TUTORIAL HOUR - 27/8

# 1 Groups over $Z$

We define the group $G_n$ for each $n \in Z$ in the following manner:
$G_n := (Z, \circ_n, n)$, where $\circ_n : Z \times Z \to Z$, $a \circ_n b = a + b - n$
Verify that each $G_n$ is indeed a group.
Is there another group structure on $Z$ that is not one of the constructed $G_n$?

# 2 Thinking about permutations

You can run the `symmetric.txt` on SAGE's online platform to examine the group structure of the set of permutations on three letters[1].

# 3 Assignment

**This is not evaluative.**

Consider a disgruntled electrician and an infinite line of sodium vapour lamps. Being disgruntled, the electrician only moves a finite number of steps away from his current location and can toggle an adjacent lamp 'ON' and 'OFF' a finite number of times.

Suppose the electrician receives a plan from a supervisor after a certain amount of work has been completed. The plan describes exactly how the electrician must traverse and toggle lamps.

The electrician, being disgruntled (again), decides to not reset the work done in previous plans when a new plan is received; the new plan is followed from the location that the previous plan left the electrician at.

Let $X$ be the set of all possible plans that the supervisor can provide, plans that have an infinite number of movements, toggles, and instructions are discarded.
The binary operation $\star : X \times X \to X$, $p_1 \star p_2$ is defined by copying the instructions of $p_2$ and inserting them after the instructions of $p_1$. This captures the electrician's behavior.
The identity element of $X$ is the 'do-nothing' plan, call it $\phi$.

Is $(X, \star, \phi)$ a group? If so, is this group commutative?
Is there a nice subset of $X$ that behaves like $(Z, +, 0)$?

[1] Can you fill the composition table of the group of permutations on four letters?

## 3.1 An example

Use this example as a way to understand the $\star$ operation.[2]
Suppose the electrician starts from a state of all lamps turned 'OFF'.

Figure 1: Start state

Suppose a plan $p_1$ is communicated to the electrician, $p_1$ describes the following process:

```
walk one step down
toggle the lamp
walk one step up
```

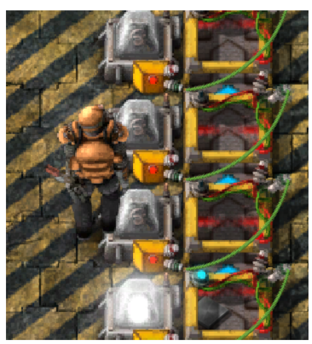

Figure 2: After Plan $p_1$

Suppose after $p_1$ has been completed implemented, the electrician receives a plan $p_2$:

```
walk one step up
toggle the lamp
walk one step down
```

[2]These images are screenshots from the video game factorio.

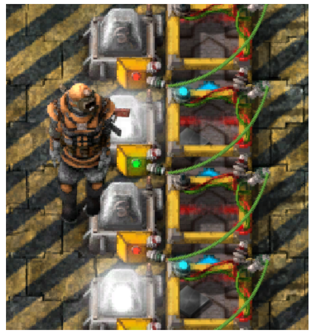

Figure 3: $p_2$, after Plan $p_1$

Note that sending the plan $p$ would have done the job:

```
walk one step down
toggle the lamp
walk one step up
walk one step up
toggle the lamp
walk one step down
```