$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \cdot \frac{1}{8} + \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \cdot \frac{3}{8} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \cdot \frac{3}{8} + \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} \cdot \frac{1}{8} =$$

$$= \begin{bmatrix} \frac{x_0}{8} \\ \frac{y_0}{8} \\ \frac{z_0}{8} \end{bmatrix} + \begin{bmatrix} x_1 \cdot \frac{3}{8} \\ y_1 \cdot \frac{3}{8} \\ z_1 \cdot \frac{3}{8} \end{bmatrix} + \begin{bmatrix} x_2 \cdot \frac{3}{8} \\ y_2 \cdot \frac{3}{8} \\ z_2 \cdot \frac{3}{8} \end{bmatrix} + \begin{bmatrix} \frac{x_3}{8} \\ \frac{y_3}{8} \\ \frac{z_3}{8} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{8}(x_0 + 3x_1 + 3x_2 + x_3) \\ \frac{1}{8}(y_0 + 3y_1 + 3y_2 + y_3) \\ \frac{1}{8}(z_0 + 3z_1 + 3z_2 + z_3) \end{bmatrix} = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix}$$

$$\vec{e}(u) = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = \vec{c}(u) + \lambda \vec{e}(u)$$

$$\lambda \in \mathbb{R}$$

~~$b(\frac{1}{2})$~~ # Görbepont

$$b(u) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{1}{8} + \begin{bmatrix} -8 \\ 8 \\ 0 \end{bmatrix} \cdot \frac{3}{8} + \begin{bmatrix} -8 \\ -8 \\ 0 \end{bmatrix} \frac{3}{8} + \begin{bmatrix} -8 \\ -8 \\ 8 \end{bmatrix} \frac{1}{8} =$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -7 \\ -1 \\ 1 \end{bmatrix}$$

Elsőrendű derivált

$$\frac{d}{du} b(u) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \left(-\frac{3}{4}\right) + \begin{bmatrix} -8 \\ 8 \\ 0 \end{bmatrix} \cdot \left(-\frac{3}{4}\right) + \begin{bmatrix} -8 \\ -8 \\ 0 \end{bmatrix} \cdot \frac{3}{4} + \begin{bmatrix} -8 \\ -8 \\ 8 \end{bmatrix} \cdot \frac{3}{4} =$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6 \\ -6 \\ 0 \end{bmatrix} + \begin{bmatrix} -6 \\ -6 \\ 0 \end{bmatrix} + \begin{bmatrix} -6 \\ -6 \\ 6 \end{bmatrix} = \begin{bmatrix} -6 \\ -18 \\ 6 \end{bmatrix}$$

$e' : \cancel{\vec{b}(u) + \lambda \vec{e}(u)}$

Szem. - 2022. 05. 09.

1. Megoldás

a.) görbepont:

$$\vec{c}(u) = \left| \left(\vec{p_0} F_0(u) + \vec{p_1} F_1(u) + \vec{p_2} F_2(u) + \vec{p_3} F_3(u) \right) \right|_{u=\frac{1}{2}}$$

$$= \vec{p_0} \cdot \frac{1}{8} + \vec{p_1} \cdot \frac{3}{8} + \vec{p_2} \cdot \frac{3}{8} + \vec{p_3} \cdot \frac{1}{8} =$$

$$= \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \cdot \frac{1}{8} + \dots = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} \quad F_0 + F_1 + F_2 + F_3 = 1$$

b.) $\left.\frac{d}{du} F_0(u)\right|_{u=\frac{1}{2}} = \frac{d}{du} (1-u)^3 = \left| -3(1-u)^2 \right|_{u=\frac{1}{2}} = -\frac{3}{4}$

$$\frac{d}{du} F_1(u) = \frac{d}{du} \left[3u(1-u)^2 \right] = 3\left[(1-u)^2 - 2u(1-u) \right] \Big| = -\frac{3}{4}$$

$\frac{1}{4}$

$$\frac{d}{du} F_2(u) = \frac{d}{du} \left[3u^2(1-u) \right] = 3\left(2u(1-u) + u^2 \right) \Big|_{u=\frac{1}{2}} = \frac{3}{4}$$

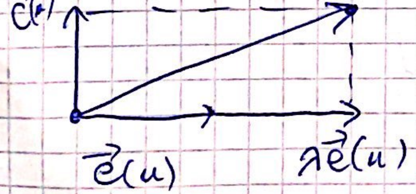
$$\frac{d}{du} F_3(u) = \left| 3u^2 \right|_{u=\frac{1}{2}} = \frac{3}{4}$$

