Quantum Entanglement and Quantum Computing ESA Report Jan 2021

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I. Abstract

Image scaling algorithms have been extensively studied in classical image processing. Scaling is done using basic image transformation methods. However, the quantum versions of the same do not exist. In the paper titled 'Quantum image scaling using nearest neighbour interpolation' the authors Jiang and Wang propose quantum algorithms and circuits to realize the quantum image scaling based on the improved novel enhanced quantum representation (INEQR) for quantum images. The quantum strategies developed in this paper initiate the research about quantum image scaling.

II. Introduction

Quantum Image Processing faces two questions:
1. How is an image stored in quantum computers?
2. How is the quantum image processed?
The FRQI method contains the colour information and corresponding position information of every pixel in an image into entangled quantum states. The image is stored by FRQI with $(1 + logN)N$ quantum states. NEQR method is developed from FRQI. It is suitable for QIP and image scaling as:

- NEQR captures information about colours and their corresponding positions in an image into normalized quantum states: $|0\rangle$ or $|1\rangle$, which is convenient for operating by using simple quantum gates.
- Colours and positions are all stored in binary quantum sequences which is similar to the representations of classical digital images. This makes it relatively easy to transplant image processing algorithms from classical computers to quantum computers.
- CNOT gates and Toffoli gates are used to link colour information and location information flexibly which makes users can process any part of the image at will.

The circuits in this note are based on NEQR. The QIP algorithm we focus on is image scaling. Image scaling is the process of resizing a digital image. Unlike its extensive study in classical image processing, no such strategies on quantum computers have emerged. Therefore, as an indispensable and basic part of QIP, quantum image scaling is necessary to be studied in-depth to promote the development of QIP.
There are two steps in image scaling:
1. Resize an image.
2. Use interpolation to give the value of newly added pixels or delete redundant pixels.
In classical image processing, the reduction or the enlargement of the size of an image is easy to realize by applying for less or more storage space. The quantum strategies proposed in the paper aimed to solve the problem that was existing- there were no QIP algorithms that can change the size of a quantum image.
Scaling is a non-trivial process that involves a trade-off between efficiency and smoothness by using different interpolation methods. Interpolation is the process of using known data to estimate values at unknown locations. The commonly used interpolation methods include nearest neighbour, bilinear, and bicubic.

III. Theoretical Bases

A. NEQR

Let the gray range of the image be 2^q. The binary sequence $C^0_{YX}C^1_{YX}...C^{q-2}_{YX}C^{q-1}_{YX}$ encodes the gray scale value $f(Y,X)$ of the corresponding pixel (Y,X):

$$f(Y,X) = C^0_{YX}C^1_{YX}...C^{q-2}_{YX}C^{q-1}_{YX}$$

where $C^k_{YX} \in \{0,1\}$ and $f(Y,X) \in \{0,1,...,2^q-1\}$ According to the NEQR, a $2^n \times 2^n$ quantum image can be written as:

$$|I\rangle = \frac{1}{2^n}\sum_{Y=0}^{2^{n-1}}\sum_{X=0}^{2^{n-1}}|f(Y,X)\rangle|YX\rangle$$
$$= \frac{1}{2^n}\sum_{Y=0}^{2^{n-1}}\sum_{X=0}^{2^{n-1}}\otimes|C^i_{YX}\rangle|YX\rangle$$

where $i = 0...q-1$.

$$|YX\rangle = |Y\rangle|X\rangle = |y_0y_1...y_{n-1}\rangle|x_0x_1...x_{n-1}\rangle$$

where $y_i, x_i \in \{0,1\}$. $|YX\rangle$ is the position information. NEQR needs $2n+q$ bits to represent a $2^n \times 2^n$ image with gray range 2^q.

B. INEQR

NEQR deals with quantum images having $2^n \times 2^n$ pixels. However, if $r_x \neq r_y$, the size of the scaled image will not be in the form of $2^n \times 2^n$. Instead, the improved NEQR (INEQR) is use to represent quantum images having $2^{n_1} \times 2^{n_2}$ pixels:

$$|I\rangle = \frac{1}{2^{\frac{n_1+n_2}{2}}}\sum_{Y=0}^{2^{n_1-1}}\sum_{X=0}^{2^{n_2-1}}|f(Y,X)\rangle|YX\rangle$$

$$= \frac{1}{2^{\frac{n_1+n_2}{2}}} \sum_{Y=0}^{2^{n_1-1}} \sum_{X=0}^{2^{n_2-1}} \otimes |C_{YX}^i\rangle |YX\rangle$$

where $i = 0...q-1$.

$$|YX\rangle = |Y\rangle |X\rangle = |y_0 y_1 ... y_{n_1-1}\rangle |x_0 x_1 ... x_{n_2-1}\rangle$$

where $y_i, x_i \in \{0,1\}$. INEQR uses Y (n_1 qubits), X (n_2 qubits) and $f(Y,X)$ (q qubits), respectively, to denote the Y-coordinate, X- coordinate and gray-scale values of a quantum image.

