Decision making among alternatives

- Types of investment proposals:
  - independent proposal
  - dependent proposal
  - contingent proposal
  - mutually exclusive proposal
- Mutually exclusive investment alternatives

Forming mutually exclusive alternatives

- Enumeration of all possible combinations of the proposals: 2 proposals $\rightarrow$ $2^K = 2^2 = 4$ alternatives

	P_1	P_2	Action
A_0	0	0	Do nothing
A_1	1	0	Accept P_1
A_2	0	1	Accept P_2
A_3	1	1	Accept P_1 Accept P_2

Forming mutually exclusive alternatives

- Enumeration of all possible combinations of the proposals: K proposals $\rightarrow$ 2^K alternatives

	P_1	P_2	P_3		P_{K-1}	P_K
A_0	0	0	0	…..	0	0
A_1	1	0	0	…..	0	0
A_2	0	1	0	…..	0	0
A_3	1	1	0	…..	0	0
A_4	0	0	1	…..	0	0
…..	…..	…..	…..	…..	…..	…..
$A(2^K-2)$	0	1	1	…..	1	1
$A(2^K-1)$	1	1	1	1	1	1

Number of zeros = 2^{k-1}

Example

	P_1	P_2	P_3
Initial investment	30	22	82
Net annual benefit	8	6	18
Salvage value	3	2	7

P_1 and P_2 are mutually exclusive

P_3 is contingent on P_1

Budget limit = 100

K=3 proposals $\rightarrow 2^K = 2^3 = 8$ alternatives

	P_1	P_2	P_3	Initial Investment	Net annual benefit	Salvage value
A_0	0	0	0	0	0	0
A_1	1	0	0	30	8	3
A_2	0	1	0	22	6	2
A_3	1	1	0	52	14	5
A_4	0	0	1	82	18	7
A_5	1	0	1	112	26	10
A_6	0	1	1	104	24	9
A_7	1	1	1	134	32	12

Decision Process - Differences between alternatives

	A_1	A_2	$A_2 - A_1$
0	-1000	-1500	-500
1	800	700	-100
2	800	1300	500
3	800	1300	500

$$A_2 = A_1 + (A_2 - A_1)$$

If $A_2 - A_1$ is economically desirable, the alternative
A_2 is preferred to A_1

If $A_2 - A_1$ is economically undesirable, the alternative
A_1 is preferred to A_2

MARR policy

The Minimum Attractive Rate of Return (MARR) is a cut-off rate representing a yield on investments that is considered minimally acceptable

The Do Nothing Alternative $\rightarrow$ $\mathbf{i}^{*}_{A_0} = \mathbf{MARR}$

The equivalent profit of the Do Nothing alternative is always zero, it is not necessary to know its cash flows since it can be assumed that the cash flows are zero

Decision rules on incremental investment (i=MARR)

If $PV(MARR)_{A2-A1} > 0 \quad \rightarrow \quad$ accept A_2

If $PV(MARR)_{A2-A1} \leq 0 \quad \rightarrow \quad$ accept A_1

If $FV(MARR)_{A2-A1} > 0 \quad \rightarrow \quad$ accept A_2

If $FV(MARR)_{A2-A1} \leq 0 \quad \rightarrow \quad$ accept A_1

Incremental cash flows pass Test 1 or Test 2?

NO → IRR method should not be applied

YES → if $i^*_{A2-A1} > MARR \rightarrow$ accept A_2

if $i^*_{A2-A1} \leq MARR \rightarrow$ accept A_1

Decision rules on total investment (i = MARR)

If $PV(i)_{A2} > PV(i)_{A1} \rightarrow$ accept A_2

If $PV(i)_{A2} \leq PV(i)_{A1} \rightarrow$ accept A_1

If $FV(i)_{A2} > FV(i)_{A1} \rightarrow$ accept A_2

If $FV(i)_{A2} \leq FV(i)_{A1} \rightarrow$ accept A_1

The IRR is calculated for each proposal, and then the proposal are ranked in descending order of IRR. We can accept all the proposals with IRR > MARR

