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% Cubic Spline Interpolation - Natural Spline
% INPUT: X and Y are the vectors of given x-coordinates and y-coordinates
%        respectively

% Baillie, Borden, Miskovitz

function reflector(X,Y,f,df)

if length(X) ~= length(Y)                % Stops if length(X) =/= length(Y)
 erro('vectors X and Y must be of same length');
end

n = length(X);                           % Number of points interpolating

% Vector h with subintervals:
h = zeros(n-1,1);                        % Step-size of x
for j = 1:n-1
 h(j) = X(j+1) - X(j);
end

% Coefficient matrix A:
A = zeros(n);                            % Creates empty matrix

% Natural Spline boundary conditions:
A(1,1)= 1;                               % First row
A(n,n) = 1;                              % Last row

for i = 2:n-1
 A(i,i-1) = h(i-1);
 A(i,i) = 2*(h(i-1)+h(i));               % Diagonal elements
 A(i,i+1) = h(i);
end

% Vector b:                              % RHS vector
b = zeros(n,1);

for i = 2:n-1
 b(i) = (3/h(i))*(Y(i+1)-Y(i)) - (3/h(i-1))*(Y(i)-Y(i-1));
end

% Coefficient vector cj:
cj = A\b;

% Coefficient vector bj:
bj = zeros(n-1,1);
for i = 1:n-1
 bj(i) = (1/h(i))*(Y(i+1)-Y(i)) - (1/3*h(i))*(2*cj(i)+cj(i+1));
end

% Coefficient vector dj:
dj = zeros(n-1,1);
for i = 1:n-1
 dj(i) = (1/(3*h(i))) * (cj(i+1)-cj(i));
end

% Making a matrix P with all polynomials
P = zeros(n-1,4);
for i = 1:n-1
 P(i,1) = dj(i);
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 P(i,2) = cj(i);
 P(i,3) = bj(i);
 P(i,4) = Y(i);
end

% ||************************************************************||
% ||  Generating Shapes and Comparing them to Original Functions  ||
% ||************************************************************||
figure
plot(X,Y,'or','LineWidth',3)    % Data Points
resolution = 20;                % 20 equally spaced points
for i = 1:n-1

 % Constructing Interpolating function for i interval
 s = @(x) Y(i) + bj(i).*(x-x(i)) + cj(i).*(x-x(i)).^2 + dj(i).*(x-x(i)).^3;
 xs = linspace(X(i),X(i+1),resolution);

hold on
plot(xs,s(xs),'k*','LineWidth',2)
hold on
plot(xs,f(xs),'g','LineWidth',2);
hold on

legend('Data Points','Reflector','Function of Shape','Location','best')
% legend('boxoff')
title('Comparing Reflector with Function of Shape using 20 equally spaced points')
xlabel('x'); ylabel('y'); hold off;
end
 xlim([-1 1]);

% ||************************************************************||
% ||  Interpolating Shape Function & Calculating Bound Error   ||
% ||************************************************************||

figure
resolution = 5;                              % 5 equally spaced points
plot(X,Y,'or','LineWidth',3)                 % Data Points
for i = 1:n-1

    % Interpolating function for interval i
s = @(x) Y(i) + bj(i).*(x-x(i)) + cj(i).*(x-x(i)).^2 + dj(i).*(x-x(i)).^3;
xs = linspace(X(i),X(i+1),resolution);

hold on
% Graph
plot(xs,s(xs),'k--','LineWidth',2)
hold on
plot(xs,f(xs),'m','LineWidth',2);
xlabel('x'); ylabel('y');
legend('Data Points','Reflector','Function of Shape','Location','best')
title('Comparing Reflector with Function of Shape using 5 equally spaced points');
hold off; xlim([-1 1]);
end

% Bound Error
for i = 1:n
bound_error = abs(f(xs)-s(xs));
fprintf('The bound error at x = %d is %f.\n',X(i),bound_error(1,i));
end
fprintf('\n\n_____________________________________________________\n\n')
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% ||************************************************||
% ||  Differentiating Function of Shape and Reflector  ||
% ||************************************************||
figure
resolution = 20;                % 20 equally spaced points
for i = 1:n-1

 % Constructing Derivative of spline function for i interval
   ds =@(x) bj(i) + 2*cj(i).*(x-x(i)) + 3*dj(i).*(x-x(i)).^2;
   xs = linspace(X(i),X(i+1),resolution);

hold on
plot(xs,ds(xs),'k*','LineWidth',2)
hold on
plot(xs,df(xs),'b','LineWidth',2)
title('Slopes of Reflector and Function of the Shape');
xlabel('x'); ylabel('y');
legend('Derivative of Spline','Derivative of Function','Location','best')
hold off; xlim([-1 1]);

end

 % Derivatives of Spline at Data Points
   der_f = df(X);
   der_s = ds(X);
for i = 1:n
fprintf('The derivative of the reflector at %d is %f.\n',X(i),der_s(1,i))
fprintf('The derivative of the function at %d is %f.\n\n',X(i),der_f(1,i));
end
fprintf('\n_____________________________________________\n\n')

% Bound Error for Derivatives
for i = 1:n
bound_error_der = abs(df(xs)-ds(xs));
fprintf('The bound error of the derivative at x = %d is %f.\n',X(i),bound_error_der(1,i));
end

fprintf('\n_____________________________________________\n')
fprintf('\n_____________________________________________\n')
end
% Sources:
% (1) https://dafeda.wordpress.com/2010/11/28/cubic-spline-interpolation-code/
% (2) https://www.math.uh.edu/~jingqiu/math4364/spline.pdf
% (3) Wolfram Alpha
% (4) MathWorks
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The bound error at x = -1 is 0.017638.
The bound error at x = -5.000000e-01 is 0.082012.
The bound error at x = 0 is 0.133121.
The bound error at x = 5.000000e-01 is 0.174633.
The bound error at x = 1 is 0.210007.


