Kotitehtävät_5.pdf

Teht 1

$ f(x) = x^4 + 5x^3 - 7x $

Laske paikallinen minini käyttäen $gradient descent$ menetelmää.

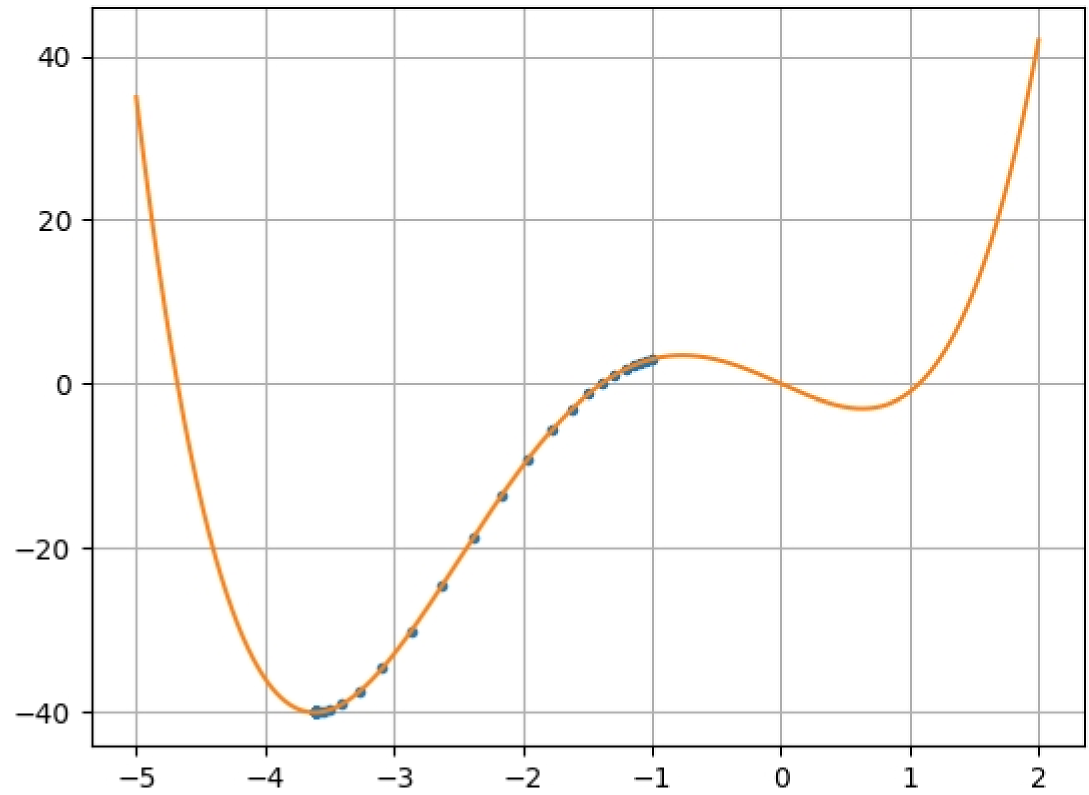
Iteraatio kaava:

$x_{i+1} = x_i - \alpha f'(x_i)$

$'f(x) = 4x^3 + 15x^2 - 7$

In [43]:
```
import numpy as np
import matplotlib.pyplot as plt

# Iteraatio pyhtonilla
# Funktio
def p(x):
    return x**4 + 5*x**3 - 7*x

# f derivaatta
def dp(x):
    return 4*x**3 + 15*x**2 - 7

x = -1
lr = 0.01
path_x = np.array([x])

for k in range(1000):
    x = x - lr*dp(x)
    path_x = np.append(path_x, x)

# Print result
print(x, "    Correct: ")

path_y = p(path_x)
plt.plot(path_x, path_y, '.')

xx = np.linspace(-5,2, 1000)
f = p(xx)
plt.plot(xx, f)
plt.grid()
plt.show()
```

```
-3.6161743618070274     Correct:
```



