

POLITECNICO
MILANO 1863

«Assortative and preferential attachment leads to core periphery» Torres, L., Puggini, L., & Moreno, V. (2023)

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Borgatti and Everett:

- Fully-connected core
- Disconnected periphery
- Indicate the core periphery as the correlation between the real adjacence matrix and the ideal one.

Gallagher

Introduce two type of core periphery models:

- Hub and spoke as generalization of the Borgatti and Everett' model
- Layered network

1 Models used

Rewiring Model

- i. Focal node select randomly with $p = n_a$
- ii. Choose Candidate
 - i. Given a uniform distribution if $p < c$
 - i. Preferential Attachment
 - ii. Otherwise randomly
- iii. Link creation
 - i. If the nodes belongs to the same group
 - i. Create a link with $p = S_a$
 - ii. Otherwise $1 - S_a = S_{ab}$
- iv. Link deleted with a random neighbors

$$\frac{\delta P_{aa}}{\delta t} = n_a * M_{aa} - n_a * T_{aa}(M_{aa} + M_{ab})$$

$$\frac{\delta P_{bb}}{\delta t} = n_b * M_{bb} - n_b * T_{bb}(M_{ba} + M_{bb})$$

$$\frac{\delta P_{ab}}{\delta t} = n_a * M_{ab} + n_b * M_{ba} - n_a * T_{ab}(M_{aa} + M_{ab}) - n_b * T_{ba} * (M_{bb} + M_{ba})$$

Growing Model

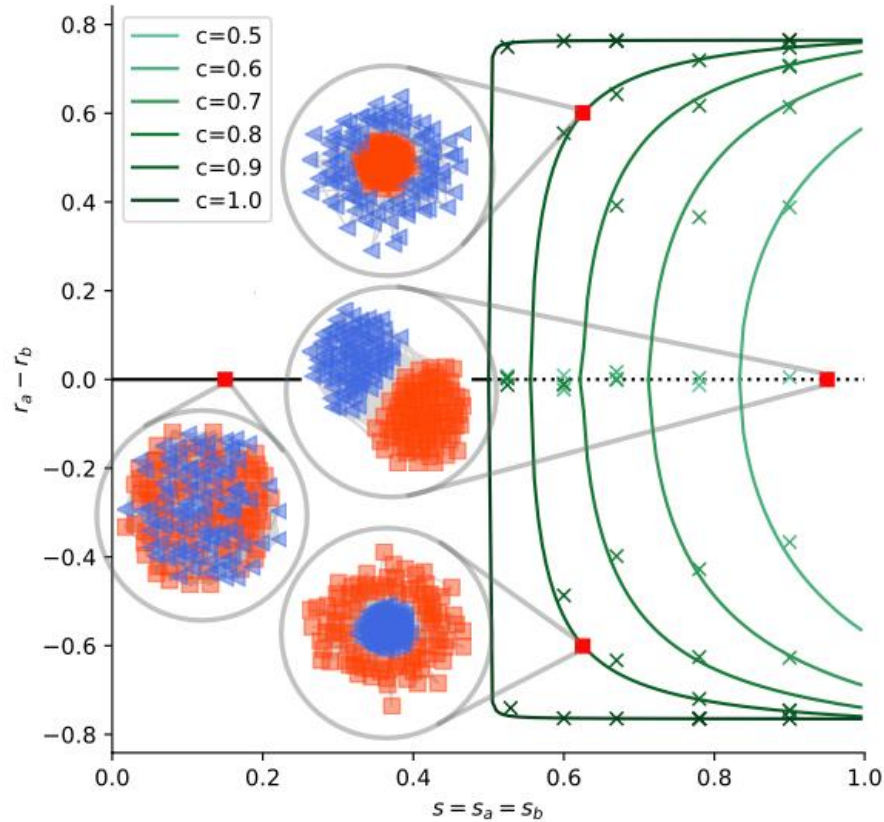
- i. Focal node added with $p = n_a$
 - ii. Choose M candidates
 - i. Given a uniform distribution if $p < c$
 - i. Preferential Attachment
 - ii. Otherwise randomly
 - iii. Link creation
 - i. If the nodes belongs to the same group
 - i. Create a link with $p = S_a$
 - ii. Otherwise $1 - S_a = S_{ab}$
- Repeated m times

$$P_{aa}^* = \frac{mn_a M_{aa}^*}{mn_a(M_{aa}^* + M_{ab}^*) + m(1 - n_a)(M_{ba}^* + M_{bb}^*)}$$

$$P_{bb}^* = \frac{m(1 - n_a)M_{bb}^*}{mn_a(M_{aa}^* + M_{ab}^*) + m(1 - n_a)(M_{ba}^* + M_{bb}^*)}$$



