TUTORIAL HOUR - 5/10

1 Required

1.1 Material

- Lecture material from 18.703 - Modern Algebra, MIT on isomorphisms.
- SageMath documentation for group theory.

1.2 Exercises

1.2.1 From Gallian

- Problems 30-40, Chapter 4, A First Course in Abstract Algebra.
- Problems 4-10, 28-39, Chapter 6, A First Course in Abstract Algebra.

1.2.2 Programming

- Write a program that takes $n \in \mathbb{N}, n > 2$ as an input and outputs the Cayley table of the Permutation group on n letters.
- Write a program that takes $n \in \mathbb{N}, n > 2$ as an input and outputs the probability of 2 being a generator of the group $U(p)$ for all primes $p < n$.
- Write a program that takes $n \in \mathbb{N}, n > 2$ as an input and checks if $U(n)$ is cyclic.
- Write a program that takes an input string that contains the letters $\{u, u^{-1}, t, d, d^{-1}\}$ and checks if the related element in the lamplighter group[1] has a finite order.

2 Additional

2.1 Not always a subgroup

Let G be a group and define $[\cdot, \cdot] : G \times G \to G$ as $[g, h] = ghg^{-1}h^{-1}$.
The set $H(G) = \{[g, h] \mid g, h \in G\}$ is not always a group.

Write a program that verifies that $H(D_4)$ is a subgroup, where D_4 is the set of symmetries of the square.

Run the `counter.txt` script on Sage's online platform to examine an instance of when $H(G)$ is not a subgroup.

[1] Refer to material provided on 27/8 and 3/9

2.2 Rational Points on a familiar shape

Let $H(\mathbb{Q}) := \{(x, y) \in \mathbb{Q}^2 \mid x^2 - y^2 = 1\}$
Define the operation $\star : H(\mathbb{Q}) \times H(\mathbb{Q}) \to H(\mathbb{Q})$ as $(x, y) \star (u, v) = \Big((xu + yv), (xv + yu)\Big)$
What parallels exist between $\Big(H(\mathbb{Q}), \star, (1, 0)\Big)$ and the group of rational points of the unit circle?

2.3 A tall order

Find at least three Rubik's Cube™ algorithms that take the most number of iterations to loop back.[2]
The algorithm `RULD` took 315 iterations to cycle back but doesn't have the largest order as the algorithm `RULDDD` takes 420 iterations to cycle back.

[2] A good starting point.