**Topics: Normal distribution, Functions of Random Variables**

**Solutions**

1). The time required for servicing transmissions is normally distributed with   
*μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

1. 0.3875
2. 0.2676
3. 0.5
4. 0.6987

Answer: We have a normal distribution with = 45 and = 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X d" 50 so the question is to find Pr(X > 50).

Pr(X > 50) = 1 - Pr(X d" 50).

>Z = (X - µ)/ = (X - 45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X d" 50) = Pr(Z d" (50 - 45)/8.0) = Pr(Z d" 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676.

So, The option (B) is correct.

2). The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.

Answer: We have a normal distribution with = 38 and = 6. Let X be the number of employees. So according to question

1. More employees at the processing center are older than 44 than between 38 and 44.

Answer: Probabilty of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X d" 44).

Z = (X -µ )/ = (X - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X d" 44) = Pr(Z d" (44 - 38)/6) = Pr(Z d" 1)=84.1345%

Probabilty that the employee will be greater than age of 44 = 100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%

Therefore the statement that More employees at the processing center are older than 44 than between 38 and 44 is TRUE.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer: Probabilty of employees less than age of 30 = Pr(X<30).

Z = (X -µ )**/** = (30 - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X d" 30) = Pr(Z d" (30 - 38)/6) = Pr(Z d" -1.333)=9.12%

So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48( or 36 employees).

Therefore the statement B of the question is also TRUE.

3). If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer: As we know that if X<" N(µ1, Ã 1^2 ), and Y <" N(µ2, Ã 2^2 ) are two independent random variables then X + Y <" N(µ1 + µ2, Ã 1^2 + Ã 2^2 ) and X " Y <" N(µ1 " µ2, Ã 1^2 + Ã 2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z <" N(aµ1 + bµ2, a^2Ã 1^2 + b^2Ã 2^2 ).

Therefore in the question

2X1~ N(2 u,4 Ã ^2) and

X1+X2 ~ N(µ + µ, Ã ^2 + Ã ^2 ) ~ N(2 u, 2Ã ^2 )

2X1-(X1+X2) = N( 4µ,6 Ã ^2)

4). Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9

Answer: Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- ¼ ) / Ã

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* Ã + ¼ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct.

5). Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
2. Specify the 5th percentile of profit (in Rupees) for the company
3. Which of the two divisions has a larger probability of making a loss in a given year?

Answer: we convert the dollars into rupees

profit 1(22.5,144)

profit 2(31.5,189)

First, we determine the mean and variance for the company

﻿μ=﻿ ﻿μ1+μ2=22.5+31.5=54﻿

﻿σ21+σ22=144+189=333﻿

﻿σ=333​=18.248﻿

second, we calculate the interval with a 95% confidence level, apply the empirical rule, add and subtract 2 deviations from the mean

﻿(μ−2σ,μ+2σ)=(54−2∗18.248,54+2∗18.246)=(17.504,90.496)﻿

Third, we determine the fifth percentile

﻿ given, μ=54

σ=18.248

P(x≤x)=0.05

first,we computed z

z=−1.64

second, we can calculate x using the formula,

z=x−μ/σ third, x can be calculated as follows:

X=Zσ+μ

fourth, putting the given values, we have

x=−1,64∗18.248+54=24.07328≈24.07

fifth,hence, the required x is24.07​

Fourth, we determine which division is most likely to lose.

(A). profit 1

﻿GivenWe use standardized normal distributionz=x−μ​/σ

μ1​=22.5, σ1​=12.0, x=0,

the probability can be calculated as

p(x<0)=p(x−μ/σ​<0−22.5​/12)=p(z<−1.88)=0.0301

Hence, the required the probability

p(x<0)=0.0301​

(B). profit 2

﻿GivenWe use standardized normal distribution

z=x−μ​/σ

μ2​=31.5, σ2​=13.748, x=0

the probability can be calculated as

p(x<0)=p(x−μ/σ​<13.7480−31.5​)=p(z<−2.29)=0.011

Hence, the required the probability

p(x<0)=0.011​

(C). Profit1 is more likely to lose in a year.