CS 332 – Spring 2021, Assignment 1

Colaborators: None

Answer 1

(a) Any strings made up of the members of the alphabet $\{0, 1\}$ who are symmetrical, single character strings are also apart of this, so is the empty string.

Examples inside the language: 101, 010.

Examples outside the language: 110, 001.

(b) Any strings that include two repeated a's, followed and preceded by any string apart of the alphabet $\{a, b\}$, as well as any strings that don't contain two repeated b's, followed and preceded by any string apart of the alphabet.

Examples inside the language: $baab$, $baaba$.

Examples outside the language: bb, bba.

Answer 2

(a) $L_3 = \{xybz \mid x, y, z \in \{a, b, c\} \wedge |x| = |y| = 1 \wedge |xybz| \geq 3\}$

(b) $L_4 = \{0x \mid x \in \{0, 1\} \wedge |0x| \,\%\, 2 = 1\} \cup \{1y \mid y \in \{0, 1\} \wedge |1y| \,\%\, 2 = 0\}$

Answer 3

(a) This is true of all strings x,y,z.

Hypothesis: For any x,y,z then $|x \circ (yz)^R| = |x| + |y| + |z|$

Base case $|x| = |y| = |z| = 1$:

$|x \circ (yz)^R| = |x| + |y| + |z| \equiv |x \circ z^R y^R| = 1 + 1 + 1 \equiv |xz^R y^R| = 3$, since $|x| = |y| = |z| = 1$ then we get that the prior statement ($|xz^R y^R| = 3$) is true, regardless of the specific character that x, y, and z represent.

Assuming that this holds for $|x| = a$, $|y| = b$, $|z| = c$, for any a,b,c, then if $|x| = a + 1$, $|y| = b + 1$, $|z| = c + 1$:

$|x \circ (yz)^R| = |x| + |y| + |z| \equiv |x \circ (yz)^R| = a + b + c + 3 \equiv |x \circ z^R y^R| = a + b + c + 3 \equiv |xz^R y^R| = a + b + c + 3$

Since we know that the reverse operation does not change length, we know that $|z| = |z^R|$ and $|y| = |y^R|$, we also know that $|x| = a + 1$, $|y| = b + 1$, $|z| = c + 1$ as per our thesis, so we can split each of these string into $x = x_{old} + x_{additionalChar}$, $y = y_{old} + y_{additionalChar}$, $z = z_{old} + z_{additionalChar}$, so that $|x_{old}| = a$, $|y_{old}| = b$, $|z_{old}| = c$, so we get:

$|xz^R y^R| = |x_{old} y_{old} z_{old}| + 3$, going back to our previous statement we get: $|x_{old} y_{old} z_{old}| + 3 = a + b + c + 3 \equiv |x_{old} y_{old} z_{old}| = a + b + c$ thus we arrive at our base case as proved above.

(b) This is true for all languages $L_1, L_2 \subseteq \sum$.

We know that concatenating two languages results in a language made up of strings where the first part is from one language, and the second part is from another. So in the most general terms for $L_1 = \{x \in \sum\}$ and $L_2 = \{y \in \sum\}$ then $L_1 \circ L_2 = L_3 = \{xy \in \sum | x \in L_1 \wedge y \in L_2\}$.

From here we are know that by the basic rules of concatenation and reversion, $(x \circ y)^R = y^R \circ x^R$ (as proven in class), so we know that $(L_1 \circ L_2)^R = L_2^R \circ L_1^R$.

(c) This is true.

As we established above, concatenating two languages results in a language where the strings are made up of two halves, the first made up of strings of the first language, and the second half made up of strings of the second language.

As such, if we concatenate a language $L_1 = \{x \in \sum\}$ and $L_2 = \{\epsilon\}$, we know that we are going to get a language $L_3 = \{x\epsilon | x \in \sum\}$, this however is the same as $\{x \in \sum\}$, since adding an empty string either at the start, or at the end of any other string just results in the same string. So we know that $L \circ \{\epsilon\} = L$.

Furthermore, we know that $\emptyset = \{\}$, so we the same thing applies, if we concatenate this language with L, we are going to get a language $L_4 = \{xy | x \in \sum \wedge y \in \{\}\}$, here again, we know that adding an empty string at the start or end of another string, just results in the saame string, so we know that $L \circ \emptyset = L$.

Thus, $L \circ \emptyset = L \circ \{\epsilon\}$.

(d) This is true:

If we assume a general $L_1 = \{x \in \sum_1\}$, $L_2 = \{y \in \sum_2\}$, and $L_3 = \{w \in \sum_3\}$, with $\sum_1, \sum_2, \sum_3 \in \sum$ then:

$L_1 \circ (L_2 \cap L_3) = L_1 \circ \{z \in (\sum_2 \cap \sum_3)\} = L_4 = \{xz | x \in \sum_1 \wedge z \in (\sum_2 \cap \sum_3)\}$

$(L_1 \circ L_2) \cap (L_1 \circ L_3) = (\{xy | x \in \sum_1 \wedge y \in \sum_2\}) \cap (\{xw | x \in \sum_1 \wedge w \in \sum_3\}) = \{xz | x \in \sum_1 \wedge z \in (\sum_2 \cap \sum_3)\}$

As such, $L_1 \circ (L_2 \cap L_3) = (L_1 \circ L_2) \cap (L_1 \circ L_3)$

Answer 4

(a) q_0

(b) $\{q_1, q_2\}$

(c) $(\{q_0, q_1, q_2\}, \{A, B\}, f(x, y) = \{\text{if } y = A \text{ then } x + 1 \text{ else } x\}, q_0, \{q_1, q_2\})$

(d) q_0, q_1, q_1, q_1, q_2, q_2

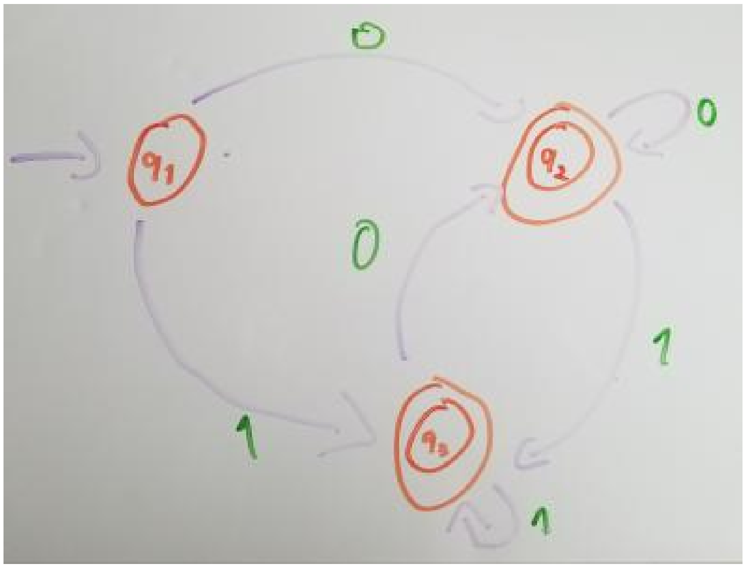
(e) No, as the final state is q_0.

(f) Yes, as the final state is q_1.

(g) The language recognized by this machine is any string that has a number of A's that is not divisible by 3 (integer division), and any number of B's.

Answer 7

(a)



(b)