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The derivative of the reflector at -1 is -4.075160.
The derivative of the function at -1 is -0.932039.
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The derivative of the reflector at -5.000000e-01 is -2.023228.
The derivative of the function at -5.000000e-01 is -0.644218.

The derivative of the reflector at 0 is -0.557562.
The derivative of the function at 0 is -0.198669.

The derivative of the reflector at 5.000000e-01 is 0.321838.
The derivative of the function at 5.000000e-01 is 0.295520.

The derivative of the reflector at 1 is 0.614971.
The derivative of the function at 1 is 0.717356.

The bound error of the derivative at x = -1 is 0.073559.
The bound error of the derivative at x = -5.000000e-01 is 0.063678.
The bound error of the derivative at x = 0 is 0.055199.
The bound error of the derivative at x = 5.000000e-01 is 0.048105.
The bound error of the derivative at x = 1 is 0.042379.

This is data for the first shape with a changed function value:
-cos(x-0.2)

The bound error at x = -1 is 0.635607.
The bound error at x = -5.000000e-01 is 0.424238.
The bound error at x = 0 is 1.019222.
The bound error at x = 5.000000e-01 is 1.225367.
The bound error at x = 1 is 1.118900.

The derivative of the reflector at -1 is 73.024840.
The derivative of the function at -1 is -0.932039.

The derivative of the reflector at -5.000000e-01 is 39.376772.
The derivative of the function at -5.000000e-01 is -0.644218.

The derivative of the reflector at 0 is 15.342438.
The derivative of the function at 0 is -0.198669.

The derivative of the reflector at 5.000000e-01 is 0.921838.
The derivative of the function at 5.000000e-01 is 0.295520.

The derivative of the reflector at 1 is -3.885029.
The derivative of the function at 1 is 0.717356.

The bound error of the derivative at x = -1 is 2.264114.
The bound error of the derivative at x = -5.000000e-01 is 1.666516.
The bound error of the derivative at x = 0 is 1.095771.
The bound error of the derivative at x = 5.000000e-01 is 0.551895.
The bound error of the derivative at x = 1 is 0.034907.

This data is for the parabola:

The bound error at x = -1 is 0.175781.
The bound error at x = -5.000000e-01 is 0.314732.
The bound error at x = 0 is 0.424665.
The bound error at x = 5.000000e-01 is 0.515625.
The bound error at x = 1 is 0.597656.

The derivative of the reflector at -1 is 8.571429.
The derivative of the function at -1 is 2.000000.

The derivative of the reflector at -5.000000e-01 is 4.071429.
The derivative of the function at -5.000000e-01 is 1.000000.

The derivative of the reflector at 0 is 0.857143.
The derivative of the function at 0 is -0.000000.

The derivative of the reflector at 5.000000e-01 is -1.071429.
The derivative of the function at 5.000000e-01 is -1.000000.

The derivative of the reflector at 1 is -1.714286.
The derivative of the function at 1 is -2.000000.

The bound error of the derivative at x = -1 is 0.147606.
The bound error of the derivative at x = -5.000000e-01 is 0.123664.
The bound error of the derivative at x = 0 is 0.103285.
The bound error of the derivative at x = 5.000000e-01 is 0.086466.
The bound error of the derivative at x = 1 is 0.073209.

This data is for the circular arc:

The bound error at x = -1 is 0.037111.
The bound error at x = -5.000000e-01 is 0.071592.
The bound error at x = 0 is 0.098786.
The bound error at x = 5.000000e-01 is 0.121690.
The bound error at x = 1 is 0.143449.

The derivative of the reflector at -1 is 2.179719.
The derivative of the function at -1 is 0.500000.

The derivative of the reflector at -5.000000e-01 is 1.045442.

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The derivative of the function at -5.000000e-01 is 0.229416.

The derivative of the reflector at 0 is 0.235245.
The derivative of the function at 0 is -0.000000.

The derivative of the reflector at 5.000000e-01 is -0.250874.
The derivative of the function at 5.000000e-01 is -0.229416.

The derivative of the reflector at 1 is -0.412913.
The derivative of the function at 1 is -0.500000.
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The bound error of the derivative at x = -1 is 0.033752.
The bound error of the derivative at x = -5.000000e-01 is 0.027215.
The bound error of the derivative at x = 0 is 0.021689.
The bound error of the derivative at x = 5.000000e-01 is 0.017180.
The bound error of the derivative at x = 1 is 0.013697.
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