The two methods lead to the same solutions:

$$\mathbf{PW(i)_{A_2} - PW(i)_{A_1} = PW(i)_{A_2-A1}}$$

$$\mathbf{PW(i)_{A_2} - PW(i)_{A_1}} = \sum_{t=0}^{n} \mathbf{F_{A_{2,t}}(1+i)^{-t}} - \sum_{t=0}^{n} \mathbf{F_{A_{1,t}}(1+i)^{-t}} =$$

$$\mathbf{F_{A_{2,0}} - F_{A_{1,0}} + F_{A_{2,1}}(1+i)^{-1} - F_{A_{1,1}}(1+i)^{-1}} + \cdots\cdots +$$

$$\mathbf{F_{A_{2,n}}(1+i)^{-n} - F_{A_{1,n}}(1+i)^{-n}} =$$

$$\mathbf{F_{A_2-A_{1,0}} + F_{A_2-A_{1,1}}(1+i)^{-1}} + \cdots\cdots \mathbf{F_{A_2-A_{1,n}}(1+i)^{-n}} =$$

$$\sum_{t=0}^{n} \mathbf{F_{A_2-A_{1,t}}(1+i)^{-t} = PW(i)_{A_2-A_1}}$$

Example

MARR=10%

t	A	B	C	D
0	-10000	-12000	-12000	-15000
1	-2500	-1500	-1200	-400
2	-2500	-1500	-1200	-400
3	1000	1500	1500	3000

$PW(10)_A = -10000 - 2500(1+0,10)^{-1} - 2500(1,10)^{-2} +$
$+ 1000(1,10)^{-3} = -13587$

we choose C

$PW(10)_B = -13476$ $PW(10)_C =$ ***-12956*** $PW(10)_D = -13440$

$PW(10)_{B-A} = -2000 + 1000(1,10)^{-1} + 1000(1,10)^{-2} +$
$+ 500(1,10)^{-3} = 111 > 0$ accept B, reject A

$PW(10)_{C-B} = 0 + 300(1,10)^{-1} + 300(1,10)^{-2} + 0 = 520 > 0$ accept C, reject B

$PW(10)_{D-C} = -3000 + 800(1,10)^{-1} + 800(1,10)^{-2} + 1500(1,10)^{-3} =$
$-485 < 0$ accept C, reject D

Inflation Example

MARR = 10%

t	A	B	C	D
0	-10000	-12000	-12000	-15000
1	-2500	-1500	-1200	-400
2	-2500	-1500	-1200	-400
3	1000	1500	1500	3000

Lack of inflation (constant money)

$PW(10)_A = -13587$ $PW(10)_B = -13476$

$PW(10)_C =$ ***-12956*** $PW(10)_D = -13440$

$f = 0 \quad i = i'$

Presence of inflation $f = 9\%$. We need to transform the constant money in current money multiplying by $(1+f)^n$

Inflation example

MARR = 10%

t	A	B	C	D
0	-10000	-12000	-12000	-15000
1	-2725	-1635	-1308	-436
2	-2970	-1792	-1426	-475
3	1295	1943	1943	3885

$PW(10)_A$= -13959 $PW(10)_B$= -13499
$PW(10)_C$= -12908 $PW(10)_D$= ***-12870*** → The choice changes

The constant dollar method provides the same results with less calculation considering the inflation-free rate i'

$$\mathbf{i' = \frac{i - f}{1 + f} = \frac{0{,}10 - 0{,}09}{1 + 0{,}09} = \frac{0{,}01}{1{,}09} = 0{,}00917}$$

