CS 132 – Spring 2020, Assignment 1

Answer 1.

$$2x_1 + 4x_2 = -4$$
$$5x_1 + 7x_2 = 11$$

By multiplying the first equation by -2.5 and adding it to the second one, we get that:

$$2x_1 + 4x_2 = -4$$
$$0x_1 + -3x_2 = 1$$

Rearranging this, we get that:

$$2x_1 + 4x_2 = -4$$
$$x_2 = -\frac{1}{3}$$

By re-plugging it into the first equation we get:

$$x_1 = -\frac{4}{3}$$

$$x_2 = -\frac{1}{3}$$

Answer 2.

$$2x_1 + 4x_2 + 4x_3 = 4$$
$$x_2 - 2x_3 = -2$$
$$2x_1 + 3x_2 = 0$$

By multiplying the second equation by -2, and then adding it to the third we get:

$$2x_1 + 4x_2 + 4x_3 = 4$$
$$x_2 - 2x_3 = -2$$
$$0x_1 - 1x_2 = 4$$

Continuing on we get that:

$$2x_1 + 4x_2 + 4x_3 = 4$$
$$x_2 - 2x_3 = -2$$
$$x_2 = -4$$

By re-plugging this onto the second equation, we get that:

$$2x_1 + 4x_2 + 4x_3 = 4$$
$$x_3 = -1$$
$$x_2 = -4$$

Finally, by plugging these values into the first equation we get that:

$$x_1 = 12$$
$$x_3 = -1$$
$$x_2 = -4$$

Hence proving that these three places meet at a point in space at (12,-4,-1)

Answer 3.

In order for this augmented matrix to be a consistent linear system, the matrix has to be able to achieve a triangular form. Since there are only two equations, and the first is already in a triangular form, then we must put the second one in said form.
To do this, we must assure that the first element is a zero, and the second one is a one. The only way to assuring the first part is by multiplying the first equation by 2, and add it to the second, thus getting an equation of the form:
$-2x_1 + (2h + 4)x_2 = 0.$
Now, in order for $2h + 4 = 1$*, then* $h = -1.5$
Then applying back-substitution we get that $x_2 = 0$ *and so:* $x_1 + 0 = -3$*, thus* $x_1 = -3$*. As such, this system of equations is consistent, and meets at (-3, 0).*

Answer 4.

Thus far every I have found of working this equation eventually leads us to a row in the augmented matrix that equal 0 0 0 = ag + bh + ck (with a b and c being rational numbers and non-zero), which means for that row to be true, g, h and k have to equal 0.

Proof:

$$\begin{bmatrix} 1 & -3 & 8 & g \\ 0 & 4 & -15 & h \\ -3 & 5 & -9 & k \end{bmatrix}$$

Multiply the first line by 3 and add it to the third:

$$\begin{bmatrix} 1 & -3 & 8 & g \\ 0 & 4 & -15 & h \\ 0 & -4 & 15 & 3g+k \end{bmatrix}$$

Add the second line to the third:

$$\begin{bmatrix} 1 & -3 & 8 & g \\ 0 & 4 & -15 & h \\ 0 & 0 & 0 & 3g+k+h \end{bmatrix}$$

Thus, 3g+k+h = 0, which is only true if g, k and h are 0. If we continue from here we get that $4x_2 - 15x_3 = 0 \equiv x_2 = \frac{15}{4}x_3$ *which then applied to the first equation gets us:* $x_1 - \frac{45}{4}x_3 + 8x_3 = 0 \equiv x_1 = \frac{13}{4}x_3$ *which is true for infinitely as many values, making the equations be consistent, with g, h and k equal to 0.*

Answer 5.

First we will establish the augmented matrix of this equation:

$$\begin{bmatrix} a & b & f \\ c & d & g \end{bmatrix}$$

Now we have to eliminate the c from the second row, for which we will multiply the top one by $-\frac{c}{a}$ *and get:*

$$\begin{bmatrix} a & b & f \\ 0 & d-\frac{bc}{a} & g \end{bmatrix}$$

With this known, we can state that, for the system to be consistent for every value of f g, $d \neq \frac{bc}{a}$

Answer 6.

- $4T_1 - T_2 + 0T_3 - T_4 = 30$
- $-1T_1 + 4T_2 - T_3 + 0T_4 = 60$
- $0T_1 - T_2 + 4T_3 - T_4 = 70$
- $-1T_1 + 0T_2 - T_3 + 4T_4 = 40$

Answer 7.

To start of with, lets name our lines, in order we get a, b, c and d, this will help us preform calculations (we will be switching the first and fourth equation as hinted):

$$\begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix}$$

Now we will begin by putting the first line into triangular form, using this math: $a = 3a + d$*:*

$$\begin{bmatrix} 1 & -1 & -3 & 11 & 120 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix}$$

Now we will put the second line into triangular form, with the following math: $b = a + b$ *AND* $b = 2c + b$*:*

$$\begin{bmatrix} 1 & -1 & -3 & 11 & 120 \\ 0 & 3 & -4 & 11 & 180 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -3 & 11 & 120 \\ 0 & 1 & 4 & 9 & 320 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix}$$

Now we will work on the third row, using this math $c = c + b$ *AND* $c = \frac{c}{8}$*:*

$$\begin{bmatrix} 1 & -1 & -3 & 11 & 120 \\ 0 & 1 & 4 & 9 & 320 \\ 0 & 0 & 8 & 8 & 390 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -3 & 11 & 120 \\ 0 & 1 & 4 & 9 & 320 \\ 0 & 0 & 1 & 1 & \frac{195}{4} \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix}$$

Finally, we will handle the last line, using this math $d = -4a + d$ *AND* $d = 3b + d$ *AND* $d = \frac{d}{-72}$*:*

$$\begin{bmatrix} 1 & -1 & -3 & 11 & 120 \\ 0 & 1 & 4 & 9 & 320 \\ 0 & 0 & 1 & 1 & \frac{195}{4} \\ 0 & 3 & 12 & -45 & -450 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -3 & 11 & 120 \\ 0 & 1 & 4 & 9 & 320 \\ 0 & 0 & 1 & 1 & \frac{195}{4} \\ 0 & 0 & 0 & -72 & -1410 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -3 & 11 & 120 \\ 0 & 1 & 4 & 9 & 320 \\ 0 & 0 & 1 & 1 & \frac{195}{4} \\ 0 & 0 & 0 & 1 & \frac{235}{12} \end{bmatrix}$$

To finish this up, we back-substitute and reach: $T_4 = \frac{235}{12} | T_3 = \frac{175}{6} | T_2 = \frac{325}{12} | T_1 = \frac{115}{6}$

Answer 8. -

Answer 9.

- *(a) 2.7755575615628914e-17*
- *(b) From a purely mathematical standpoint, no, as the result is 0*
- *(c) From the point of view of a CS student, and after reading the handouts, this answer makes sense, as the 64-bit processor does not support the amount of bits in its floating point representation to hold the exact value of* $\frac{2}{70}$*, and, as such, will produce an error in any calculation that involves that value, which he has already rounded to the nearest re-presentable value.*