2.1 How much preferential attachment and assortative influenced the CP



For $s \leq 0.5$ no detailed distinction between Core and periphery

- Both group are disassortative

For $s > 0.5$ preferential attachment led the bifurcation between core and periphery

- Two possible unstable solution (a (b) core if $r_a - r_b > (<) 0$)
- The higher the c the more accentuated is the distinction

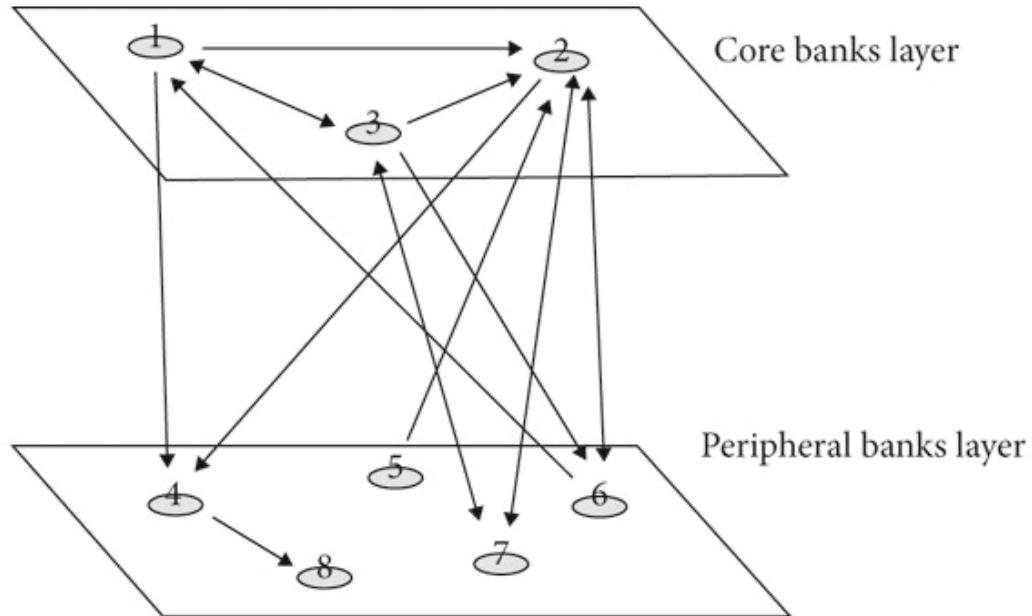
Core quality indicator

$$r_a(\alpha) = r_{X,Y}(\alpha) = \frac{(P_{aa} + \alpha P_{ab})\rho - (n_a^2 + 2\alpha n_a(1 - n_a))\rho}{\sqrt{\rho(1 - \rho)}\sqrt{(n_a^2 + 2\alpha n_a(1 - n_a))(1 - n_a^2 - 2\alpha n_a(1 - n_a))}}$$

$$r_g = \max r_g(\alpha)$$

Where do the models evolve to?

