

Anova example

Manishanker

Let's say we have 3 samples of people at different situations and we are measuring the stress levels.

Analysis of Variance

levels of stress

sample

2
3
7
2
6

normal

1
2
3
4
5

sample

10
8
7
5
10

announced layoffs

sample

10
13
14
13
15

during layoffs

All these are same 5 people. And what we are looking for is the impact of layoff in the form of stress levels at different points of time.

Calculate sum of squares for each sample individually, i.e. the variance

Analysis of Variance

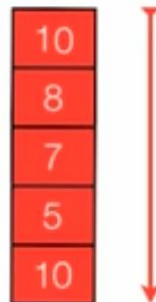
levels of stress

variance



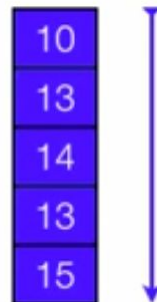
sum of squares

variance



sum of squares

variance



sum of squares

Sum of Squares Within Groups

Next calculation is sum of squares between groups. This would consider all the values

Analysis of Variance

levels of stress

variance

2
3
7
2
6

variance

10
8
7
5
10

variance

10
13
14
13
15



variance between groups

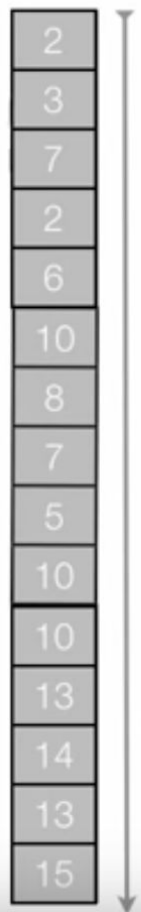
sum of squares

Sum of Squares Between Groups

Also consider all these samples combined as one large big sample and calculate
Total sum of squares

