Indicators

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Net Present Value

$$NPV = -F_0 + \frac{F_1}{(1+r)} + \frac{F_2}{(1+r)^2} + \cdots + \frac{F_N}{(1+r)^N}$$

$$NPV = \sum_{t=0}^{N} \frac{F_t}{(1+r)^t}$$

- F_0, initial investment
- F_t, cash flows during the period t {1 … N}
- r, cost opportunity of capital
- N, lifetime

If **NPV > 0** the project is accepted

If **NPV < 0** the project is not accepted

Why NPV is the best method?

- NPV considers the time value of money
- NPV uses cash flows
- NPV uses all cash flows of a project and an opportunity cost of capital
- If we have two projects A e B, **NPV (A+B) = NPV (A) + NPV (B)**

Exercise

Investment cost = 900,000 €. Net cash flows are equal to: 120,000 € in the first year, 250,000 € and 800,000 € in the second and third year, respectively. Opportunity cost of capital = 9%.

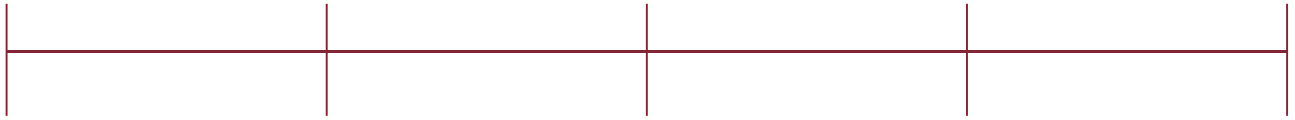
a) Is the project profitable?

b) If opportunity of capital = 12%, the project is economically feasible?

Year	Cash Flow	PV (9%)	PV (12%)
0	-900,000	-900,000	-900,000
1	120,000	110,092	107,143
2	250,000	210,420	199,298
3	800,000	617,747	569,424
NPV		38,259 €	-24,134 €

NPV with different discount rates

Cash Flow	300,000	400,000	500,000	600,000
Rate of return	10%	11%	12%	13%



Investment -1,000,000

-1,000,000
300,000/(1.10) = 272,727
400,000/(1.10*1.11) = 327,600
500,000/(1.10*1.11*1.12)=365,625
600,000/(1.10*1.11*1.12*1.13)=388,275
NPV=354,228

Alternative Indicators

- Payback Time
- Discounted Payback Time
- Internal Rate of Return
- Profitability Index

Payback Time

- A project is accepted if PBT < cutoff period.
- PBT > cutoff period a project is not accepted.

$$\sum_{t=0}^{PBT} F_t = 0$$

Exercise

PBT and NPV can provide different solutions.

Three projects (A, B, C) with a cost opportunity of capital = 10%.

Cutoff period is fixed equal to 2 years.

If the cutoff period is 3 years, what are the challenges?

Projects	F_0	F_1	F_2	F_3	PBT	NPV 10%
A	-2000	500	500	5000	3	+2624
B	-2000	500	1800	0	2	-58
C	-2000	1800	500	0	2	50

Discounted payback time

- A project is accepted if DPBT < cutoff period.
- DPBT > cutoff period a project is not accepted.

$$\sum_{t=0}^{DPBT} \frac{F_t}{(1+r)^t} = 0$$

Exercise

PBT and DPBT can provide different results. In the following example, the cost opportunity of capital is fixed equal to 10%.

Year	Cash Flow	PV
0	-100	-100
1	80	73
2	20	17
3	40	10

Internal Rate of Return

- IRR > cost opportunity of capital the project is accepted
- IRR < cost opportunity of capital the project is noaccepted