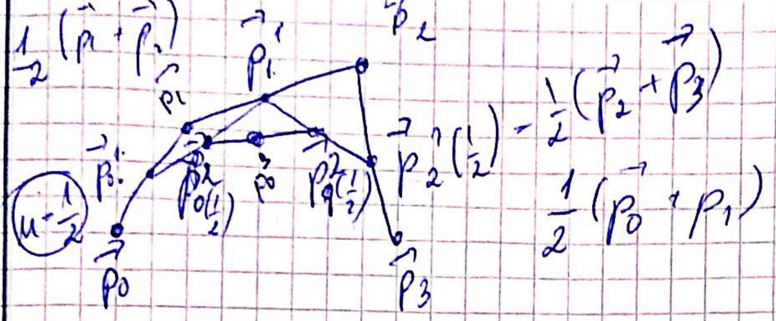
$$F_0 + F_1 + F_2 + F_3 = 0$$

$$\vec{e}(u) = \vec{p_0} \frac{d}{du} F_0(u) + \vec{p_1} \frac{d}{du} F_1(u) + \vec{p_2} \frac{d}{du} F_2(u) + \vec{p_3} \frac{d}{du} F_3(u) =$$

$$= -\frac{3}{4} \vec{p_0} - \frac{3}{4} \vec{p_1} + \frac{3}{4} \vec{p_2} + \frac{3}{4} \vec{p_3} = \dots = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = \vec{c}(u) + \lambda \vec{e}(u)$$

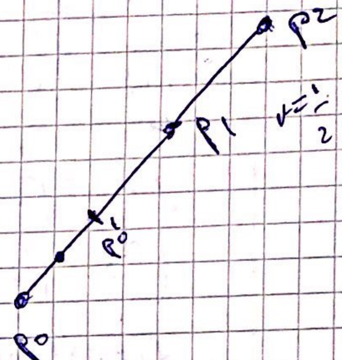
$\lambda \in \mathbb{R}$





pl.

$$(1-u)\vec{p} + u\vec{q}\,\Big|_{u=\frac{1}{3}} = \qquad u\in[0,1]$$



$$\vec{p}_0^{\,2} = \frac{1}{2}\left(\vec{p}_0^{\,1}\left(\tfrac{1}{2}\right) + \vec{p}_1^{\,1}\left(\tfrac{1}{2}\right)\right) = \frac{1}{4}\vec{p}_0$$

$$\rightarrow \frac{1}{2}\left(\frac{1}{2}(\vec{p}_0+\vec{p}_1) + \frac{1}{2}(\vec{p}_1+\vec{p}_2)\right) =$$

$$= \frac{1}{4}\vec{p}_0 + \frac{2}{4}\vec{p}_1 + \frac{1}{4}\vec{p}_2$$

$$\vec{p}_0^{\,3}\left(\frac{1}{2}\right) = \frac{1}{8}\vec{p}_0 + \frac{3}{8}\vec{p}_1 + \frac{3}{8}\vec{p}_2 + \frac{1}{8}\cdot\vec{p}_3$$

2.) Tekintsük az $u_0 = 0 < u_1 = \frac{1}{3} < u_2 = \frac{2}{3} < u_3 = 1$ csomóértékekhez tartozó interpolálandó $\vec{d}_0(x_0, y_0, z_0)$, $\vec{d}_1(x_1, y_1, z_1)$, $\vec{d}_2(x_2, y_2, z_2)$ és $d_3(x_3, y_0, z_3)$ adatpontokat.

Hat. meg annak a $[0,1]$-es harmadfokú Bézier görbének a kontrollpontjait, amely teljesíti a megadott interpolációs feltételeket.

Megoldás:

harmadfokú Bézier görbe

$$\vec{c}(u) = \vec{p}_0(1-u)^3 + \vec{p}_1 3u(1-u)^2 + \vec{p}_2 3u^2(1-u) + \vec{p}_3 u^3$$

$$\vec{c}(0) = \vec{d}_0 = \vec{p}_0$$

$$\vec{c}\left(\frac{1}{3}\right) = \vec{d}_1 = \frac{8}{27}\vec{p}_0 + \frac{12}{27}\vec{p}_1 + \frac{6}{27}\vec{p}_2 + \frac{1}{27}\vec{p}_3$$

$$\vec{c}\left(\frac{2}{3}\right) = \vec{d}_2 = \frac{1}{27}\vec{p}_0 + \frac{6}{27}\vec{p}_1 + \frac{12}{27}\vec{p}_2 + \frac{8}{27}\vec{p}_3$$

$$\vec{c}(1) = \vec{d}_3 = \vec{p}_3$$

$$\begin{cases} \vec{d}_0 = \vec{p}_0 \\ \vec{d}_3 = \vec{p}_3 \end{cases} \qquad HF$$

LinearCombination3::UpdateDataForInterpolation

3.)

