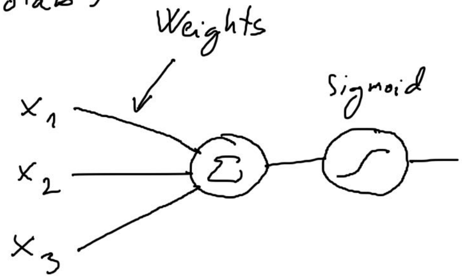
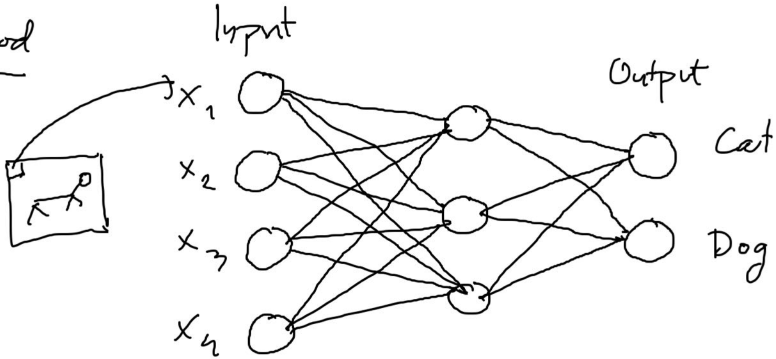
- Matrices (Neural Networks)
- Python, numpy, Anaconda (Colab)
  - Jupyter Notebook
- Linear Regression
- Derivative (one variable)
- Gradient (many variables)
- Gradient Descent Method
- Logistic Regression
- Neural Network
  - Backpropagation





Matrices

$$A = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & & \\ 2 & & \\ 3 & & \end{array} \quad A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 5 \end{bmatrix}$$

Matrix

3×2

Rows Columns

$A(2,1) = a_{21} = 0$

Element of a matrix

Square Matrix

- type $n \times n$

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -7 & 10 \\ -5 & 3 & 7 \end{bmatrix} \quad 3 \times 3$$

Diagonal Matrix

- square matrix
- $b_{ij} = 0$, if $i \neq j$

$$B = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -9 \end{bmatrix}$$

Main diagonal

Column vector

- type $n \times 1$

$$C = \begin{bmatrix} 10 \\ 5 \\ -2 \\ 0 \end{bmatrix} \quad 4 \times 1$$

Row vector

- type $1 \times n$

$$D = \begin{bmatrix} -11 & 5 & 3 \end{bmatrix} \quad 1 \times 3$$

Zero Matrix

- all elements are zero
- $A + O = A$

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$O = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Identity matrix

- always a square matrix
- $AI = IA = A$
- $I(i,j) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 3 \times 3$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad 2 \times 2$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 4 \times 4$$

Scalar multiplication

A matrix can be multiplied by a scalar (= real number)

$$\boxed{(\lambda A)(i,j) = \lambda A(i,j)}$$

Ex. $A = \begin{bmatrix} 3 & 1 \\ -5 & 0 \\ 7 & 10 \end{bmatrix} \Rightarrow 3A = \begin{bmatrix} 3\cdot 3 & 3\cdot 1 \\ 3\cdot(-5) & 3\cdot 0 \\ 3\cdot 7 & 3\cdot 10 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ -15 & 0 \\ 21 & 30 \end{bmatrix}$

$B = \begin{bmatrix} 9 \\ -4 \end{bmatrix} \Rightarrow -2B = \begin{bmatrix} -2\cdot 9 \\ -2\cdot(-4) \end{bmatrix} = \begin{bmatrix} -18 \\ 8 \end{bmatrix}$

Sum

- Matrix sum is calculated elementwise
- Matrices must be of the same type

Ex. $A = \begin{bmatrix} 1 & 0 \\ 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 10 & -1 \\ 5 & 3 \end{bmatrix}$

$$A + B = \begin{bmatrix} 1 & 0 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 10 & -1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1+10 & 0-1 \\ 5+5 & -2+3 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 11 & -1 \\ 10 & 1 \end{bmatrix}}}$$

Ex.