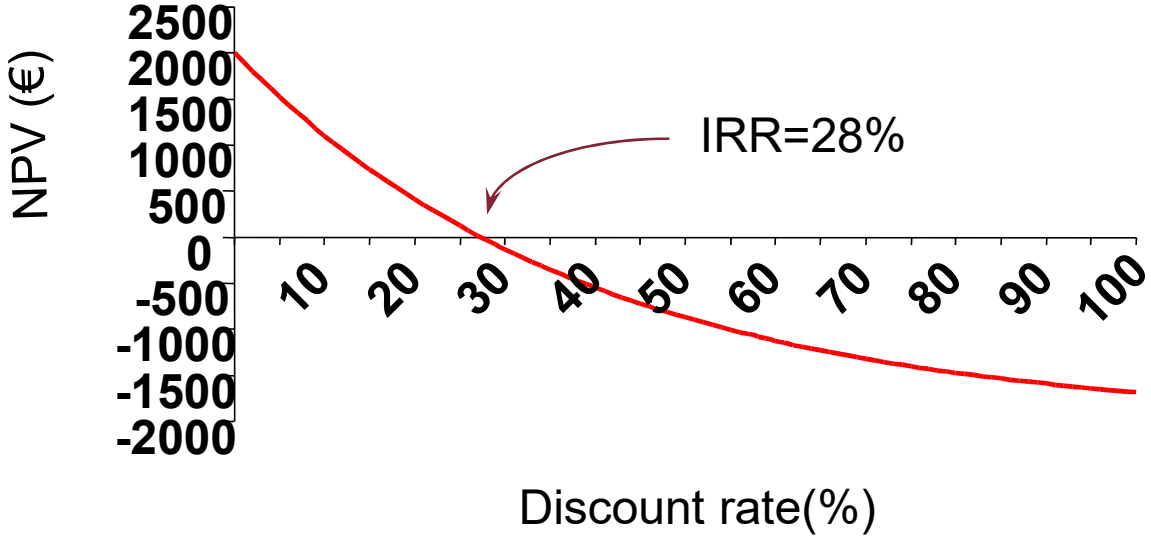
$$\sum_{t=0}^{N} \frac{F_t}{(1 + IRR)^t} = 0$$

- IRR is a profitability measure
- Cost opportunity of capital is a profitability standard