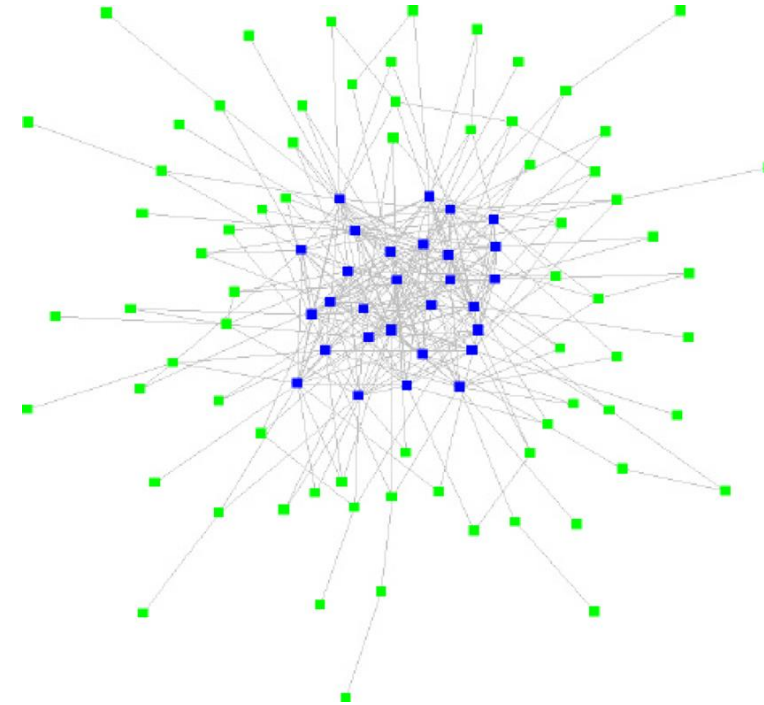
Growing models tends to Layered Network



$$\Omega g \approx 0$$

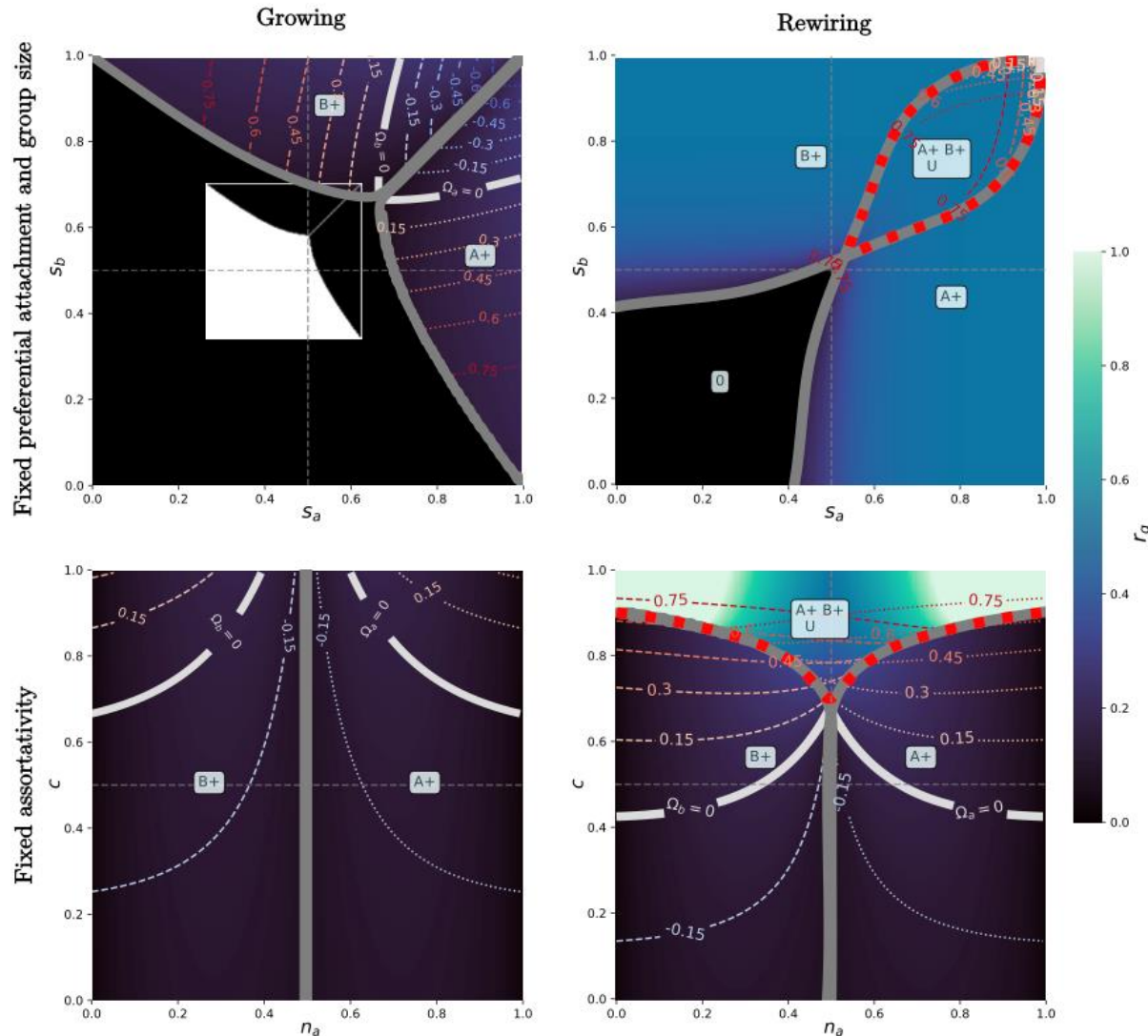
$$\Omega_a = \frac{\rho_{ab} - \rho_b}{\rho_{ab} + \rho_b},$$