Analysis of Variance

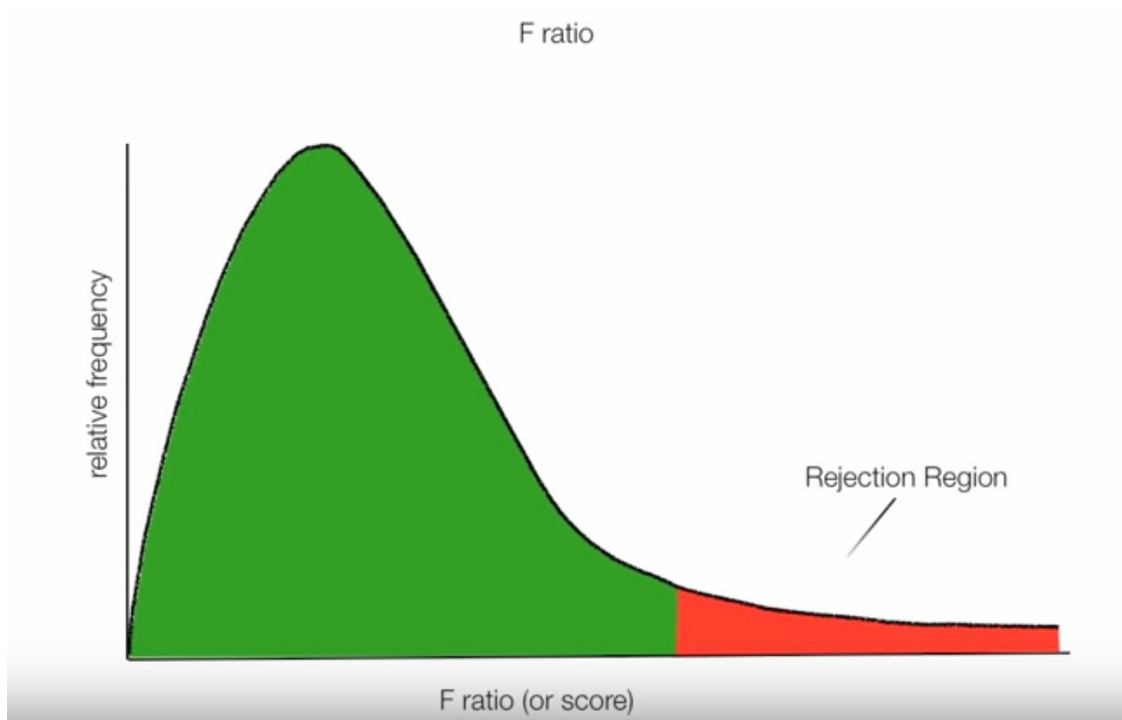
levels of stress



variance from the mean

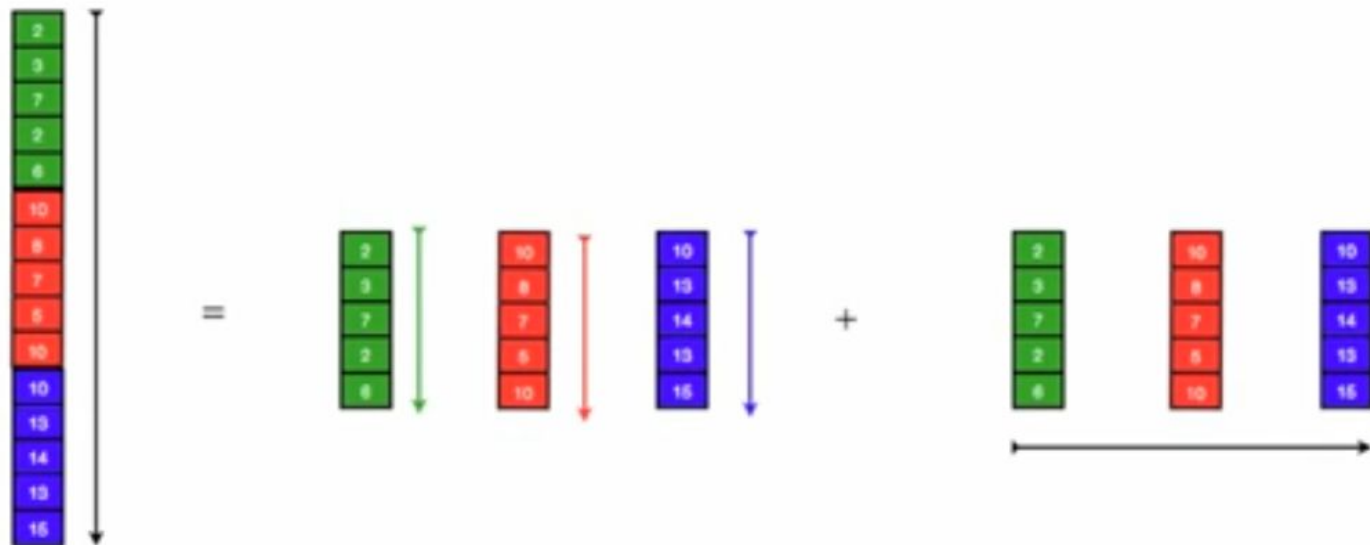
Total Sum of Squares

Calculate the F-ratio and see where it falls.



Analysis of Variance

F Ratio



$$\begin{array}{rccccccc} \text{Total Sum of Squares} & = & \text{Sum of Squares Within Groups} & + & \text{Sum of Squares Between Groups} \\ 257.3 & = & 54 & + & 203.3 \end{array}$$

Analysis of Variance

sample

2
3
7
2
6

sample

10
8
7
5
10

sample

10
13
14
13
15

Total Sum of Squares = Sum of Squares Between Groups + Sum of Squares Within Groups

Analysis of Variance

Sum of Squares Within Groups

sample

2	- 4 =	-2^2	4
3	- 4 =	-1^2	1
7	- 4 =	3^2	9
2	- 4 =	-2^2	4
6	- 4 =	2^2	4
			<hr/>
			22

sample

10	- 8 =	2^2	4
8	- 8 =	0^2	0
7	- 8 =	-1^2	1
5	- 8 =	-3^2	9
10	- 8 =	2^2	4
			<hr/>
			18

sample

10	- 13 =	-3^2	9
13	- 13 =	0^2	0
14	- 13 =	1^2	1
13	- 13 =	0^2	0
15	- 13 =	2^2	4
			<hr/>
			14