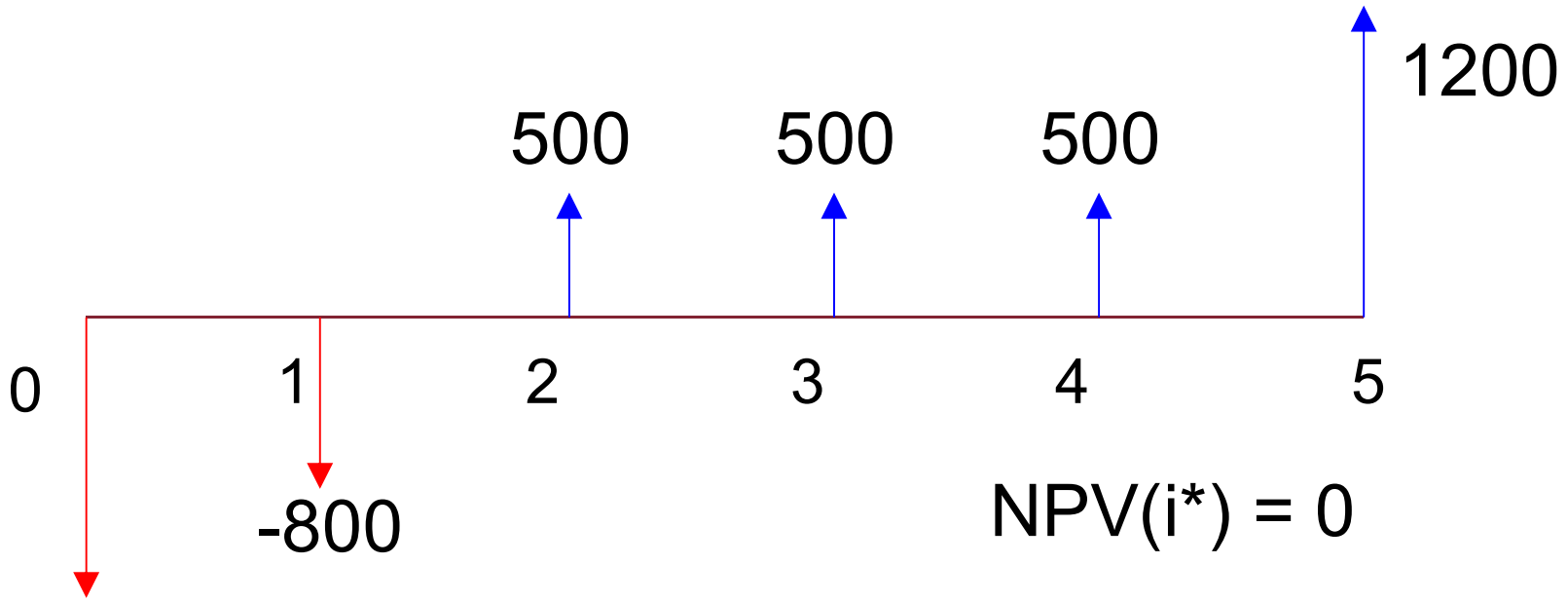
Exercise

F_0	F_1	F_2
-4000	+2000	+4000

$$\text{NPV} = -4000 + \frac{2000}{(1+\text{IRR})^1} + \frac{4000}{(1+\text{IRR})^2}$$



Interpolation



NPV(i*) = 0

for i* = 0 NPV(0) = 900 > 0

for i* = 12% NPV(12) = 39 > 0

for i* = 13% NPV(13) = -12 < 0

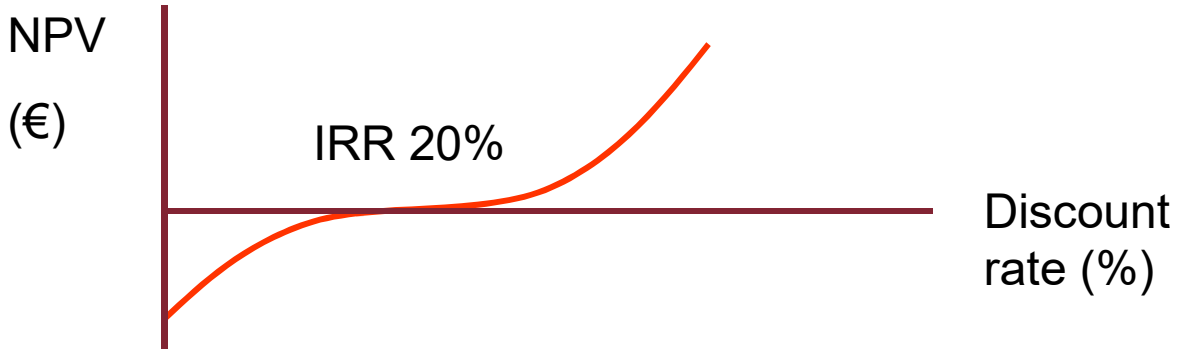
Interpolation $\mathbf{i^*} = 12 + 1\left[\frac{39-0}{39-(-12)}\right] = 12{,}8$ i* = 12.8%