$$\mathbf{P} = \mathbf{F}\frac{1}{(1+\mathbf{i})^{\mathbf{n}}}$$

$$\mathbf{P} = \mathbf{F'}\frac{1}{(1+\mathbf{i'})^{\mathbf{n}}}$$

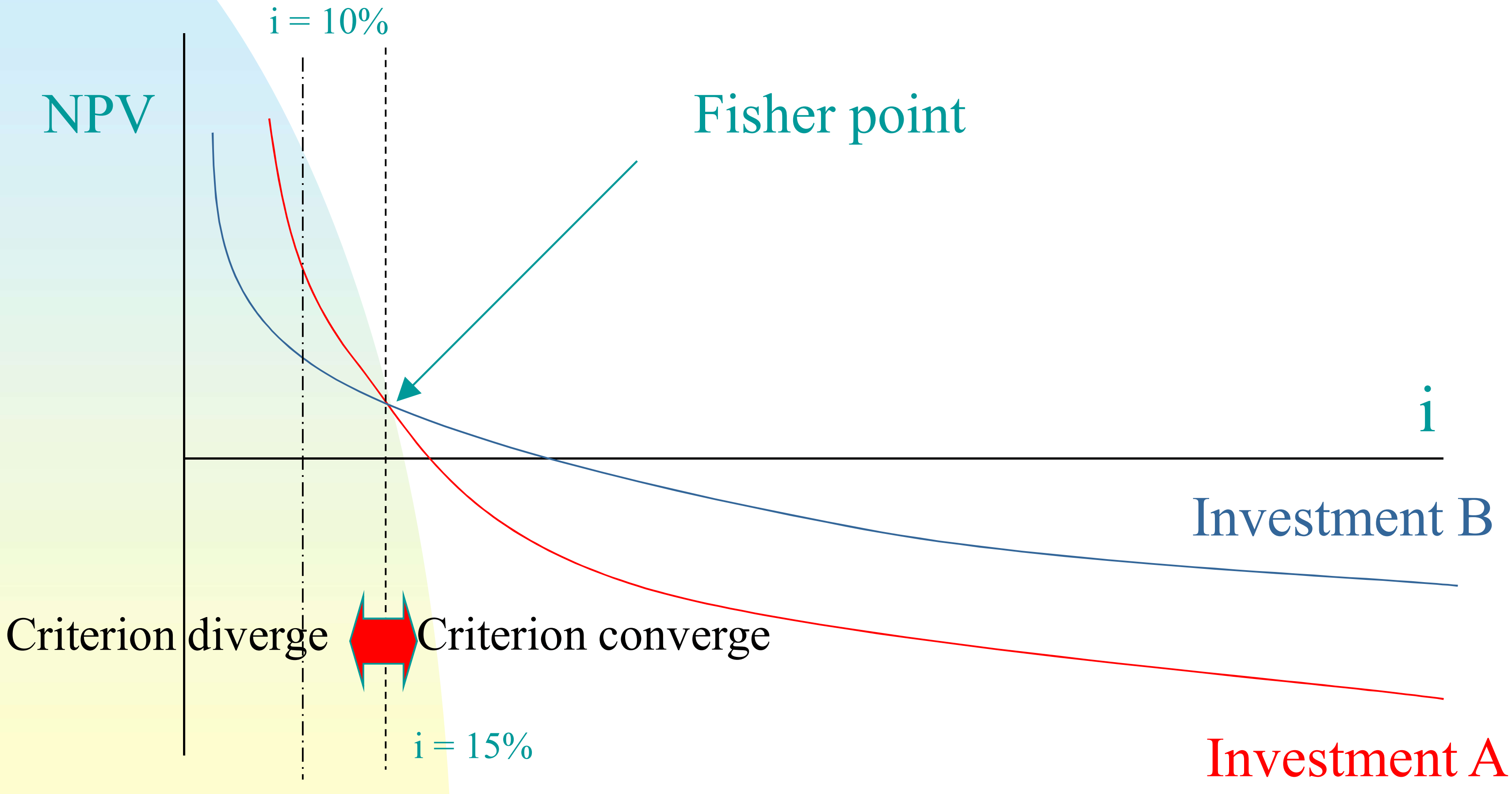
$$\mathbf{F'} = \mathbf{F}\frac{1}{(1+\mathbf{f})^{\mathbf{n}}}$$

$$\mathbf{P} = \mathbf{F'}\frac{1}{(1+\mathbf{i'})^{\mathbf{n}}} = \mathbf{F}\frac{1}{(1+\mathbf{f})^{\mathbf{n}}}\frac{1}{(1+\mathbf{i'})^{\mathbf{n}}}$$