Sum of **S**quares **W**ithin Groups = 22 + 18 + 14 = 54

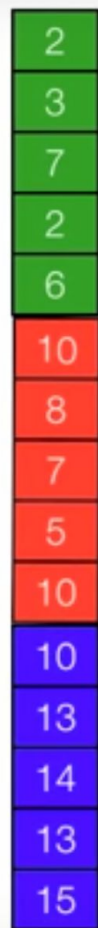
observation		mean	observation - mean	(observation - mean) ²
2	-	8.3	= -6.3	40.1
3	-	8.3	= -5.3	28.4
7	-	8.3	= -1.3	1.8
2	-	8.3	= -6.3	40.1
6	-	8.3	= -2.3	5.4
10	-	8.3	= 1.7	2.7
8	-	8.3	= -0.3	0.1
7	-	8.3	= -1.3	1.8
5	-	8.3	= -3.3	11.1
10	-	8.3	= 1.7	2.8
10	-	8.3	= 1.7	2.8
13	-	8.3	= 4.7	21.8
14	-	8.3	= 5.7	32.1
13	-	8.3	= 4.7	21.8
15	-	8.3	= 6.7	44.4

Total Sum of Squares

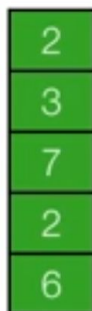
$$\text{SST} = 257.3$$

Analysis of Variance

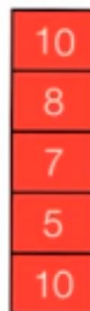
Sum of Squares Between Groups



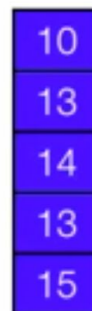
mean



mean



mean



mean

1. $\text{mean} - \text{mean}$

$\text{mean} - \text{mean}$

$\text{mean} - \text{mean}$

2. $(\text{mean} - \text{mean})^2$

$(\text{mean} - \text{mean})^2$

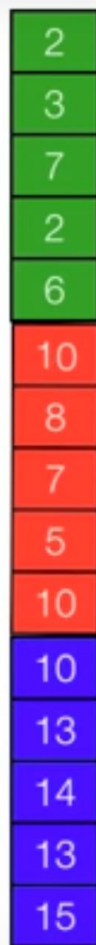
$(\text{mean} - \text{mean})^2$

3. $(\text{mean} - \text{mean})^2 + (\text{mean} - \text{mean})^2 + (\text{mean} - \text{mean})^2$

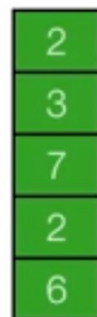
4. $(\text{mean} - \text{mean})^2 + (\text{mean} - \text{mean})^2 + (\text{mean} - \text{mean})^2 \times 5$

