Tutorial - 4

A-1 → $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3$
$b = 2$, $n^{\log_b a} = n^{\log_2 3}$

$n^{\log_2 3} < n^2$ (case 3)

$\therefore$ $T(n) = \Theta(n^2)$

A-2 → $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4$, $b = 2$ $n^{\log_2 4} = n^2 = f(n)$

$\therefore$ $\Theta(n^2 \log n)$

A-3 → $T(n) = T\left(\frac{n}{2}\right) + 2^n$.

$a = 1$, $b = 2$, $n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (case 3)

$\Rightarrow$ $T(n) = \Theta(2^n)$

A-4 → This can't be solved by using M.T because a and b, is dependent on n.

A-5 → $T(n) = 16T\left(\frac{n}{4}\right) + n$

$a = 16,\ b = 4,$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$\therefore\ n^2 > f(n) - n$ (case 1)

$T(n) = \Theta(n^2)$

A-6 → $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

$a = 2,\ b = 2$

$n^{\log_b a} = n^{\log_2 2} = n^1 = n$

$n\log^k n = f(n) = n\log n$ (case-2)

$\therefore\ f(n) = \Theta(n\log^2 n)$

A-7 → $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

M.T can't be applied because f(n) is not in a polynomial function.

A-8 → $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2,\ b = 4$

$n^{\log_4 2} = n^{0.5}$

$n^{0.5} < n^{0.51} = f(n)$ (case3)

$\therefore\ T(n) = \Theta(n^{0.51})$

A-9 → $T(n) = 0.5\,T\left(\frac{n}{2}\right) + \frac{1}{n}$

M.T cant be applied because $a < 1$

A-10 → $T(n) = 16\,T\left(\frac{n}{4}\right) + n!$

↳ $n!$ can be written as n^n polynomial.

$a = 16$, $b = 4$,

$n^{\log_b a} = n^{\log_4 a} = n^2$

$n^2 < n!$. (case 3)

$\Rightarrow T(n) = \Theta(n^n) = \Theta(n!)$

A-11 → $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$a = 4$, $b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > \log n = f(n)$ (case 1)

$T(n) = \Theta(n^2)$

A-12 → $T(n) = \mathrm{sqrt}(n)\,T\left(\frac{n}{2}\right) + \log n$.

M.T. can't be applied ~~because $\sqrt{n}$ is~~.

A-13 → $T(n) = 3T\left(\frac{n}{2}\right) + n$

$a = 3$, $b = 2$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > n$ (1^{st} Case)

$\Theta(n^{\log_2 3})$.

A-14 → $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$a = 3, \quad b = 3$

$n^{\log_b a} = n^{\log_3 3} = n^1$

$n > \sqrt{n}$ (case I)

$T(n) = \Theta(n)$

A-15 → $T(n) = 4T\left(\frac{n}{2}\right) + cn$

$a = 4, b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > cn$

$\Rightarrow \quad T(n) = \Theta(n^2)$

A-16 → $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$a = 3, \quad b = 4$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$ (case 3)

$T(n) = \Theta(n\log n)$

A-18) $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

$a = 6,\ b = 3$

$n^{\log_3 6} = n^2$

$n^2 < n^2 \log n$ (case 3)

$\therefore\ T(n) = \Theta(n^2 \log n)$

A-19 → $T(n) = 4T\left(\frac{n}{2}\right) + n/\log n$

$a = 4,\ b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$ (case 1)

$T(n) = \Theta(n^2)$

A-20 → $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$

M.T. is not applicable since $f(n)$ is negative.

A-24 → $7T\left(\frac{n}{3}\right) + n^2$

$a = 7,\ b = 3$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$ (case 3)

$T(n) = \Theta(n^2)$

A-22 → $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

M.T. isn't applicable since the condition is violated in Case 3.