IRR problem: investment or debt

Project	F_0	F_1	IRR (%)	NPV 10%
A	-1000	+1500	50	+364
B	+1000	-1500	50	-3640

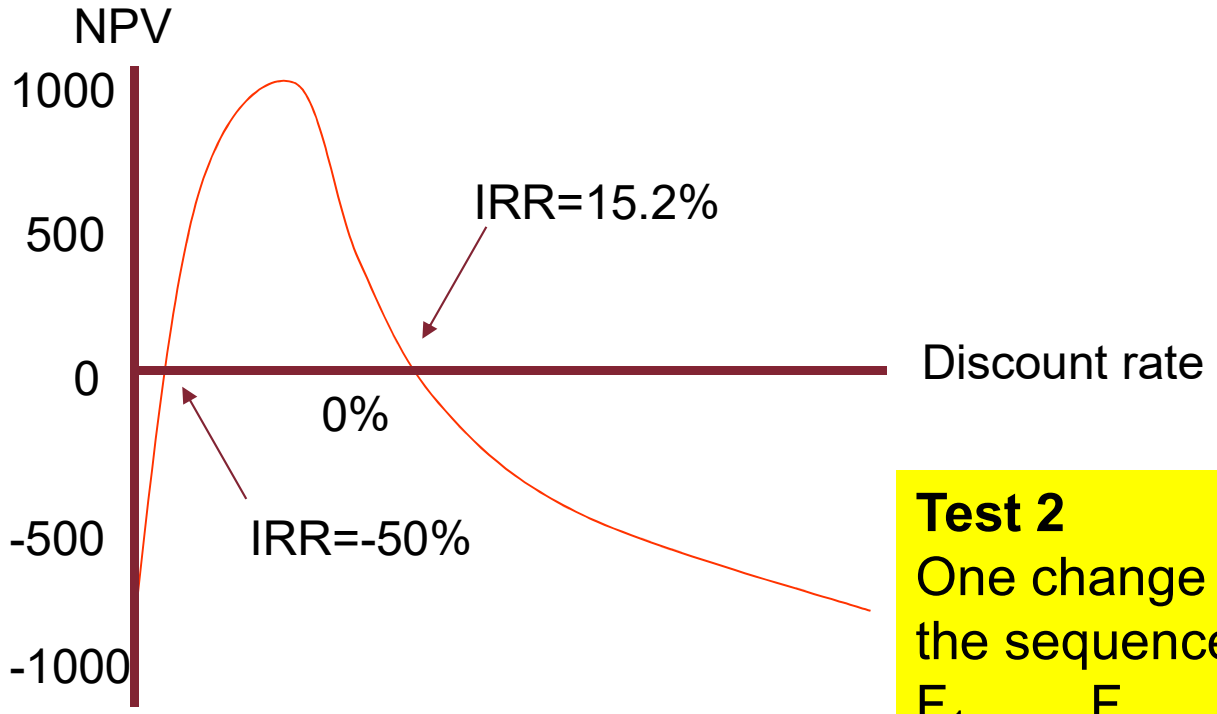
C_0	C_1	C_2	C_3	*IRR*	*NPV al 10%*
+ 1.000	− 3.600	4.320	−1. 728	+ 20 %	− 0, 75

Test 1
$F_0 < 0$



IRR problem – Multiple values

C_0	C_1	C_2	C_3	C_4	C_5	C_6
−1.000	+800	+150	+150	+150	+150	−150



Test 2
One change in sign in the sequence F_0, F_1,……F_n

IRR problem – Alternative projects

IRR does not consider the dimension of the investment.

Project	F_0	F_1	IRR (%)	NPV 10%
E	-10000	20000	100	+8182
F	-20000	35000	75	+11818

Solution: the use of IRR associated to the incremental flows.

Project	F_0	F_1	IRR (%)	NPV 10%
F-E	-10000	15000	50	+3636

IRR problem – Multiple discount rates

What is the discount rate to compare with IRR?

$$NPV = -F_0 + \frac{F_1}{(1 + r_1)} + \frac{F_2}{(1 + r_2)^2} + \cdots$$

Profitability Index

- The project is accepted if PI > 0
- If PI < 0, the project is not accepted

$$\text{PI} = \frac{\text{NPV}}{\text{Initial Investment}}$$

Exercise

Firm can invest in project A or in both projects B and C.

Project	F_0	F_1	F_2	NPV 10%
A	-10	+30	+5	+21
B	-5	+5	+20	+16
C	-5	+5	+15	+12

Project	Investiment	NPV	PI
A	10	21	2.1
B	5	16	3.2
C	5	12	2.4

Profitability Index problem

PI cannot be used when more than one resource is rationed.

Project	F_0	F_1	F_2	NPV 10%	PI
A	-10	+30	+5	+21	2.1
B	-5	+5	+20	+16	3.2
C	-5	+5	+15	+12	2.4
D	0	-40	+60	+13	0.4

Weighted Average Profitability Index

Exercise

A total sum of 300,000 € can be invested. What are the projects to choose?

Project	NPV	Investment	PI
A	230,000	200,000	1.15
B	141,250	125,000	1.13
C	194,250	175,000	1.11
D	162,000	150,000	1.08

Weighted Average PI (A) = 1.15 * (200/300) + 0 * (100/300) = 0.77

Weighted Average PI (BC) = 1.13 * (125/300) + 1.11 * (175/300) + 0 * (0/300) = 1.12

Weighted Average PI (BD) = 1.13 * (125/300) + 1.08 * (150/300) + 0 * (25/300) = 1.01