1 Financial Engineering: Modeling and Optimization in Commodity Markets

1.1 Introduction

Financial engineering encompasses the application of mathematical and computational techniques to analyze, design, and optimize financial products and strategies. In the realm of commodity markets, where prices are influenced by diverse factors such as supply-demand dynamics, geopolitical events, and macroeconomic indicators, financial engineering plays a crucial role in effectively managing risks and maximizing returns. This document explores the application of advanced financial engineering principles in modeling and optimizing investment portfolios in commodity markets.

1.2 Fetching Commodity Data

The foundation of any financial analysis lies in the availability of high-quality data. The provided code leverages the `yfinance` library to fetch historical price data for selected commodities from Yahoo Finance. This dataset serves as the basis for subsequent analysis and modeling.

1.3 Modeling Price Dynamics

Commodity prices exhibit stochastic behavior, often modeled using stochastic processes. One such widely used model is Geometric Brownian Motion (GBM), which assumes that the logarithm of commodity prices follows a Brownian motion with drift and volatility. The dynamics of a commodity price $S(t)$ can be described by the following stochastic differential equation (SDE):

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$$

where:

- μ is the drift rate, representing the average rate of return of the commodity.
- σ is the volatility, representing the standard deviation of the returns.
- $W(t)$ is a Wiener process, representing random market fluctuations.

The code employs GBM to simulate correlated price paths for selected commodities, capturing their stochastic nature and interdependencies.

1.4 Risk Assessment

Effective risk management is essential in commodity trading. Two fundamental risk measures utilized in financial engineering are Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR):

- **Value-at-Risk (VaR)** represents the maximum potential loss of a portfolio within a specified confidence level over a given time horizon. It is calculated as the quantile of the portfolio's return distribution.

$$\text{VaR}_\alpha = \text{quantile}_\alpha(R)$$

- **Conditional Value-at-Risk (CVaR)** provides a measure of the expected shortfall beyond the VaR level. It represents the average loss of the portfolio given that the losses exceed the VaR threshold. Mathematically, it is defined as the conditional expectation of the loss beyond the VaR level.

$$\text{CVaR}_\alpha = E[R \mid R < \text{VaR}_\alpha]$$

The code calculates these risk measures to assess the downside risk of the commodity portfolio at a specified confidence level.

1.5 Portfolio Optimization

Portfolio optimization aims to construct an investment portfolio that achieves a desirable trade-off between risk and return. The code formulates an objective function for portfolio optimization, considering both the expected return and the risk associated with the portfolio. Specifically, it seeks to maximize the expected return while simultaneously minimizing the risk, subject to certain constraints such as weight bounds and sum-to-one constraints. This optimization problem can be mathematically formulated as follows:

$$\max_{\mathbf{w}} \left(\mathbf{w}^T \boldsymbol{\mu} - \lambda \sqrt{\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}} \right)$$

subject to:

$$\sum_{i=1}^{n} w_i = 1$$

$$0 \leq w_i \leq 1$$

where:

- $\mathbf{w}$ is the vector of portfolio weights.
- $\boldsymbol{\mu}$ is the vector of expected returns.
- $\boldsymbol{\Sigma}$ is the covariance matrix of returns.
- λ is the risk aversion parameter, controlling the trade-off between risk and return.

The code employs optimization techniques such as constrained optimization to find the optimal portfolio weights that maximize the objective function.