$x_0 = 1$ ja $\alpha = 0.01$

$x_{min} = 0.631954$

$x_0 = -1$ ja $\alpha = 0.01$

$x_{min} = -1.631954$

Tehtävä 2

$f(x, y) = 3x^2 + 3xy + 2y^2 + 3x - 5y$

$f_x(x, y) = 6x + 3y + 3$ $f_y(x, y) = 3x + 4y - 5$

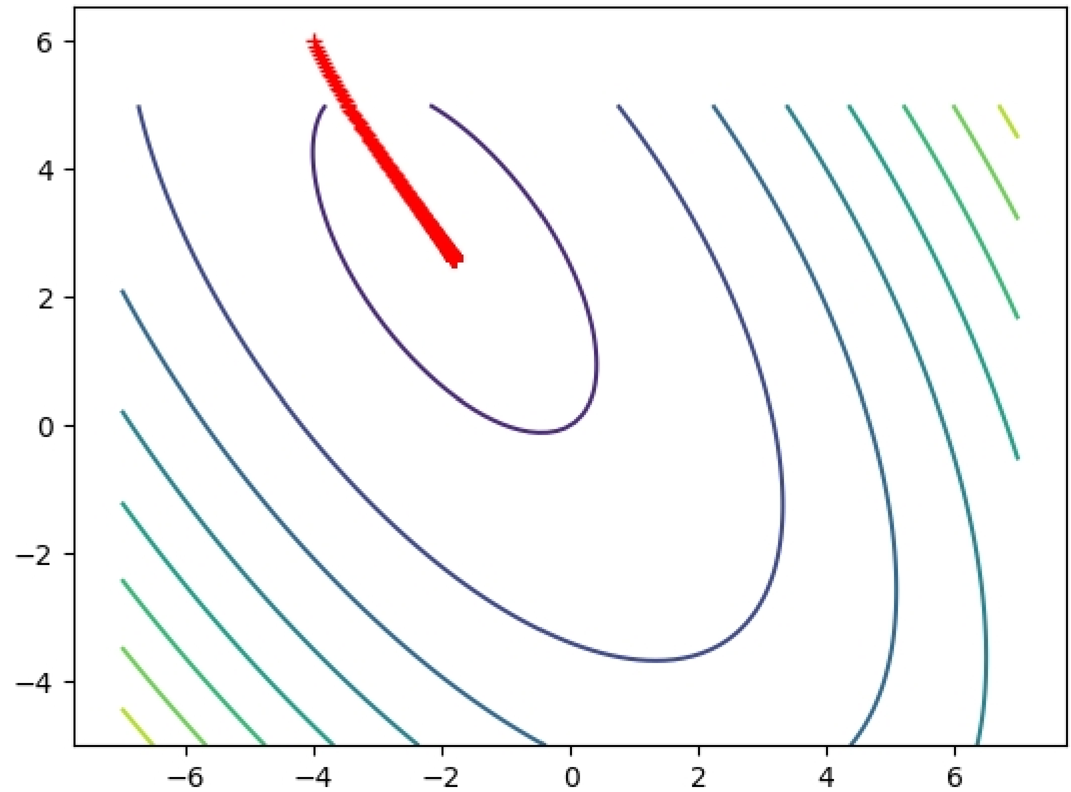
$\nabla f(x, y) = f_x(x, y)\hat{i} + f_y(x, y)\hat{j}$

$\nabla f(x, y) = (6x + 3y + 3)\hat{i} + (3x + 4y - 5)\hat{j}$

In [44]:
```
def df(x, y):
    return 3*x**2 + 3*x*y + 2*y**2 + 3*x - 5*y

def df(x, y):
    dx = 6*x + 3*y + 3
    dy = 3*x + 4*y - 5
    return dx, dy

lr = 0.01
x=-4
y=6

path_x = np.array([x])
path_y = np.array([y])

for k in range(1000):
    dx, dy = df(x,y)
```

```
    x = x - lr*dx
    y = y - lr*dy
    path_x = np.append(path_x, x)
    path_y = np.append(path_y, y)

print("x = ", x, "     y = ", y)

xx = np.linspace(-7, 7, 100)
yy = np.linspace(-5, 5, 100)
(X, Y) = np.meshgrid(xx, yy)
Z = 3*X**2 + 3*X*Y + 2*Y**2 + 3*X - 5*Y
plt.contour(X, Y, Z)
plt.plot(path_x, path_y, 'r+')
plt.axis('equal')
plt.show()
```

x = -1.800000020818124   y = 2.600000028883604



Teht3

a)

$f_i(x) = w_ix + b_i$

$i=1, 2$

$\frac{\partial}{\partial w_1} f_2(f_1(x))$

$f_1(x) = w_1x + b_1$

$f_2(x) = w_2x + b_2$

$$ \begin{aligned} \frac{\partial}{\partial w_1} f_2(f_1(x)) &= \frac{\partial}{\partial w_1} f'_2(w_1x+b_1) \cdot f'_1(x) \\ &= f'_2(w_1x+b_1) \cdot f'_1(x) \\ &= x \cdot w_2 \\ \end{aligned} $$

$ \begin{aligned} \frac{\partial}{\partial b_1} f_2(f_1(x)) &= \frac{\partial}{\partial b_1} f'_2(w_1x+b_1) \cdot f'_1(x) \\ &= f'_2(w_1x+b_1) \cdot f'_1(x) \\ &= w_2 \cdot 1 \\ \end{aligned} $

c)

en jaksa