Commodity Options Pricing: Black-Scholes Model and Greeks Calculation

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1 Introduction

Commodity options are financial derivatives that provide the holder with the right, but not the obligation, to buy or sell a commodity at a predetermined price within a specified time frame. The Black-Scholes model is a widely used mathematical model for pricing options. It calculates the theoretical price of European-style options based on various factors, including the current commodity price, strike price, time to expiration, risk-free interest rate, and volatility. Additionally, option Greeks are essential measures used to assess the sensitivity of option prices to changes in these factors.

2 Black-Scholes Pricing Calculation

The Black-Scholes formula for pricing European call and put options on commodities is as follows:

2.1 Call Option Price

$$C = S_0 N(d_1) - K e^{-rt} N(d_2)$$

where:

- C = Call option price
- S_0 = Current commodity price
- $N(x)$ = Cumulative distribution function of the standard normal distribution
- $d_1 = \frac{\ln(S_0/K) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$
- $d_2 = d_1 - \sigma\sqrt{T}$
- K = Strike price
- r = Risk-free interest rate
- T = Time to expiration
- σ = Volatility of the commodity

2.2 Put Option Price

$$P = K e^{-rt} N(-d_2) - S_0 N(-d_1)$$

where all variables are the same as in the call option case.

3 Option Greeks Calculation

Option Greeks are measures of the sensitivity of option prices to changes in various factors. The key Greeks are as follows:

- **Delta (Δ)**: Measures the rate of change of option price with respect to changes in the price of the underlying commodity.
$$\Delta = N(d_1) \quad \text{(for call option)}$$
$$\Delta = N(d_1) - 1 \quad \text{(for put option)}$$
- **Gamma (Γ)**: Measures the rate of change of Delta with respect to changes in the price of the underlying commodity.
$$\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$$
- **Theta (Θ)**: Measures the rate of change of option price with respect to changes in time to expiration.
$$\Theta = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rt}N(-d_2) \quad \text{(for call option)}$$
$$\Theta = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rt}N(-d_2) \quad \text{(for put option)}$$
- **Vega (ν)**: Measures the rate of change of option price with respect to changes in volatility.
$$\nu = S_0 \sqrt{T} N'(d_1)$$
- **Rho (ρ)**: Measures the rate of change of option price with respect to changes in the risk-free interest rate.
$$\rho = KTe^{-rt}N(d_2) \quad \text{(for call option)}$$
$$\rho = -KTe^{-rt}N(-d_2) \quad \text{(for put option)}$$

4 Conclusion

The Black-Scholes model provides a powerful framework for pricing commodity options and assessing their sensitivity to various factors through option Greeks. By understanding these concepts, traders and risk managers can make informed decisions in the commodity markets.

5 References

- Black, F., Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. Journal of Political Economy.
- Hull, J. C. (2018). Options, Futures, and Other Derivatives. Pearson.