TUTORIAL HOUR - 16/11

1 Required

1.1 Material

- An article on group actions by Professor Ryan Vinroot.
- More on normal subgroups and quotient groups by Professor Bruce Ikenaga.

1.2 Exercises

1.2.1 Gallian

- Chapter 7, 10 - 20, A First Course in Abstract Algebra
- Chapter 9, 1 - 10, A First Course in Abstract Algebra

1.2.2 Herstein

- Section 2.6, 1 - 6, 8 - 13, Topics in Algebra

1.2.3 Programming

- Read more about the `coset()` method to compute cosets of a subgroup H of a group G.
- Let $\mathbb{X} :=$ all permutations of $(1, 2, 3, 4)$.
  Define an action $S_4 \times \mathbb{X} \to \mathbb{X}$ as: $\Big(\sigma, (i_1.i_2, i_3, i_4)\Big) \mapsto (\sigma(i_1), \sigma(i_2), \sigma(i_3), \sigma(i_4))$.
  Use SageMath to compute the orbit of $(1, 2, 3, 4)$.
- Given a group G list out all normal groups and compute the associated quotient group.

2 Additional

2.1 3D Rotations

Let $SO(3) := \{A \in Mat_{3\times3}(\mathbb{R})|\det(A) = 1\}$ and let $S^2 := \{(x, y, z) \in \mathbb{R}^3|x^2 + y^2 + z^2 = 1\}$.

Define a map $SO(3) \times S^2 \to S^2$ as $\Big(A, (x, y, z)\Big) \mapsto A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Is this a group action?
If A is a non-identity element, define $Inv(A) := \Big\{(x, y, z) \in S^2|\Big(A, (x, y, z)\Big) = (x, y, z)\Big\}$.
Does the cardinality of $Inv(A)$ depend on A?

Given another non-identity element B and $(x, y, z) \in Inv(A)$, consider $B \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

Can you find a matrix C such that $Inv(C)$ contains this element?

2.2 MC Escher and groups

The Algebraic Escher by Professor Marjorie Senechal that connects topics in group theory with MC Escher's works.