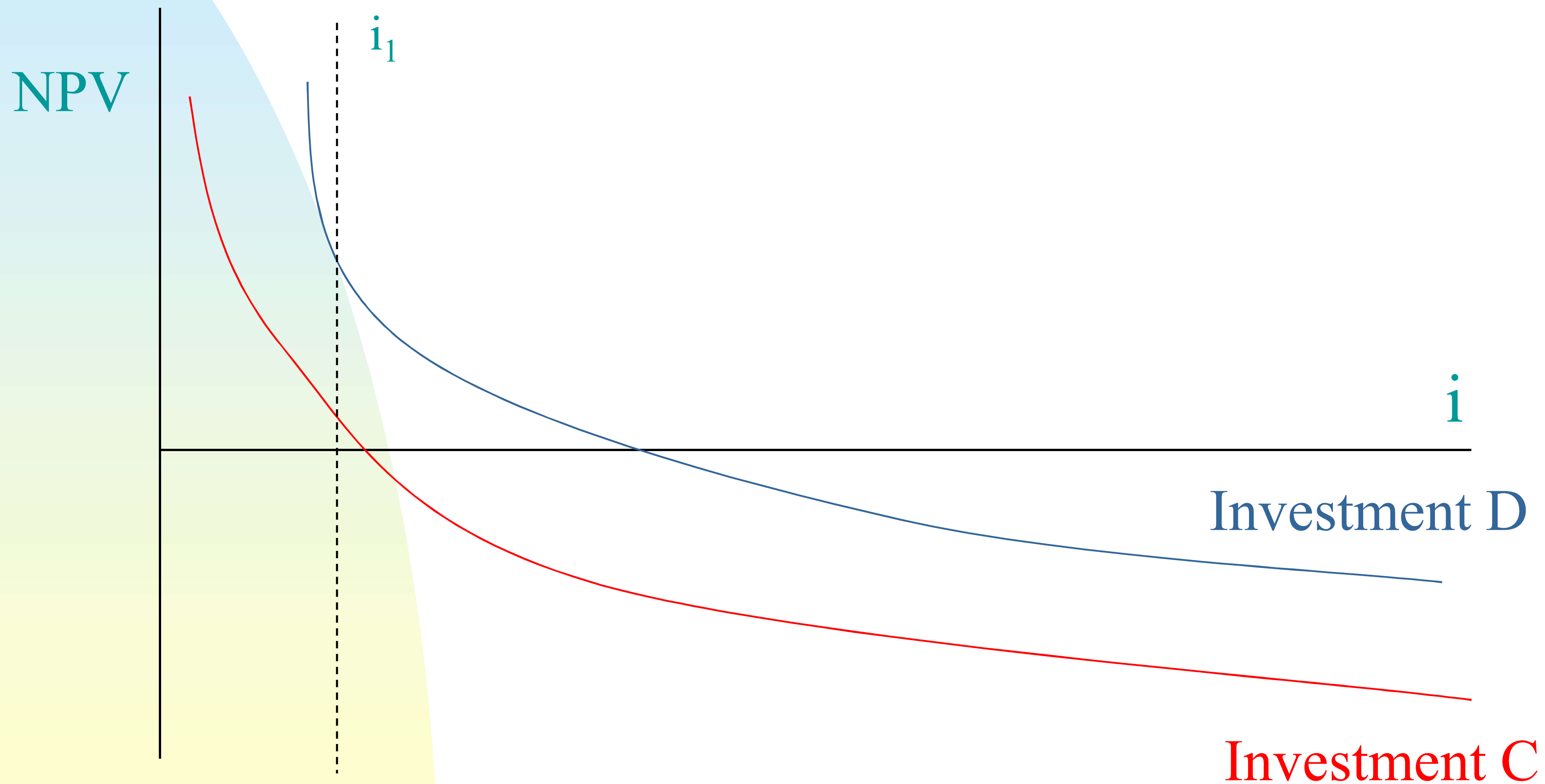
$$(1+\mathbf{i}) = (1+\mathbf{f})(1+\mathbf{i'})$$

$$i' = \frac{i-f}{1+f}$$

NPV – IRR relationship



NPV – IRR relationship



Comparing alternatives with unequal lives

What's happened if we have to compare alternative with unequal service lives?

The time period over which the alternatives are to be compared is usually referred to as the study period or planning horizon n*.

Example

Alternatives	**A**	**B**	**C**
Initial Cost	4000	16000	20000
Annual Cost	6400	1400	1000
Lifetime	6 years	3 years	4 years

A, B and C all fulfill the same objective, but for a different number of years; select the least costly for i=7%