IV. Quantum image scaling using the nearest neighbour interpolation

Image scaling, which resizes an image, is one of the most common image processing operations. It uses interpolation to create new or delete redundant pixels.Among the interpolation methods, nearest neighbour is the simplest and fastest one. The nearest neighbor algorithm works in two directions and can be decomposed as shown below.

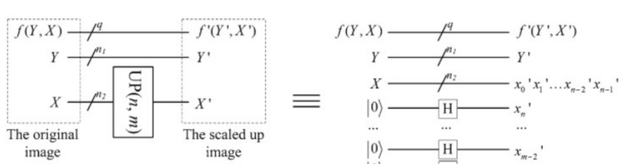
$$I' = S(I, r_x, r_y) = S_y(S_x(I, r_x), r_y) = S_x(S_y(I, r_y), r_x)$$

where S is the scaling function, I is the original image, I is the scaled image, r_x and r_y are the horizontal and vertical scaling ratio. S can be decomposed into S_x and S_y , which are the horizontal and vertical scaling function. The principles of S_x and S_y are exactly the same, and they are commutative. That is to say image scaling is the combination of two one-dimensional scaling. We study the quantum image scaling in one direction firstly to simplify the issue and then offer the complete circuit by giving some examples.

This paper assumes that a $2^{n_1} \times 2^{n_2}$ image is scaled to $2^{m_1} \times 2^{m_2}$ and the scale ratio $r_y = 2^{m_1-n_1}$ and $r_x = 2^{m_2-n_2}$, where n_1, n_2, m_1, m_2 are all non-negative integers. Based on the INEQR, (Y, X), $f(Y, X)$ and (Y', X'), $f(Y', X')$ are employed to notate the colour and site information of the original and the scaled image.

A. Quantum image scaling up circuit

The quantum circuit for image scaling up on one direction is notated by a quantum module UP(n, m) with n, m representing that an image is scaled up from 2^n to 2^m.

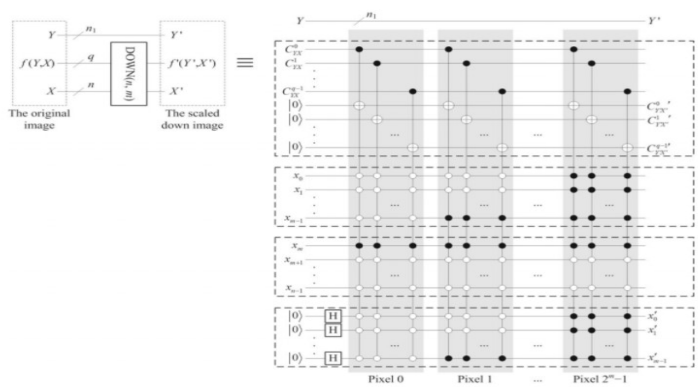


The structure of UP(n, m) is shown in the figure, where m>n.
Steps:
1. Prepare $m-n\,|0\rangle$ qubits as the new position qubits of X axis.
2. Add $m-n$ Hadamard gates to derive $x_n^{'}, x_{n+1}^{'}, ...x_{m-2}^{'} x_{m-1}^{'}$.

Theorem 1: The effect of the quantum module UP(n, m) is to repeat the value of every pixel 2^{m-n} times.
The theorem deals with 2 problems:
1. The number of pixels that one pixel in the original image is enlarged to. One pixel X in the original image is enlarged to 2^{m-n} pixels.
2. The colour value of the new pixels. UP(n, m) does nothing to the color value $C_{YX}^0 C_{YX}^1 ... C_{YX}^{q-2} C_{YX}^{q-1}$, which implies that the color value is remained as unchanged, i.e., is repeated because the color information and the location information are entangled.

B. Quantum image scaling down circuit

The quantum circuit for image scaling down on one direction is notated by a quantum module DOWN(n, m), with n, m representing an image is scaled down from 2^n to 2^m. The structure of DOWN(n, m) is shown in the figure, where m<n.



Steps:
1. Add $q\,|0\rangle$ qubits as the new colour qubits and $m\,|0\rangle$ qubits as the new X position qubits of the scaled down image.
2. Add m Hadamard gates to derive $x_0^{'}, x_1^{'}, ...x_{m-2}^{'} x_{m-1}^{'}$.
3. Using CNOT gates, copy $f(Y, X)$ to $f'(Y', X')$ when $x_m x_{m+1} ... x_{n1}$ = 10 ... 0.

Theorem 2: The effect of the quantum module DOWN(n, m)is to scale down an image using nearest neighbour interpolation.

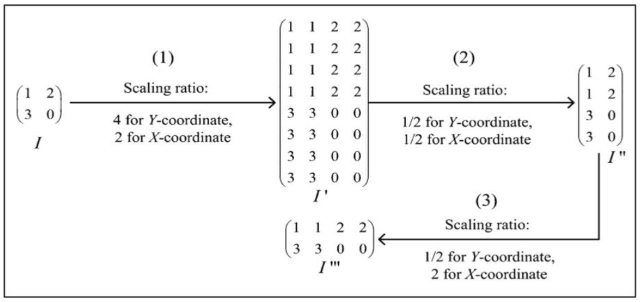
C. Network complexity

1) Upscaling: If the scaling ratio is 2^{m-n}, only $m-n$ Hadamard gates can solve the issue.

