TUTORIAL HOUR - 27/8

1 Groups over Z

We define the group G_n for each $n \in Z$ in the following manner:
$G_n := (Z, \circ_n, n)$, where $\circ_n : Z \times Z \to Z$, $a \circ_n b = a + b - n$
Verify that each G_n is indeed a group.
Is there another group structure on Z that is not one of the constructed G_n?

2 Thinking about permutations

You can run the file `symmetric.txt` on SAGE's online platform to examine the group structure of the set of permutations on three letters[1].

3 Assignment

This is not evaluative.

Consider a disgruntled electrician and an infinite line of sodium vapour lamps. Being disgruntled, the electrician only moves a finite number of steps away from the current location and can toggle an adjacent lamp 'ON' and 'OFF' a finite number of times.

Suppose the electrician receives a plan from a supervisor after a certain amount of work has been completed. The plan describes exactly how the electrician must traverse and toggle lamps.

The electrician, being disgruntled (again), decides to not reset the work done in previous plans when a new plan is received; the new plan is followed from the location that the previous plan left the electrician at.

Let X be the set of all possible plans that the supervisor can provide, plans that have an infinite number of movements, toggles, and instructions are discarded.
The binary operation $\star : X \times X \to X$, $p_1 \star p_2$ is defined by copying the instructions of p_2 and inserting them after the instructions of p_1. This captures the electrician's behavior.
X has a plan with no instructions, call it ϕ.

Is $(X, \star, \phi)$ a group? If so, is this group commutative?
Is there a nice subset of X that behaves like $(Z, +, 0)$?

[1] Can you fill the composition table of the group of permutations on four letters?

3.1 Valid instructions

A supervisor can include a finite number of these instructions in a plan:

```
walk one step down
walk one step up
toggle the lamp
```

3.2 An example

Use this example as a way to understand the $\star$ operation.[2]
Suppose the electrician starts from a state of all lamps turned 'OFF'.

Figure 1: Start state

Suppose a plan p_1 is communicated to the electrician, p_1 describes the following process:

```
walk one step down
toggle the lamp
walk one step up
```

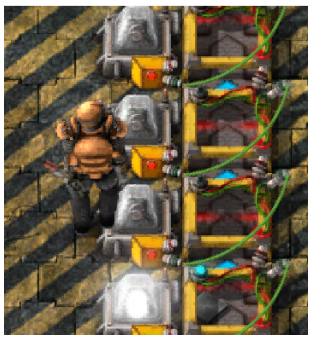

Figure 2: After Plan p_1

[2]These images are screenshots from the video game factorio.

Suppose after p_1 has been completed, the electrician receives a plan p_2:

```
walk one step up
toggle the lamp
walk one step down
```

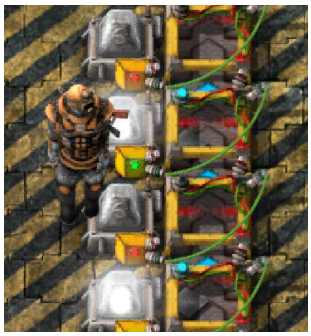

Figure 3: p_2, after Plan p_1

Note that sending the plan p would have done the job:

```
walk one step down
toggle the lamp
walk one step up
walk one step up
toggle the lamp
walk one step down
```