$$\vec{s}(u,v) = \begin{bmatrix} F_0(u) & F_1(u) & F_2(u) & F_3(u) \end{bmatrix} \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} \\ p_{10} & p_{11} & p_{12} & p_{13} \\ p_{20} & p_{21} & p_{22} & p_{23} \\ p_{30} & p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} F_0(v) \\ F_1(v) \\ F_2(v) \\ F_3(v) \end{bmatrix}$$

harmadfokú Bézier folt

TensorProductSurface3::CalculatePartialDerivatives

::GenerateImage

a.) felületi pont

b.) u és v irányú parciális deriváltak

c.) normálvektor és érintősík

a.) $\vec{s}\left(\frac{1}{2},\frac{1}{2}\right) = \begin{bmatrix} \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{bmatrix} \cdot P \cdot \begin{bmatrix} \frac{1}{8} \\ \frac{3}{8} \\ \frac{3}{8} \\ \frac{1}{8} \end{bmatrix} =$

$= \frac{1}{64}\begin{bmatrix} 1 & 3 & 3 & 1 \end{bmatrix} P \cdot \begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix} = \dots = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}$

b.)

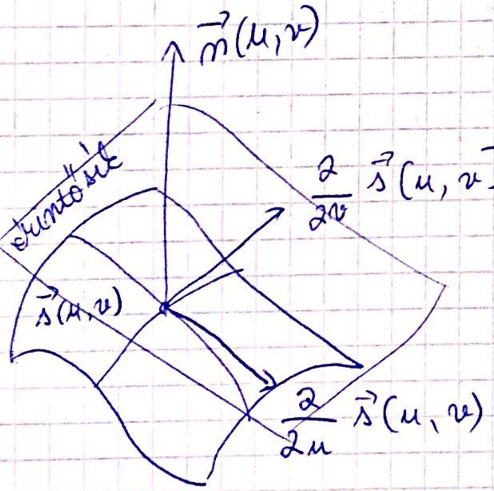
$\frac{\partial}{\partial u}\vec{s}\left(\frac{1}{2},\frac{1}{2}\right) = \begin{bmatrix} -\frac{3}{4} & -\frac{3}{4} & \frac{3}{4} & \frac{3}{4} \end{bmatrix} \cdot P \begin{bmatrix} \frac{1}{8} \\ \frac{3}{8} \\ \frac{3}{8} \\ \frac{1}{8} \end{bmatrix} =$

$= \frac{3}{32}\begin{bmatrix} -1 & -1 & 1 & 1 \end{bmatrix} P \begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix} = \dots = \begin{bmatrix} s^u_x \\ s^u_y \\ s^u_z \end{bmatrix}$

tükörvált.

$\frac{\partial}{\partial v}\vec{s}\left(\frac{1}{2},\frac{1}{2}\right) = \frac{3}{32}\begin{bmatrix} 1 & 3 & 3 & 1 \end{bmatrix} P \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} = \dots = \begin{bmatrix} s^v_x \\ s^v_y \\ s^v_z \end{bmatrix}$

c.)



$$\vec{n}(u,v) = \left| \frac{\partial}{\partial u}\vec{s}(u,v) \times \frac{\partial}{\partial u}\vec{s}(u,v) \right|_{\left(\frac{1}{2},\frac{1}{2}\right)} =$$

$$\left(\tfrac{1}{2},\tfrac{1}{2}\right)$$

$$= \det\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ s^u_x & s^u_y & s^u_z \\ s^v_x & s^v_y & s^v_z \end{pmatrix} = \vec{i}\,\underbrace{\left(s^u_y \cdot s^v_z - s^u_z \cdot s^v_y\right)}_{=\,n_x} + \vec{j}\,n_y + \vec{k} \cdot n_z = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

érintő sík:

$$\left\langle \begin{bmatrix} x - s_x \\ y - s_y \\ z - s_z \end{bmatrix}, \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \right\rangle = 0$$, mert ~~a~~ merőlegesek egymásra

$$\alpha x + \beta y + \gamma z + \delta = 0$$

$$(x - s_x) n_x + (y - s_y) n_y + (z - s_z) n_z = 0$$

$$\underset{\alpha}{x n_x} + \underset{\beta}{y n_y} + \underset{\gamma}{z n_z} \underset{\delta}{-\left(n_x s_x + n_y s_y + n_z \cdot s_z\right)} = 0$$

1.) Hat meg a kontrollpontokat

$\vec{p}_0(x_0, y_0, z_0)$; $\vec{p}_1(x_1, y_1, z_1)$, $\vec{p}_2(x_2, y_2, z_2)$, $\vec{p}_3(x_3, y_3, z_3)$

$\{[u+(1-u)]^3 = 1^3 = 1 \quad \forall u \in [0,1]\}$

és az $F_0(u) = (1-u)^3$

$F_1(u) = 3u(1-u)^2$

$F_2(u) = 3u^2(1-u)$

$F_3(u) = u^3 \qquad u \in [0,1]$

bázis függvények lineáris kombinációjával leírt görbenek az $u = \frac{1}{2}$ paraméteréhez tartozó

a.) görbepontokat

b.) érintővektort és az érintő egyenes egyenletét