Answer 1.

(A)

Flow in A*: in:* $x_1 = 40 + x_3 + x_4$

Flow in B*: in:* $200 = x_1 + x_2$

Flow in C*: in:* $x_2 + x_3 = 100 + x_5$

Flow in D*: in:* $x_4 + x_5 = 60$

If we re-organize this into a matrix we get that:

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

With that we can re-organize into a system of equation:

$$\begin{cases} x_1 = 40 + x_3 + x_4 \\ x_2 = 200 - x_1 \\ x_3 = 100 + x_5 - x_2 \\ x_4 = 60 - x_5 \\ x5 - free \end{cases}$$

B

If $x_4 = 0$ *then* $x_5 = 60$*, so applying the system from A we get that:*

$$\begin{cases} x_1 = 40 + x_3 \\ x_2 = 200 - x_1 \\ x_3 = 100 + 60 - x_2 \\ x_4 = 0 \\ x5 = 60 \end{cases}$$

C *With* $x_4 = 0$*, then the minimum value of* x_1 *is 40.*

Answer 2.

***Flow in A**: in:* $x_1 = 100 + x_2$
***Flow in B**: in:* $x_2 + 50 = x_3$
***Flow in C**: in:* $x_3 = 120 + x_4$
***Flow in D**: in:* $x_4 + 150 = x_5$
***Flow in E**: in:* $x_5 = 80 + x_6$
***Flow in F**: in:* $x_6 + 100 = x_1$
If we re-organize this into a matrix we get that:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ -1 & 0 & 0 & 0 & 0 & 1 & -100 \end{bmatrix} \equiv \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

With this we get the following system of equations where x_6 is free:

$$\begin{cases} x_1 = 100 + x_2 \\ x_2 = x_3 - 50 \\ x_3 = 120 + x_4 \\ x_4 = x_5 - 150 \\ x5 = 80 + x_6 \\ x_6 - free \end{cases}$$

The smallest value for x_6 is 0, since it can never be negative.

Answer 3.

For these vectors to be dependent, then the following matrix must be true:

$$\begin{bmatrix} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & -8 & 1 & 0 \end{bmatrix}$$

If we solve this system, we get the following:

$$\begin{bmatrix} 1 & -4 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{4}{5} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Since there are pivot values for every column and no free variables, the equations are linearly independent

Answer 4.

For these vectors to be dependent, then the following matrix must be true:

$$\begin{bmatrix} -1 & -2 & 0 \\ 4 & -8 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 2 & 0 \\ 0 & -16 & 0 \end{bmatrix}$$

As we can see, there are no free variables, and the only solution is the trivial one, hence the vectors are linearly independent.

Answer 5.

If we solve the matrix we get that:

$$\begin{bmatrix} -4 & -3 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 1 & 0 & 3 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

As we can see, there are no free variables, and the only solution is the trivial one, hence the vectors are linearly independent.

Answer 6.

If we solve the matrix we get that:

$$\begin{bmatrix} 1 & -3 & 3 & -2 & 0 \\ -3 & 7 & -1 & 2 & 0 \\ 0 & 1 & -4 & 3 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & -3 & 3 & -2 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

As we can see, there are no free variables, and the only solution is the trivial one, hence the vectors are linearly independent.

Answer 7.

(A)

If we put the vectors in a matrix and solve it we get that:

$$\begin{bmatrix} 1 & -2 & 2 \\ -5 & 10 & -9 \\ -3 & 6 & h \end{bmatrix} \equiv \begin{bmatrix} 1 & -2 & 2 \\ 0 & 0 & 1 \\ -3 & 6 & h \end{bmatrix}$$

Since we already reached an impossibility here, then no matter the value of h, v_3 isn't in the span of v_1 and v_2.

(B)

If we put this into an augmented matrix, and solve it we get that:

$$\begin{bmatrix} 1 & -2 & 2 & 0 \\ -5 & 10 & -9 & 0 \\ -3 & 6 & h & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & -2 & 2 & 0 \\ 0 & 0 & h+6 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The only value for which the system is linearly dependent is when $h = -6$.

Answer 8.

If we put the vectors in a matrix and solve it we get that:

$$\begin{bmatrix} 2 & -6 & 8 & 0 \\ -4 & 7 & h & 0 \\ 1 & -3 & 4 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & -3 & 4 & 0 \\ 0 & 1 & \frac{12+h}{-5} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since we have a free variable and a pivot in every column then the system is linearly dependent.

Answer 9.

If we put the vectors in a matrix and solve it we get that:

$$\begin{bmatrix} 1 & -5 & 1 & 0 \\ -1 & 7 & 1 & 0 \\ 3 & 8 & h & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & h+26 & 0 \end{bmatrix}$$

Since a free variable is required for the system to be linearly dependent, then $h = -26$ so that the last variable becomes a free variable, hence we have a linearly dependent system.

Answer 10.

$$\begin{bmatrix} 1 & * & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Answer 11.

$$\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Answer 12.

5

Answer 13.

A1

$AxIP = [17, 13, 15, -3.552713678800501e - 15, 1, 4.]$

$AxVS = [17, 13, 15, -3.552713678800501e - 15, 1, 4]$

A2

$AxIP = [10, 9, 8, 7, 6, 5]$

$AxVS = [10, 9, 8, 7, 6, 5]$