

# 1 Required

## 1.1 Material

- [Permutation Puzzles](#) by Dr. Jamie Mulholland as a part of [MATH 304](#) at Simon Fraser University.

## 1.2 Exercises

### 1.2.1 Gallian

- Supplementary Exercises for Chapters 1 - 4, 1 - 10, A First Course in Abstract Algebra
- Chapter 5, 70 - 79, A First Course in Abstract Algebra

### 1.2.2 Programming

- Given a list of generators of a group (of order  $< 256$ ) can you express every element of the group as a minimal product of the generators? A product is deemed minimal if no smaller product produces the same element.
- Given a finite group  $G$  find the longest chain of subgroups  $\{H_i\}_{i=0}^n$  such that
  - $H_0 := \{e\}$
  - $H_n := G$
  - $H_{i-1} \subsetneq H_i$  for all  $1 \leq i \leq n$

# 2 Additional

## 2.1 Furloughing non-performers

Define  $GenSet(G) := \{S \subseteq G \mid S \text{ generates } G\}$  as the set of all possible generating sets of a group  $G$ . Observe  $GenSet(G) \neq \emptyset$ , as  $G \in GenSet(G)$ .

We call an element  $g$  of  $G$  a *non-generator* if for all generating sets  $S$  containing  $g$ ,  $S \setminus \{g\}$  is still a generating set.

Define  $NotGen(G) := \{g \in G \mid g \text{ is a non-generator}\}$ .

Compute the following:

- $NotGen(S_3)$
- $NotGen(\mathbb{Z})$
- $NotGen(\mathbb{Z}/4\mathbb{Z})$

Are all of these groups? Is  $NotGen(G)$  always a subgroup of  $G$ ?