NPVof Original Alternatives

NPV(A, 6 yrs) = 4 + 6.4(P/A, 7%, 6) = 4 +(6.4)(4.76) = \$34,460

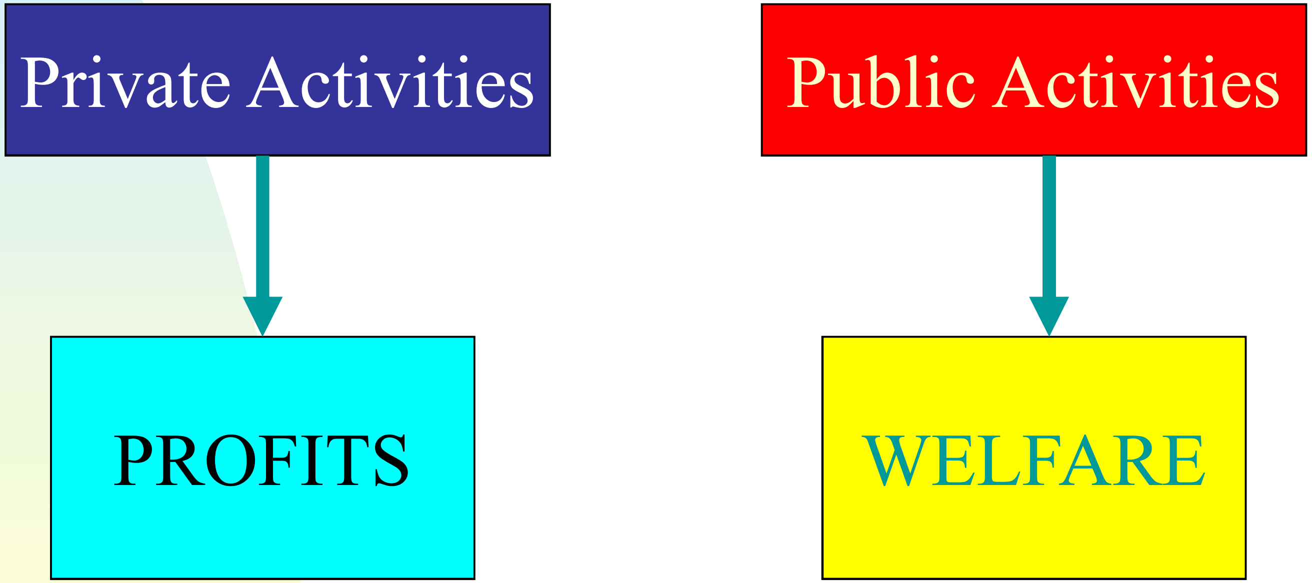
NPV(B, 3 yrs) = 16 + 1.4 (P/A, 7%, 3) = 16 +(1.4)(2.62) = \$19,680

NPV(C, 4 yrs) = 20 + 1 (P/A, 7%, 4) = 20 + 1(3.4) = \$23,400

These are the PWs of costs over the actual lifetimes.

Best alternative: B

Evaluating Public Activities
Benefit-Cost Analysis



Benefit-Cost Analysis

BC(i) = Equivalent Benefits / Equivalent Costs

BC(i) = 1	represents the minimum justification for an expenditure by a public agency
Benefits =	All the advantages, less disadvantages to the user
Costs =	All the disbursements, less any savings to the sponsor

Benefit-Cost Analysis

- Aggregated benefit-cost ratio

$$R_A = \frac{B}{C} = \frac{\sum_{t=0}^{n} b_t (1+i)^{-t}}{\sum_{t=0}^{n} c_t (1+i)^{-t}} = \frac{B}{I + C'} > 1$$

- Net benefit-cost ratio

$$R_N = \frac{B - C'}{I} > 1$$

Expected net gain per unit money invested

The incremental benefit-cost analysis will again provide results consistent with NPV (or PW)

Consistent base

$$if \qquad BC(i)_{A_2-A_1} > 1 \rightarrow PW(i)_{A_2} > PW(i)_{A_1}$$

Accept A_2

$$\frac{B(i)_{A_2} - B(i)_{A1}}{I(i)_{A_2} - I(i)_{A1} + [C(i)_{A_2} - C(i)_{A_1}]} > 1$$

$$B(i)_{A_2} - B(i)_{A1} > I(i)_{A_2} - I(i)_{A1} + C(i)_{A_2} - C(i)_{A_1}$$

$$B(i)_{A_2} - I(i)_{A_2} - C(i)_{A_2} > B(i)_{A_1} - I(i)_{A1} - C(i)_{A_1}$$

$$PW(i)_{A_2} > PW(i)_{A_1}$$

Example

	Annual Equivalent Benefits	Annual Equivalent Costs	B/C
A_1	182000	91500	1.90
A_2	167000	79500	2.10
A_3	115000	78500	1.46
A_4	95000	50000	1.90

	Incremental annual benefits	Incremental annual costs	Incremental B/C	Decision
$A_4 - A_0$	95000	50000	1.90	Accept A_4
$A_3 - A_4$	20000	28500	0.70	Accept A_4
$A_2 - A_4$	72000	29500	2.44	Accept A_2
$A_1 - A_2$	15000	12000	1.25	Accept A_1

Benefit-Cost Analysis deficiencies

- identifying costs
- identifying benefits:

primary and secondary benefits

↓

negative and positive externalities

- valuation of benefits in monetary terms