix}}$$

As for the Basis of Nul A we must solve Ax = 0:

$$\boxed{\begin{bmatrix}1 & -3 & 6 & 9 & 0\\0 & 0 & 4 & 5 & 0\\0 & 0 & 0 & 0 & 0\end{bmatrix} \equiv \begin{bmatrix}1 & -3 & 0 & \frac{3}{2} & 0\\0 & 0 & 1 & \frac{5}{4} & 0\\0 & 0 & 0 & 0 & 0\end{bmatrix} \equiv \begin{cases} x_1 = 3x_2 - \frac{3}{2}x_4 \\ x_2 - free \\ x_3 = -\frac{5}{4}x_4 \\ x_4 - free \end{cases} \equiv x_2 \times \begin{bmatrix}3\\1\\0\\0\end{bmatrix} + x_4 \times \begin{bmatrix}-\frac{3}{2}\\0\\-\frac{5}{4}\\1\end{bmatrix}}$$

$$\boxed{\text{The Basis of Nul A} = \begin{bmatrix}3\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-\frac{3}{2}\\0\\-\frac{5}{4}\\1\end{bmatrix}}$$

Answer I

The basis of Col A are the pivot columns of A so:

$$\boxed{\text{The Basis of Col A} = \begin{bmatrix}3\\-2\\-5\\-2\end{bmatrix}, \begin{bmatrix}-1\\2\\9\\6\end{bmatrix}, \begin{bmatrix}3\\7\\3\\3\end{bmatrix}}$$

As for the Basis of Nul A we must solve Ax = 0:

$$\boxed{\begin{bmatrix}3 & -1 & 7 & 0 & 6 & 0\\0 & 2 & 4 & 0 & 3 & 0\\0 & 0 & 0 & 1 & 1 & 0\\0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \equiv \begin{bmatrix}1 & 0 & 3 & 0 & \frac{15}{6} & 0\\0 & 1 & 2 & 0 & \frac{3}{2} & 0\\0 & 0 & 0 & 1 & 1 & 0\\0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \equiv \begin{cases}x_1 = -3x_3 - \frac{15}{6}x_5\\x_2 = -2x_3 - \frac{3}{2}x_5\\x_3 - free\\x_4 = -x_5\\x_5 = free\end{cases} \equiv x_3 \times \begin{bmatrix}-3\\-2\\1\\0\\0\end{bmatrix} + x_5 \times \begin{bmatrix}-\frac{15}{6}\\-\frac{3}{2}\\0\\-1\\1\end{bmatrix}}$$

$$\boxed{\text{The Basis of Nul A} = \begin{bmatrix}-3\\-2\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-\frac{15}{6}\\-\frac{3}{2}\\0\\-1\\1\end{bmatrix}}$$

Answer J

a. $\boxed{\text{True}}$

This is true as R^2 fulfill every requirement to be a subspace of R^3, as it contains the 0 point, and each Vector in R^3 can be transformed into R^2, and for every u and v in R^2 u+v is also in R^2, and the same goes for any v scaled up by a scalar c.

b. $\boxed{\text{True}}$

As the basis for Nul A will be the solution of Ax = 0.

c. $\boxed{\text{False}}$

An infinite-dimensional space is spanned by a vector, or group of vectors, of infinite-size, not by an infinite set.

d. $\boxed{\text{False}}$

As per the definition of a Basis, if S spans V, then S is a basis of V if and only if S is an independent set.

e. $\boxed{\text{True}}$

Yes, as any combination that forms a 3 dimensional subspace in R^3, will eventually form R^3 itself by a combination of addition and scalars.

Answer K

Finding Col A is easy, as it's simply the pivot columns of A, so:

$$\boxed{\text{Col A} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 1 \\ 1 \end{bmatrix}}$$

$$\boxed{\text{The dimension of Col A is 3}}$$

As for Nul A, we must first solve Ax = 0:

$$\begin{bmatrix} 1 & -2 & 9 & 5 & 4 & 0 \\ 0 & 1 & -3 & 0 & -7 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 3 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 & -7 & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{cases} x_1 = -3x_3 \\ x_2 = 3x_3 + 7x_5 \\ x_3 - free \\ x_4 = 2x_5 \\ x_5 - free \end{cases} \equiv x_3 \begin{bmatrix} -3 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 7 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\boxed{\text{The Basis of Nul A} = \begin{bmatrix} -3 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 0 \\ 2 \\ 1 \end{bmatrix}}$$

$$\boxed{\text{The dimension of Nul A is 2}}$$

Answer L

Finding Col A is easy, as it's simply the pivot columns of A, so:

$$\boxed{\text{Col A} = \begin{bmatrix} 1 \\ 5 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ -9 \\ -9 \\ 5 \end{bmatrix}}$$

$$\boxed{\text{The dimension of Col A is 2}}$$

As for Nul A, we must first solve Ax = 0:

$$\begin{bmatrix} 1 & 2 & -4 & 3 & 3 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 2 & 0 & -5 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{cases} x_1 = -2x_2 + 5x_4 \\ x_2 - free \\ x_3 = 2x_4 \\ x_4 - free \\ x_5 = 0 \end{cases} \equiv x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 5 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{\text{The Basis of Nul A} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}}$$

$$\boxed{\text{The dimension of Nul A is 2}}$$

Answer M

To find the basis for the column space H, we must first row reduce the matrix formed by these vector:

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ -1 & -3 & 2 & 4 & -8 \\ -2 & -1 & -6 & -7 & 9 \\ 5 & 6 & 8 & 7 & -5 \end{bmatrix} \equiv \begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & 1 & -2 & -3 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\text{The Basis of H} = \begin{bmatrix} 1 \\ -1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ -1 \\ 6 \end{bmatrix}}$$

$$\boxed{\text{The dimension of H is 2}}$$

Answer N

$\boxed{\text{rank A + dim Nul A = n where n is the number of columns of A, so: } \boxed{rankA + 3 = 5 \equiv rankA = 2}}$

Answer O

A rank 1 matrix has a one-dimensional column space, hence A =

$$\boxed{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

Answer P

As per Rank Theory $\boxed{rankA + dimNulA = n \equiv colA = 6 - 5 = 1}$

Answer Q

As per Rank Theory dim Col A + dim Nul A = n, so, if we know that n = 4, we have that dim Col A + dim Nul A = 4.

At most this matrix can have 4 pivots, so, if A spans R^4, $\boxed{dimNulA = 0}$

Answer R

The matrix would be a 6x8, since it's made up of 6 equations with 8 unknowns.
Since we also know that it has 2 free variables, and that dim Nul A will be equal to the number of free variables, then $dimNulA = 2$.
We also know that rank A = n - dim Nul A, so $rankA = 6$.

$\boxed{\text{Putting this together, we know that Col A is a six dimensional subspace of } R^6\text{, but ColA = 6.}}$

$\boxed{\text{This means that Ax = b has a solution for every b, and we cannot change any constants to make it inconsistent.}}$