ISA ASSIGNMENT -1
QUANTUM ENTANGLEMENT AND QUANTUM COMPUTING (UE17EC424)

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I. QUESTION

Show that the arbitrary single-qubit state

$$|\Psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|1\rangle$$

is represented by the unit vector n^ in spherical coordinate system,

$$\hat{\boldsymbol{n}} = \cos(\phi)\sin(\theta)\hat{\boldsymbol{x}} + \sin(\phi)\sin(\theta)\hat{\boldsymbol{y}} + \cos(\theta)\hat{\boldsymbol{z}}$$

when plotted on Bloch sphere.

Write a python program to display the Bloch - sphere and represent a state input by the user, in phase.

II. INTRODUCTION

In quantum mechanics and computing, the Bloch sphere is a geometrical representation of the pure state space of a two-level quantum mechanical system (qubit), named after the physicist Felix Bloch. [1]
The Bloch sphere is a unit 2-sphere, with antipodal points corresponding to a pair of mutually orthogonal state vectors. The north and south poles of the Bloch sphere are typically chosen to correspond to the standard basis vectors |0> and |1> respectively, which in turn might correspond e.g. to the spin-up and spin-down states of an electron. This choice is arbitrary, however. The points on the surface of the sphere correspond to the pure states of the system, whereas the interior points correspond to the mixed states. The Bloch sphere may be generalized to an *n*-level quantum system, but then the visualization is less useful.

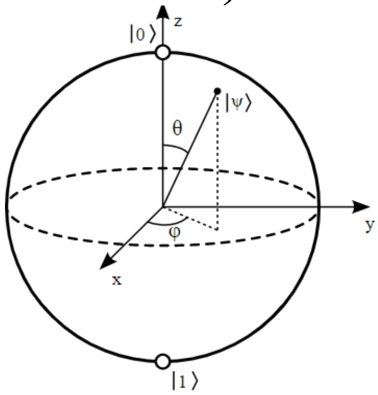

Fig 1. Bloch Sphere representation

III. DERIVATION

One can define any arbitrary qubit state

$$|\Psi> = Cos\left(\frac{\theta}{2}\right)\Big|0> + Sin\left(\frac{\theta}{2}\right)e^{i\phi}\Big|1>$$

where θ and ϕ are real numbers. The points $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$ define a point on unit 3D sphere.[2]

The Bloch sphere is a generalisation of the representation of complex number z with $|z|^2 = 1$.
If z = x + iy where x and y are real, then

$$|z|^2 = z^*z$$
$$= (x + iy)(x - iy)$$
$$|z|^2 = x^2 + y^2 \qquad - (1)$$

In polar coordinates,
x = r Cos(θ), y = r Sin(θ) so,
z = r (Cos(θ) + i Sin(θ))

Using Euler's identity,
$e^{i\theta} = \text{Cos}(\theta) + i\,\text{Sin}(\theta)$
we have,
$z = r\,e^{i\theta}$ as r = 1,

$$z = e^{i\theta} \qquad - (2)$$

A general qubit state can be written as,

$| \Psi > = \alpha |0> + \beta |1>$ where α and $\beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$

We can express the state as polar coordinates,

$$| \Psi > = r_\alpha e^{i\phi\alpha} |0> + r_\beta e^{i\phi\beta} |1> \qquad - (3)$$

As $|\alpha|^2$ and $|\beta|^2$ are the only measurable values, multiplying eq 3 with $e\text{-}i^{\phi\alpha}$ does not change the state.

Therefore,

$$| \Psi > = r_\alpha |0> + r_\beta e^{i(\phi\beta - \phi\alpha)} |1>$$

Let $\phi_\beta - \phi_\alpha = \phi$

Thus,

$$| \Psi > = r_\alpha |0> + r_\beta e^{i\phi} |1> \qquad - (4)$$

We have a normalisation constraint,

$$< \Psi | \Psi > = 1$$

Expressing eq 4 in rectangular coordinates,

$$| \Psi > = r_\alpha |0> + (x + iy)|1> \qquad - (5)$$

Normalising constraints,

$$| r_\alpha |^2 + | x + iy |^2 = r_\alpha^2 + (x + iy)(x- iy)$$
$$= r_\alpha^2 + x^2 + y^2 =1$$

This is the equation for a sphere in 3D!

Let $r_\alpha^2 = z^2$

The corresponding polar coordinates are

$$x = r \text{ Sin } (\theta)\text{Cos } (\phi)$$
$$y = r \text{ Sin } (\theta)\text{Sin } (\phi)$$
$$z = r \text{ Cos } (\theta)$$

As Bloch sphere deals with a unit circle

$$r =1$$

Therefore, as any point on a Bloch sphere is a point on the unit circle, we can represent any arbitrary single qubit state as a point on the unit circle in the form,

$$\hat{n}= \text{Sin}(\theta)\text{Cos}(\phi)\, \hat{x} + \text{Sin}(\theta)\text{Sin}(\phi)\, \hat{y} + \text{Cos}(\theta)\hat{z} \qquad - (6)$$

IV. RESULTS AND ERRORS

The Bloch sphere representation of any state is plotted on python using pylab and qutip libraries.[3] The inputs of the state can be done either in terms of its x,y,z coordinates or through the phase angles ϕ,θ. The plot and state will be displayed. Error could happen in inputting the phase angles as they should be in radians.

V. REFERENCES

[1] Bloch, Felix (Oct 1946). "Nuclear induction". Phys. Rev. 70 (7–8): 460–474. Bibcode:1946PhRv...70..460B. doi:10.1103/physrev.70.460

[2] https://www.quantiki.org/wiki/bloch-sphere

[3] P.D. Nation, J.R. Johansson, A.J.G. Pitchford, C. Granade, and A.L. Grimsmo. (2011). *Qutip 4.1 Reference Manual*. Scotts Valley, CA: CreateSpace