Rewiring models tends to Hub and Spoke



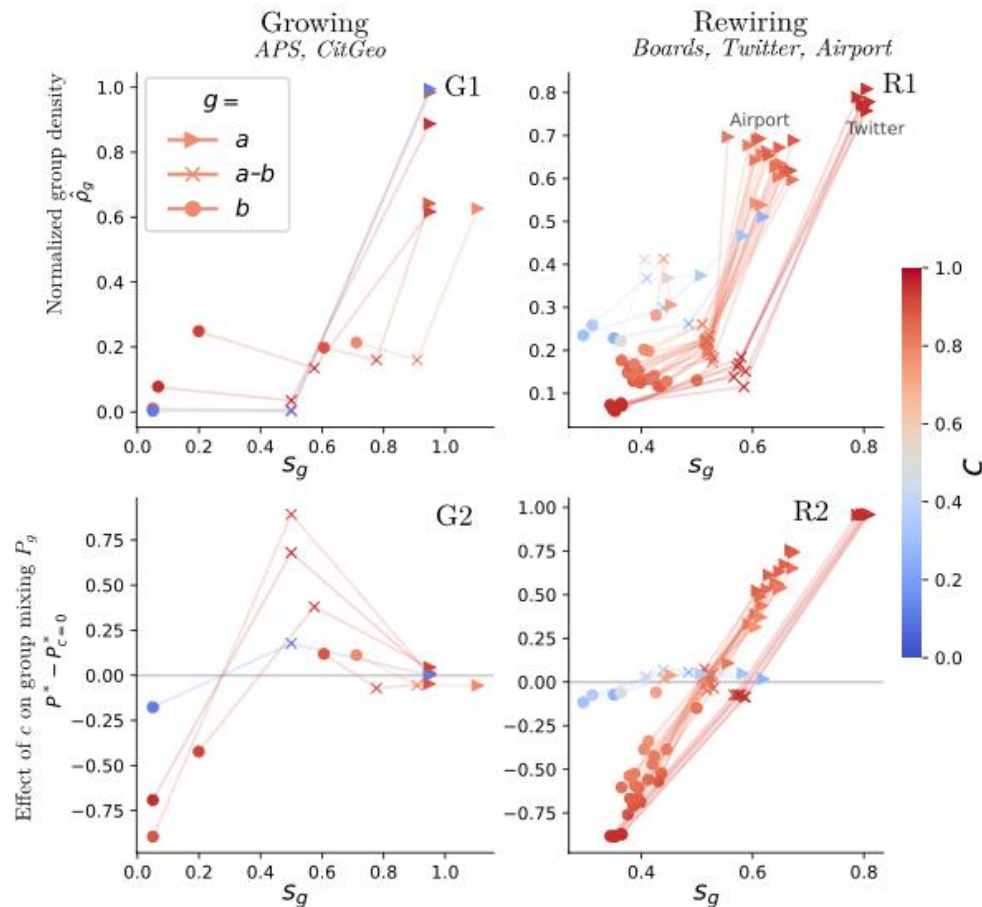
$$\Omega g \approx 1$$

2.2 How much preferential attachment and assortative influenced the CP



- Dotted red lines isolate place where a or b can become either core
- Top:
 - Assortativity can improve core periphery distinction ($c = 0.95$, $n_a = N_a/N = 0.5$)
- Bottom
 - Preferential attachment minority group to become core (rewiring model)
- In the growing model $\Omega g \approx 0$
- In the rewiring model $\Omega g > 0$

