1 Recover a relation from a partition

Suppose a set S can be nicely partitioned into mutually disjoint subsets S_α where $\alpha \in \Lambda$[1].
Is it possible to construct an equivalence relation $\sim$ on S such that the set of equivalence classes of $\sim$ is identical to the set $\{S_\alpha \mid \alpha \in \Lambda\}$?

2 Enforcing a group structure

Let X be the set of all plans that a supervisor can provide[2]. Define the equivalence relation $\sim$ on X as $p \sim q$ if and only if p and q generate the same set-up when executed separately from the start state.

Figure 1: Start state

Exercises:
If $p \sim p'$ and $q \sim q'$, verify if $p \star q \sim p' \star q'$.

Let $(X/\sim)$ be the set of all equivalence classes.
Define $\widetilde{\star} : (X/\sim) \times (X/\sim) \to (X/\sim)$ as $[p] \,\widetilde{\star}\, [q] = [p \star q]$.

Is $\Big((X/\sim), \widetilde{\star}, [\phi] \Big)$ a group[3]?
Is there a nice subset of $(X/\sim)$ that behaves like $(Z, +, 0)$?

[1]Remember you can have n partitions, countably infinite partitions, and an uncountably infinite number of partitions. We'll use Λ as a placeholder since the exact number of partitions is context dependant. Λ is formally called an index set.

[2]Look at the material provided on 27/8 for more details.
[3]It is a group, you can read more about it here.