Università degli Studi di Milano

The bizzarre story of Laughlin'ansatz

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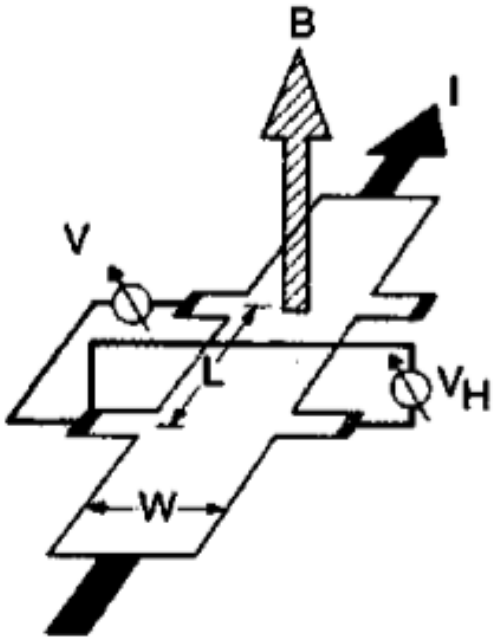
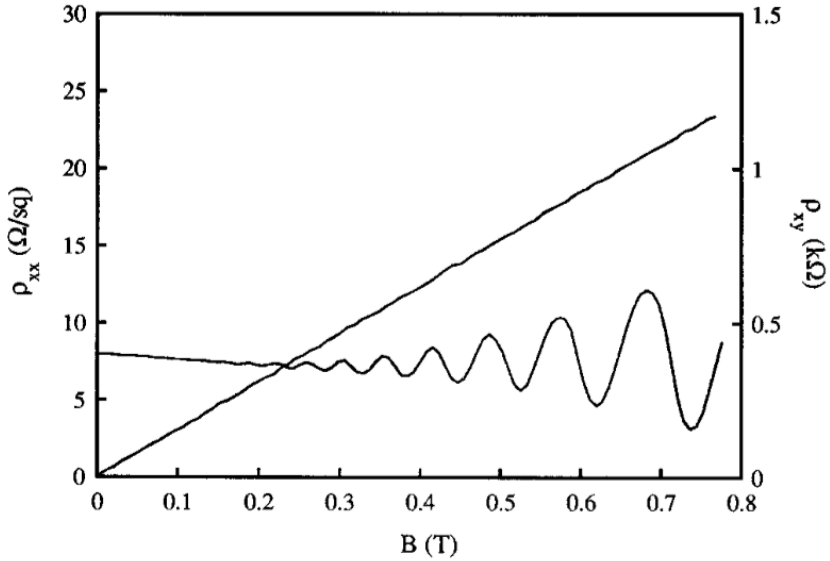
14/08/2016

Once upon a time. . . Classical Hall effect



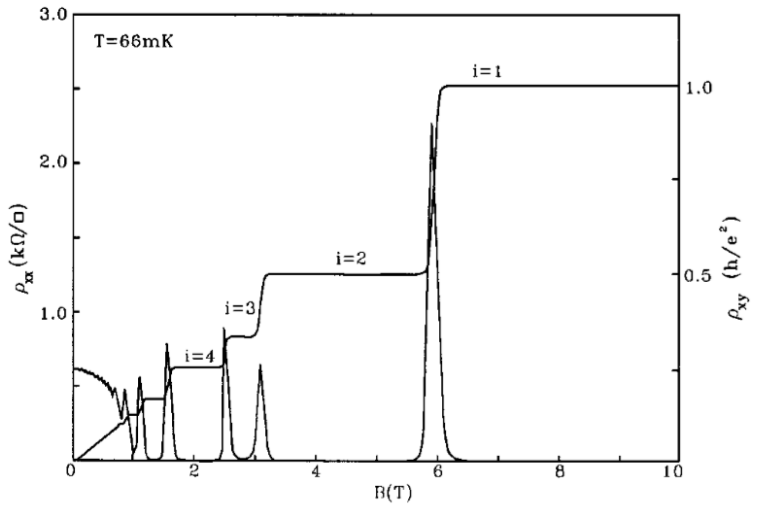
[Hall, 1879]



$$\rho_{xy} = \frac{B}{\rho e c}$$

Plot twist! Plateaux everywhere

$$B \sim 1 - 10\,\mathrm{T} \qquad T \sim 100\,\mathrm{mK}$$



Integer quantum Hall effect

[Von Klitzing, Nobel 1985]

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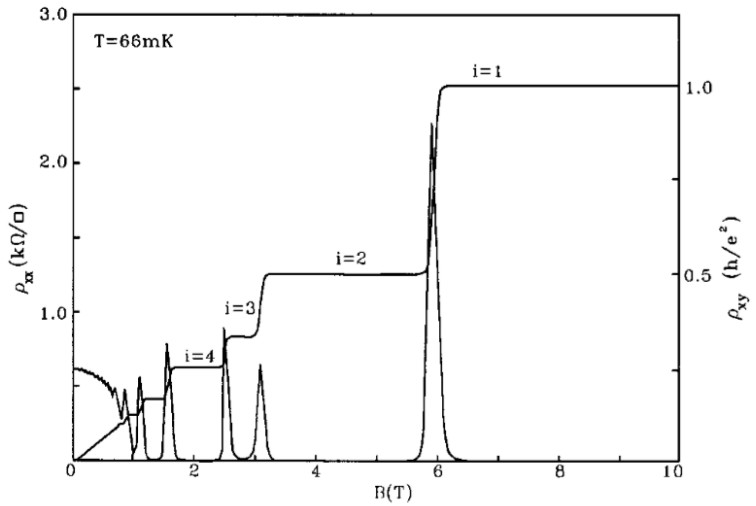