$$C = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \quad D = \begin{bmatrix} -7 \\ 2 \\ 5 \end{bmatrix}$$

$$C + D = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + \begin{bmatrix} -7 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1-7 \\ 0+2 \\ 3+5 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \\ 8 \end{bmatrix}$$

$$\boxed{(A+B)(i,j) = A(i,j) + B(i,j)}$$

Ex.

$$A = \underset{2 \times 3}{\begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 1 \end{bmatrix}}, \quad B = \underset{2 \times 3}{\begin{bmatrix} 10 & 1 & 2 \\ -3 & 0 & 0 \end{bmatrix}}$$

$$3A - 2B = \begin{bmatrix} 3\cdot 1 & 3\cdot 0 & 3\cdot 3 \\ 3\cdot(-2) & 3\cdot 1 & 3\cdot 1 \end{bmatrix} + \begin{bmatrix} -2\cdot 10 & -2\cdot 1 & -2\cdot 2 \\ -2\cdot(-3) & -2\cdot 0 & -2\cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 9 \\ -6 & 3 & 3 \end{bmatrix} + \begin{bmatrix} -20 & -2 & -4 \\ 6 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3-20 & 0-2 & 9-4 \\ -6+6 & 3+0 & 3+0 \end{bmatrix} = \underline{\underline{\begin{bmatrix} -17 & -2 & 5 \\ 0 & 3 & 3 \end{bmatrix}}}$$

Sum notation

Ex.

$$\sum_{i=1}^{5} (2i+3) = (2\cdot 1+3) + (2\cdot 2+3) +$$

$$+ (2\cdot 3+3) + (2\cdot 4+3)$$

$$+ (2\cdot 5+3)$$

$$= 5 + 7 + 9 + 11 + 13$$

$$= \underline{\underline{45}}$$

$$\sum_{j=1}^{4} a_{ij} = a_{i1} + a_{i2} + a_{i3} + a_{i4}$$

Ex.

a/ $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{64}$

$$= \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^6}$$

$$= \sum_{k=0}^{6} \frac{1}{2^k}$$

b/ $10 - 20 + 30 - 40 + 50 = 10 \cdot 1 - 10 \cdot 2 + 10 \cdot 3 - 10 \cdot 4 + 10 \cdot 5$

$$= \sum_{k=1}^{5} (-1)^{k+1} \, 10k$$

c/ $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \sum_{k=1}^{4} \frac{(-1)^{k+1}}{k}$

c/ $a_{1j} + a_{2j} + a_{3j} + \dots + a_{nj} = \sum_{i=1}^{n} a_{ij}$

Matrix Product

$$\underset{m \times p}{A} \cdot \underset{p \times n}{B} = \underset{m \times n}{C}$$

Must be the same

$$\boxed{(AB)(i,j) = \sum_{k=1}^{p} A(i,k)\, B(k,j)}$$

Ex. a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 4 \end{bmatrix}$

2×2 $\qquad\qquad$ 2×3

Same !

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 4 \end{bmatrix}$$

$\boxed{\text{BA is not defined !}}$

$$= \begin{bmatrix} 1\cdot 1 + 2\cdot(-2) & 1\cdot 0 + 2\cdot 1 & 1\cdot 3 + 2\cdot 4 \\ 3\cdot 1 + 4\cdot(-2) & 3\cdot 0 + 4\cdot 1 & 3\cdot 3 + 4\cdot 4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 11 \\ -5 & 4 & 25 \end{bmatrix} \qquad 2 \times 3$$

Ex

$$A = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 \cdot 5 & 2 \cdot (-3) \\ 1 \cdot 5 & 1 \cdot (-3) \end{bmatrix} = \begin{bmatrix} 10 & -6 \\ 5 & -3 \end{bmatrix}$$

$2 \times 1 \qquad 1 \times 2$

$$BA = \begin{bmatrix} 5 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \cdot 2 - 3 \cdot 1 \end{bmatrix} = \begin{bmatrix} 10 - 3 \end{bmatrix} = \begin{bmatrix} 7 \end{bmatrix}$$