Practical experiment on growing and rewiring model

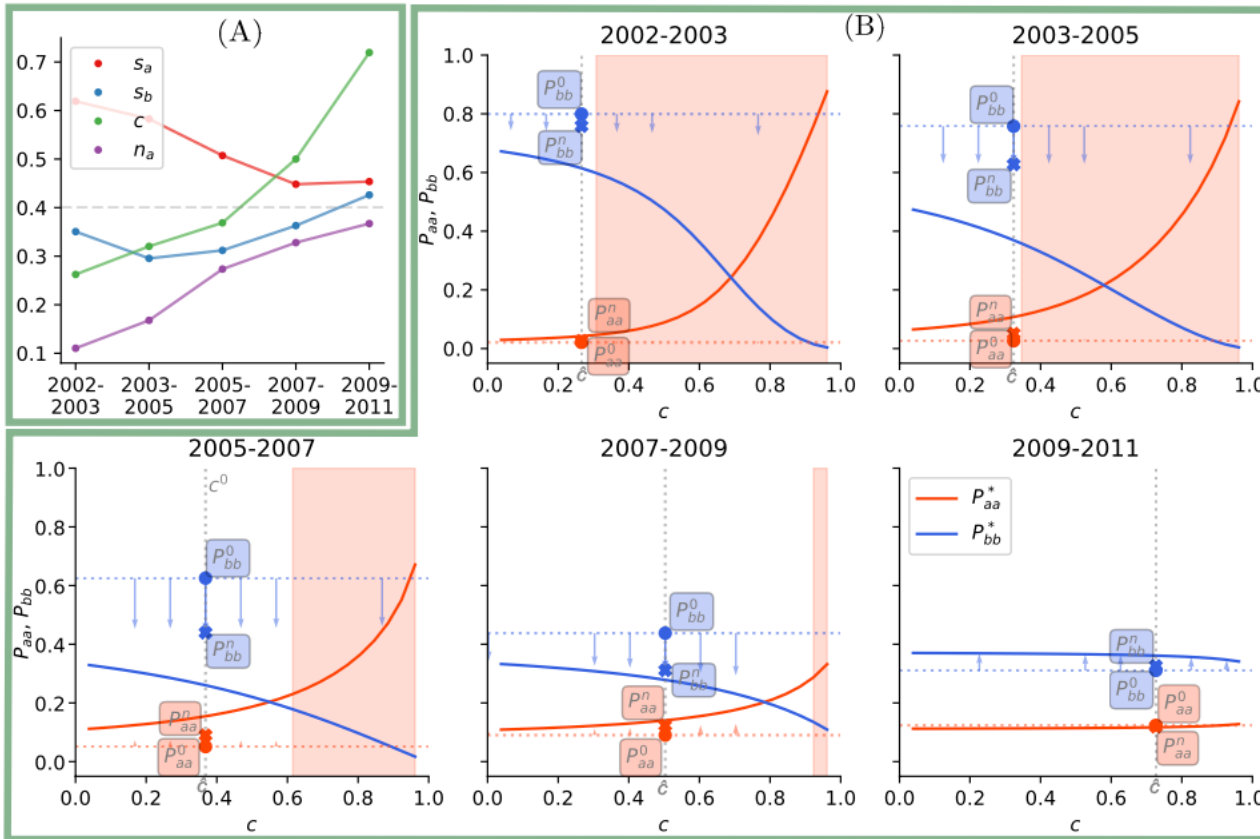


“Notably, for the rewiring model preferential attachment vastly increases the fraction of links within the core at the expense of links within the periphery”

- LEFT: growing model, citation network based on geography and subfield within physics
 - $0 < S_g < 0.5$ preferential attachment leads to more intra-core links
 - Then increasing in the S_g leads to more inter group (tends to Layered network)
- RIGHT: rewiring model, twitter discussion about climate change (core political actors)
 - $S_g > 0$ (core assortative despite the periphery tends to connect with core nodes)
 - Increase the number of links inside the core at the expense of the intra groups links. \Rightarrow hub and spoke \Rightarrow **high GINI/CP coefficient**
- $P^* - P^*_{c=0} > 0$ LOSS of Link Intragroup
- $P^* - P^*_{c=0} < 0$ GAIN of link
- P^* fixed point of P for fixed value



Variations of the parameters



Due to some constraints on the theoretical models (deletions might not follow creation, assortativity definition...) the result may deviate from the seen one.

Study case: Boards network, boards of director in Norway 2002-2011.

- Link between two nodes => belongs to the same board
- A: evolutions of the four fitted parameters.
- B: evolution of Group mixing matrix
- Policy applied: Increase of the minority group by the end of 2008 (starts in 2006)
- The real policy tends to the fixed point of the rewiring model
- Alternative policy applied by the author: leave other parameters untouched, and increase (decrease) preferential attachment. (Restricting the link between people). RED and BLUE arrow.

GROWING MODEL

- **Structure:** Tends to Layered network
- **Effect of preferential attachment:** High preferential attachment leads to more inter-group links at the expense of the intra-core ones.
- **Comparison with rewiring:** Equal to the rewiring model but gain links over time and has no deletion step.
- **Influence on the CP-CNN** maximizing the log likelihood over the preferential attachment leads the CP hidden layer to have some nodes(core) which extracts more features than peripheral nodes.

REWIRING MODEL

- **Structure:** Tends to Hub and Spoke
- **Effect of preferential attachment:** High preferential attachment boosts the intra-core links
- **Comparison with growing:** high preferential attachment can lead minority group to become core
- Link deletion «shifts» the links from one group to another over time.

References

Torres, L., Puggini, L., & Moreno, V. (2023). *Assortative and preferential attachment lead to core-periphery networks*. arXiv preprint arXiv:2305.15061.

Available at: <https://arxiv.org/abs/2305.15061>

Zhao, L., Dai, H., Wu, Z., Zhu, D., & Liu, T. (2023). CP-CNN: Core-Periphery Principle Guided Convolutional Neural Network. *ArXiv preprint arXiv:2304.10515*.

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