Tutorial - 2

A-1 →

```
void func (int n)
{ int j=1, i=0
  while (i<n)
  { i+=j;
    j++;
  }
}
```

$j=1 \quad i=0+1$

$j=2 \quad i=0+1+2$

$j=3 \quad i=0+1+2+3$

$\vdots$

$i>=n$

$0+1+2+3+\dots+k>=n$

$\frac{k(k+1)}{2}>=n$

$k^2>=n$

$k>=\sqrt{n}$

$\Rightarrow O(\sqrt{n})$

A-2 → $T(n) = \{ T(n-1) + T(n-2)$

$T(0) = T(1) = 1$

$f(n-1) \approx f(n-2)$

$\Rightarrow T(n) = 2T(n-1) \Rightarrow T(n-1) = 2T(n-2)$

$T(n) = 2(2(T(n-2))) \Rightarrow T(n) = 2^k T(n-k)$

$\because T(0) = 1 \Rightarrow n-k=0 \Rightarrow n=k$

$T(n) = 2^n T(n-n) = 2^n \cdot 1 = T(n) = 2^n$

$\Rightarrow \underline{O(2^n)}$

A-3 →

- $O(n(\log n))$

```
for (i=0 ; i<n ; i++)
    for (j=1; j<n ; j=j*2)
        O(1)
```

- $O(n^3)$

```
for (i=0 ; i<n ; i++)
    for (j=0; j<n; j++)
        for(k=0; k<n; k++)
            O(1)
```

- $O(\log(\log n))$

```
for (i=1; i<=n; i*=2)
    for( j=1; j<=n ; j*= 2)
        O(1)
```

A-4 → $T(n) = T(n/4) + T(n/2) + cn^2$

Let's assume $T(n/2) >= T(n/4)$

So, $T(n) = 2T\left(\frac{n}{2}\right) + cn^2$

applying M.T- $\left(T(n) = aT\left(\frac{n}{b}\right) + f(n)\right)$

$a = 2$, $b = 2$, $f(n) = n^2$

$c = \log_b^a = \log_2^2 = 1$

$n^c = n$

Compare n^c and $f(n) = n^2$

$f(n) > n^c$, So, $T(n) = \theta(n^2)$

A-5 for (i = 1; i <= n; i++)

    for(j = 1; j < n; j += i)

        O(1)

i	j		
1	1, 2, 3, 4, 5 . . . k	= O(n)	
2	1, 3, 5, . . . k	= O(n)	
3	1, 4, 7 . . . k	= O(n)	$\times n$
⋮	⋮		
n	⋮		

$\Rightarrow O(n^2)$

A-6 for (i = 2; i <= n; i = pow(i, k))

    O(1)

Complexity of pow(i, k) = O(logN)

= logk

i
2
2^k
2^{k^2}
2^{k^3}
⋮
$2^{k^M} > n$

$\log(2^{k^M}) > \log n$

$k^M \log_2 2 > \log n$

$k^M > \log n$

$M \log k > \log\log(n)$

$M > \dfrac{\log(\log(n))}{\log(k)}$

$\Rightarrow T(C) = O(\log(\log(n)))$

A-8 (a) 100, $\log\log n$, $\log n$, $\sqrt{n}$, n, $\log(n!)$, $n\log n$, n^2, 2^n,

$n!$, $\underbrace{2^{2n}, 4^n}_{\text{Same}}$

A-8 (b) 1, $\log(\log(n))$, $\sqrt{\log(n)}$, $\log(n)$, $\log 2n$, $2\log(n)$,

n, $2n$, $4n$, $\log(n!)$, $n\log(n)$, n^2, 2^{2^n}, $n!$

A-8 (c) 96, $\log_8 N$, ~~$\log\log$~~, $\log_2 N$, ~~$n\log$~~ $5n$,

$n\log_6(n)$, $n\log_2(n)$ $\log(n!)$, $O(8n^2)$,

$7n^3$, 8^{2n}, $n!$

A-7 Quick Sort RR $= T(n) = T(k) + T(n-k-1) + n$

$T(n) = T(0) + T(n-1) + n$

$T(n) = T(n-1) + n$

$= T(n-2) + n-1 + n$

$= T(n-k) + (n-k+1) + \dots n-1+n$

$n-k = 0 \Rightarrow k = n$

$\Rightarrow$ Ft: $1+2+3+\dots+n-2+n-1+n$

$\Rightarrow O(n^2)$