$1 \times 2 \qquad 2 \times 1$

1×1

Ex $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 2 + 0\cdot 3 & 1\cdot 1 + 0\cdot 0 \\ 2\cdot 2 - 1\cdot 3 & 2\cdot 1 - 1\cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + 1\cdot 2 & 2\cdot 0 + 1\cdot (-1) \\ 3\cdot 1 + 0\cdot 2 & 3\cdot 0 + 0\cdot (-1) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 \\ 3 & 0 \end{bmatrix}$$

(i) $A + B = B + A$

(ii) $\lambda(A + B) = \lambda A + \lambda B$

(iii) $\lambda A + \mu A = (\lambda + \mu) A$

(iv) $A + (B + C) = (A + B) + C$

(v) $A(BC) = (AB)C$

(vi) $A + O = O + A = A$

(vii) $AI = IA = A$

Transpose A^T

Ex. a) $A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 3 & 3 \end{bmatrix}$ 2×3

$$A^T = \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ 5 & 3 \end{bmatrix} \quad 3 \times 2$$

$$\boxed{A^T(i,j) = A(j,i)}$$

b)

$$C = \begin{bmatrix} 0 \\ 5 \\ 7 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 0 \\ 5 \\ 7 \end{bmatrix}^T = \begin{bmatrix} 0 & 5 & 7 \end{bmatrix}$$

$$(C^T)^T = \begin{bmatrix} 0 & 5 & 7 \end{bmatrix}^T = \begin{bmatrix} 0 \\ 5 \\ 7 \end{bmatrix} = C$$

$$\boxed{\begin{aligned} (A^T)^T &= A \\ (A+B)^T &= A^T + B^T \\ (AB)^T &= B^T A^T \end{aligned}}$$

Inverse Matrix

Identity matrix I

$$\boxed{AI = IA = A \quad \text{for any matrix } A}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{I}(i,j) = \begin{cases} 1 & , \text{ if } i = j \\ 0 & , \text{ if } i \neq j \end{cases}$$

$\underline{\text{Ex}}$ a) $\frac{5}{3} = 5 \cdot \frac{1}{3} = 5 \cdot 3^{-1}$

b) $3 \cdot 3^{-1} = 3 \cdot \frac{1}{3} = \frac{3}{3} = 1$

the inverse number of 3

Let A be a square matrix. If there exists a matrix B such that

$$\boxed{AB = BA = I}$$

then B is the inverse of A and we denote $B = A^{-1}$.

If a square matrix A has an inverse matrix, then the inverse is unique.

If $AB = BA = I$ and $AC = CA = I$, then

$$B = BI = B(AC) = (BA)C = IC = C \quad \square$$

Example

$$\begin{cases} 3x + 2y + z = 1 \\ -x + 5y + 2z = 9 \\ 2x + y - z = 5 \end{cases}$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 5 & 2 \\ 2 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 9 \\ 5 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$AX = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 5 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 3x + 2y + z \\ -x + 5y + 2z \\ 2x + y - z \end{bmatrix}}_{3 \times 1} = \underbrace{\begin{bmatrix} 1 \\ 9 \\ 5 \end{bmatrix}}_{3 \times 1} = B$$

$$\begin{cases} 3x + 2y + z = 1 \\ -x + 5y + 2z = 9 \\ 2x + y - z = 5 \end{cases}$$

$$\Leftrightarrow \boxed{AX = B} \quad \Big| A^{-1}\cdot$$

$$\Leftrightarrow A^{-1}AX = A^{-1}B$$

$$\Leftrightarrow IX = A^{-1}B$$

$$\Leftrightarrow \boxed{X = A^{-1}B}$$

If A has an inverse A^{-1}

$$\boxed{3x = 5} \quad \Big| \frac{1}{3}\cdot$$

$$\Leftrightarrow \frac{1}{3}\cdot 3x = \frac{1}{3}\cdot 5$$

$$\Leftrightarrow 1\cdot x = \frac{5}{3}$$

$$\Leftrightarrow \boxed{x = \frac{5}{3}}$$

Function	Python
$\sin(x)$	np.sin(x)
$\cos(x)$	np.sin(x)
x^3	x**3
e^x	np.exp(x)
$\ln(x)$	np.log(x)
$\sqrt{x}$	np.sqrt(x)
$\lvert x \rvert$	np.abs(x)