TUTORIAL HOUR - 27/9

1 Required

1.1 Material

An expository article about subgroups of cyclic groups by Dr. Keith Conrad.

1.2 Exercises

Problems 20-24, 33-39, 53-54, Chapter 3, A First Course in Abstract Algebra.
Problems 11-18 and 74-76, Chapter 4, A First Course in Abstract Algebra.

1.3 Rational points on the unit circle

1.3.1 Material

An AMS article about the group of rational points on the unit circle by Dr. Lin Tan.

1.3.2 Visualization tool

You can run the code in `rationalCircle.txt` on SAGE's online platform to visualize how the cyclic subgroup generated by one rational points covers the unit circle.

2 Additional

2.1 Extension of material provided on 15/9

Let $\mathbb{Z}_{2^\infty} := \{z \in \mathbb{C} \mid |z| = 1 \text{ and } z^{2^k} = 1 \text{ for some } k \in \mathbb{N}\}$.
Verify: $\left(\mathbb{Z}_{2^\infty}, \times, 1\right)$ is a group.

Prove that any finite subgroup of the group $\left(\mathbb{Z}_{2^\infty}, \times, 1\right)$ is cyclic.
Do these finite subgroups look like $\mathbb{Z}/2^k\mathbb{Z}$?
Is the group $\mathbb{Z}_{2^\infty}$ a limiting value to the sequence of groups $\mathbb{Z}/2\mathbb{Z} \subset \mathbb{Z}/2^2\mathbb{Z} \subset \mathbb{Z}/2^3\mathbb{Z} \subset \ldots$?