Kotitehtavat_4.pdf

1. Laske derivaatta f ′(x), kun f (x)

a) $f(x) = x^2e^{2x}$

$f(x) = x^2e^{2x}\ u(x) \quad = x^2, \quad v(x) = e^{2x}\ u'(x) = 2x, \quad v'(x) = 2e^{2x}\ f'(x) \quad = u'(x)v(x) + u(x)v'(x)\ f'(x) = 2xe^{2x} + x^2 \cdot 2e^{2x} \quad = 2xe^{2x} + 2x^2e^{2x}$

b) $f(x) = (2x + 4)^5$

$f(x) = (2x+4)^5\ u(x) \quad = 2x + 4, \quad n = 5\ f'(x) = n \cdot u(x)^{n-1} \cdot u'(x)\ u'(x) \quad = 2\ f'(x) = 5(2x+4)^4 \cdot 2 \quad = 10(2x+4)^4$

c) $f(x) = \frac{1}{1+e^-(ax+b)}$

$f(x) = \frac{1}{1 + e^-(ax+b)}\ u(x) \quad = 1 + e^-(ax+b)\ f'(x) = \frac{0 \cdot u(x) - 1 \cdot u'(x)}{u(x)^2}\ u'(x) \quad = -e^-(ax+b) \cdot a\ f'(x) = \frac{ae^-(ax+b)}{(1+e^-(ax+b))^2}$

2. Laske derivaatta f ′(x), kun f (x)

a) $10sin(x^2)$

$f(x) = 10sin(x^2)\ u(x) \quad = x^2\ f'(x) = 10 \cdot cos(u(x)) \cdot u'(x)\ u'(x) \quad = 2x\ f'(x) = 10 \cdot cos(x^2) \cdot 2x \quad = 20x \cdot cos(x^2)$

b) e^{cosx}

$f(x) = e^{cosx}\ u(x) \quad = cosx\ f'(x) = e^{u(x)} \cdot u'(x)\ u'(x) \quad = -sinx\ f'(x) = e^{cosx} \cdot (-sinx)\ f'(x) \quad = -sinx \cdot e^{cosx}$

c) $ln(1 - x^2)$

$f(x) = ln(1-x^2)\ u(x) \quad = 1 - x^2\ f'(x) = \frac{1}{u(x)} \cdot u'(x)\ u'(x) \quad = -2x\ f'(x) = \frac{-2x}{1-x^2}$

3. Laske funktion f osittaisderivaatat, kun $f(x, y)$

a) $xy^2 + 3x$

$f(x,y) = xy^2 + 3x\ f_x(x,y) \quad = \frac{\partial}{\partial x}(xy^2 + 3x)\ = y^2 + 3\ f_y(x,y) \quad = \frac{\partial}{\partial y}(xy^2 + 3x)\ = 2xy$

b) $ln\frac{x}{y}$

$f(x,y) = ln\frac{x}{y}\ f_x(x,y) \quad = \frac{\partial}{\partial x}(ln(x) - ln(y))\ = \frac{1}{x}\ f_y(x,y) \quad = \frac{\partial}{\partial y}(ln(x) - ln(y))\ = -\frac{1}{y}$

c) $\frac{sinx}{cosy}$

$f(x,y) = \frac{sinx}{cosy}\ f_x(x,y) \quad = \frac{\partial}{\partial x}\left(\frac{sinx}{cosy}\right)\ = \frac{cosx \cdot cosy}{cosy \cdot cosy} \quad = \frac{cosx}{cosy}\ f_y(x,y) = \frac{\partial}{\partial y}\left(\frac{sinx}{cosy}\right) \quad = \frac{sinx \cdot siny}{cos^2y}\ = -\frac{sinx \cdot siny}{cos^2y}$

4. Laske funktion f gradientti ∇f, kun

a) $f(x, y) = 2x^2y - 5y + 4xy^2$

$f(x,y) = 2x^2y - 5y + 4xy^2\ {}_xf(x,y) \quad = 4xy + 4y^2\ {}_yf(x,y)\ = 2x^2 - 5 + 8xy\ \nabla f(x,y) \quad =_x f(x,y)\hat{i} +_y f(x,y)\hat{j}\ = (4xy + 4y^2)\hat{i} + (2x^2 - 5 + 8xy)\hat{j}$

$\nabla f(2,1) = (4 \cdot 2 \cdot 1 + 4 \cdot 1^2)\hat{i} + (2 \cdot 2^2 - 5 + 8 \cdot 2 \cdot 1)\hat{j} \quad = 8\hat{i} + (8 - 5 + 16)\hat{j}\ = 8\hat{i} + 19\hat{j}$

b) $f(x, y) = \sqrt{x^2 + y^2}$

$f(x,y) = \sqrt{x^2+y^2}\ {}_xf(x,y) \quad = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x\ = \frac{x}{\sqrt{x^2+y^2}}\ {}_yf(x,y) \quad = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y\ = \frac{y}{\sqrt{x^2+y^2}}\ \nabla f(x,y) \quad = \frac{x}{\sqrt{x^2+y^2}}\hat{i} + \frac{y}{\sqrt{x^2+y}}$

$\nabla f(2,1) = \frac{2}{\sqrt{2^2+1^2}}\hat{i} + \frac{1}{\sqrt{2^2+1^2}}\hat{j} \quad = \frac{2}{\sqrt{5}}\hat{i} + \frac{1}{\sqrt{5}}\hat{j}\ = \frac{2}{\sqrt{5}}\hat{i} + \frac{1}{\sqrt{5}}\hat{j}$

5. Laske funktion f gradientti ∇f, kun

$f(x, y, z) = x^3 + 3xy^2 - 5y^2z + 2z^4$

$f(x,y,z) = x^3 + 3xy^2 - 5y^2z + 2z^4\ {}_xf(x,y,z) \quad = 3x^2 + 3y^2\ {}_yf(x,y,z)\ = 6xy - 10yz\ {}_zf(x,y,z) \quad = -5y^2 + 8z^3\ \nabla f(x,y,z)\ = (3x^2 + 3y^2)\hat{i} + (6xy$

$\nabla f(1,3,0) = (3 \cdot 1^2 + 3 \cdot 3^2)\hat{i} + (6 \cdot 1 \cdot 3 - 10 \cdot 3 \cdot 0)\hat{j} + (-5 \cdot 3^2 + 8 \cdot 0^3)\hat{k} \quad = 12\hat{i} + 18\hat{j} - 45\hat{k}$