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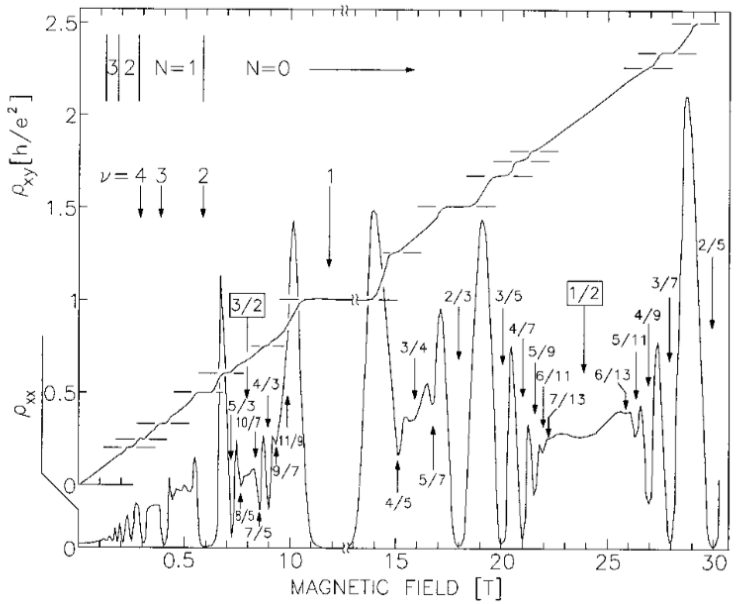
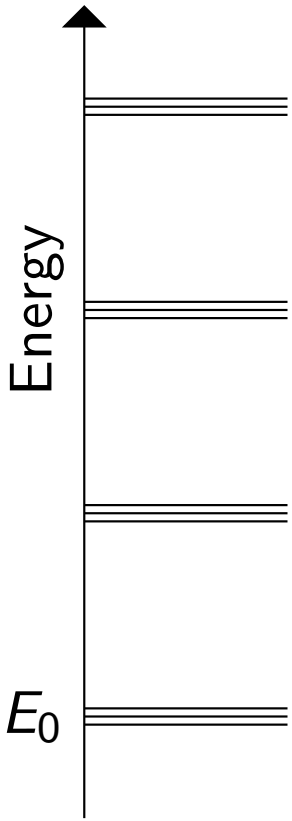


Fractional quantum Hall effect

[Laughlin, Störmer, Tsui, Nobel 1988]

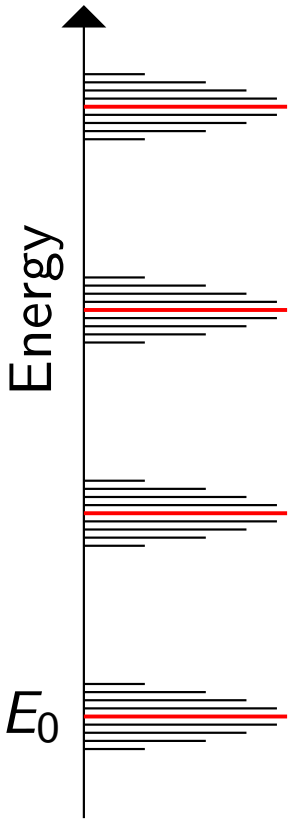
IQHE: Landau levels and impurities



Independent electrons

- Harmonic oscillator spectrum
- Level spacing $\propto$B
- High degeneracy $\propto$B

IQHE: Landau levels and impurities



Independent electrons

- Harmonic oscillator spectrum
- Level spacing $\propto$B
- High degeneracy $\propto$B
- **Impurities** induce broadening
- **Impurities** induce localization

Black: localized states
Red: conducting states

And what about FQHE?

What is known

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What's the problem

- No exact solution for Coulomb interaction
- No perturbative approach is possible

Variational approach: Laughlin's ansatz for the ground state

- E_0 has a basis of monomials
  $\Rightarrow$ Look for a **polynomial** (apart from a gaussian factor)
- H has cylindrical symmetry
  $\Rightarrow$ Look for a **homogeneous polynomial**
- Coulomb interaction is two-body and repulsive
  $\Rightarrow$ Look for **functions of** $(z_i - z_j)$
- Electrons are fermions
  $\Rightarrow$ Look for an **antisymmetric function**

Variational approach:
Laughlin's ansatz for the ground state

$$\psi_L(z_1, \ldots, z_n) = \prod_{i<j}(z_i - z_j)^q e^{-\sum_i \frac{|z_i|}{4}^2}$$

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- q variational parameter ...

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Not exact but $\sim 99\%$ overlap with computed ground state for $1/q$ plateaux

Why is Laughlin's ansatz so good?

Not clear!

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Recursive formula for second quantized form

1 to 1 correspondence: factored states $\Leftrightarrow$ Laughlin's wavefunctions

[Bernevig & Haldane, 2008]

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Work in progress:

Explicit second quantized form

Understand correlations between electrons

Look for explicit operator that creates Laughlin's wavefunctions from a factored state