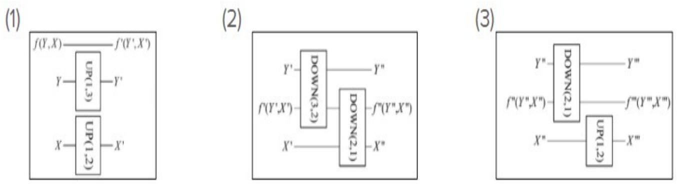
2) Downscaling: The network complexity depends very much on what is considered to be an elementary gate. In this paper, Control-NOT gate is the basic unit. the quantum module DOWN(n, m) can be simulated by $q \times 2^m \times (12(n+m)+1)$ CNOT gates.

D. Examples for quantum image scaling

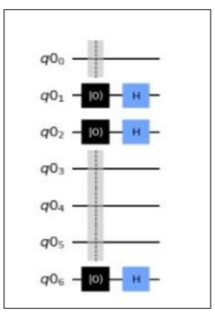
A simple example for image scaling using nearest neighbour interpolation is given in the figure.

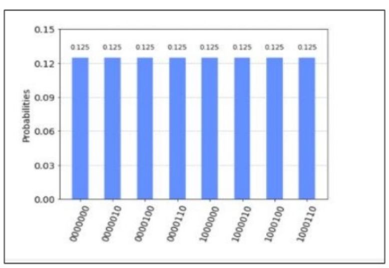


To implement the example, we make use of upscaling(1), downscaling(2) and up and down scaling(3) circuits.

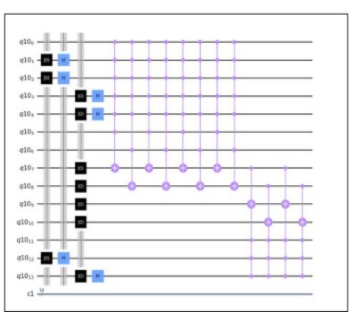


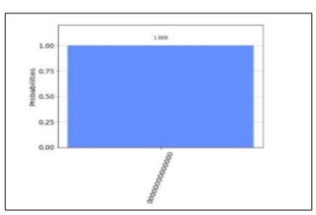
The IBM Quantum Lab is used to simulate these circuits and the outputs are observed:
Circuit (1): Here we take n1 and n2 = 1 , q = 2 ; ry = 4 and rx = 2.



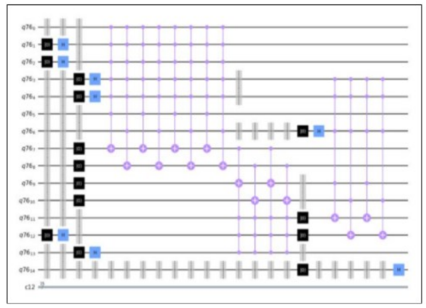


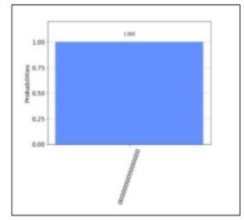
Circuit (2): Here we take the output of upscaling with ry = rx = 1/2.





Circuit (3):Here we take the output of down scaling with ry = 1/2, rx = 2.





V. Verification and Errors

The circuits for up scaling and down scaling have been executed and verified as shown above. It is seen that in the case of up scaling, the pixel values multiply in accordance to the r_x and r_y values. In the case of down scaling, it is seen that the pixel values truncate to a single pixel value as seen in the histogram of the state vectors. As the results for the same are not mentioned in the paper explicitly, these are the outputs drawn from the circuits on the IBM Quantum Lab.

VI. Conclusion

In this paper, the quantum image scaling circuits using nearest neighbour are proposed which can scale images from $2^{n_1} \times 2^{n_2}$ to $2^{m_1} \times 2^{m_2}$. Firstly, NEQR is improved into INEQR to store a quantum image sized $2^{n_1} \times 2^{n_2}$. Then quantum circuits that can scale a quantum image are proposed. Some quantum circuit examples to illustrate the concepts in a detailed manner. This paper has contributed to the start of research of quantum image scaling.

VII. Future Scope

Future work may include:
1. Implementation of more complex scaling algorithms, that is using bilinear and bicubic interpolation.
2. Quantum counterparts to classical image processing transforms like restoration and enhancement.
3. Design quantum image scaling methods if the scale ratio is not in the form 2^r.

References

[1] Venegas-Andraca, S.E., Bose, S.: Storing, processing and retrieving an image using quantum mechan- ics. In: Proceedings of the SPIE Conference on Quantum Information and Computation, pp. 137–147 (2003)

[2] Le, P.Q., Dong, F., Hirota, K.: A flexible representation of quantum images for polynomial preparation, image compression and processing operations. Quantum Inf. Process. 10(1), 63–84 (2011)

[3] Zhang, Y., Lu, K., Gao, Y.H., Wang, M.: NEQR: a novel enhanced quantum representation of digital images. Quantum Inf. Process. 12(12), 2833–2860 (2013)