Analysis of Variance

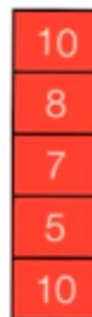
Sum of Squares Between Groups



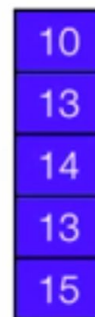
mean



$$4 - 8.3 = (-4.3)^2$$



$$8 - 8.3 = (-.3)^2$$



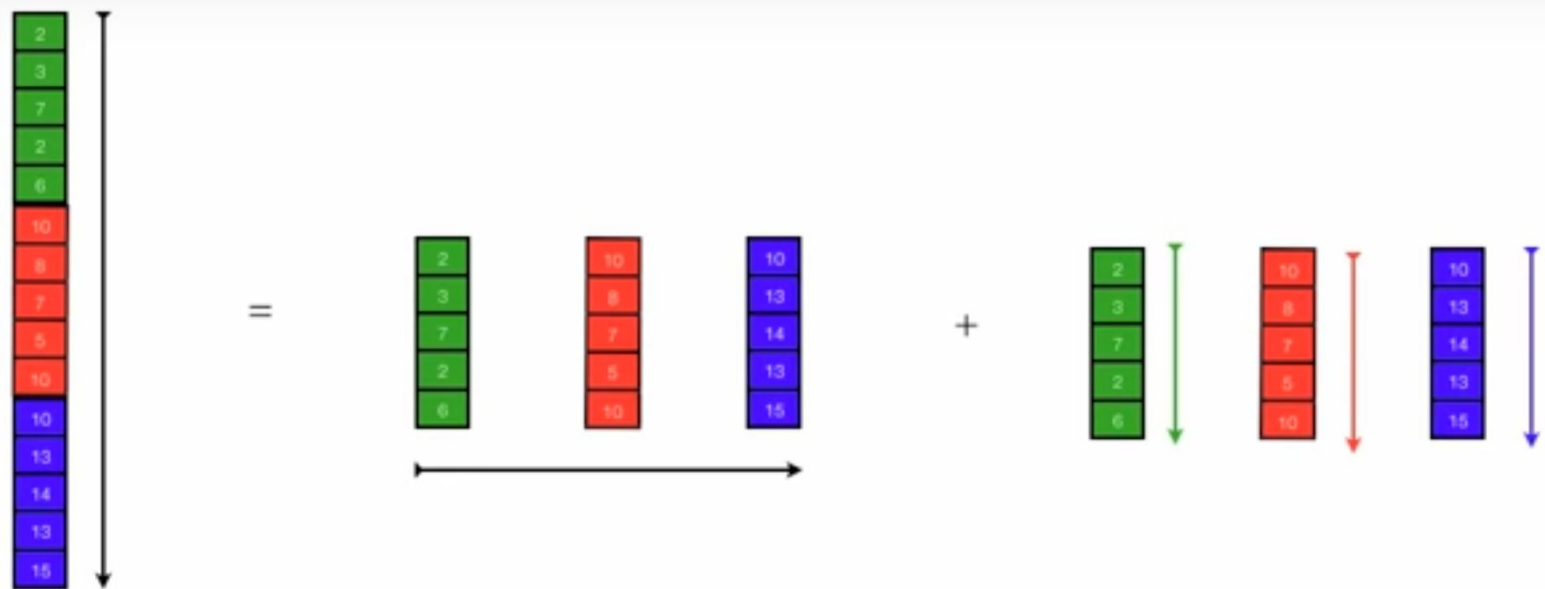
$$13 - 8.3 = (4.7)^2$$

$$18.8 + .1 + 21.8 = 40.7$$

$$40.7 \times 5 = 203.3$$

mean

8.3

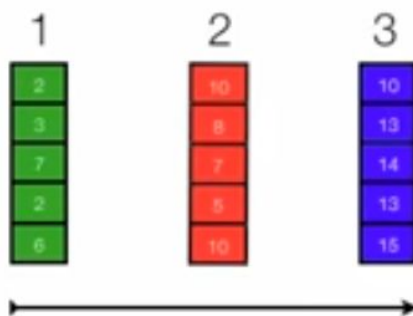


$$\begin{array}{rclcl}
 \text{Total Sum of Squares} & = & \text{Sum of Squares Between Groups} & + & \text{Sum of Squares Within Groups} \\
 257.3 & = & 203.3 & + & 54
 \end{array}$$

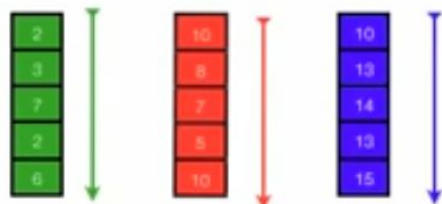
Final Calculations

$$\frac{\text{Sum of Squares Between Groups}}{\text{degrees of freedom}} = \frac{203.3}{2}$$

$$\begin{aligned} \text{Groups} - 1 \\ 3 - 1 = \end{aligned}$$



Final Calculations



$$\frac{\text{Sum of Squares Within Groups}}{\text{degrees of freedom}} = \frac{54}{12}$$

Final Calculations

$$\frac{\text{Sum of Squares Between Groups}}{\text{degrees of freedom}} = \frac{203.3}{2} = 101.667$$

$$F = \frac{101.667}{4.5} = 22.59$$

$$\frac{\text{Sum of Squares Within Groups}}{\text{degrees of freedom}} = \frac{54}{12} = 4.5$$

Final Calculations

$$F(2,12) = 22.59, p < .05$$

$$\frac{\text{Sum of Squares Between Groups}}{\text{degrees of freedom numerator}} = \frac{203.3}{2} = 101.667$$

$$F = \frac{101.667}{4.5} = 22.59$$

$$\frac{\text{Sum of Squares Within Groups}}{\text{degrees of freedom denominator}} = \frac{54}{12} = 4.5$$

F Distribution $F(2,12) = 22.59, p < .05$

degrees of freedom numerator

degrees of freedom denominator

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	161.5	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	246.0	248.0	249.1	250.1
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25

