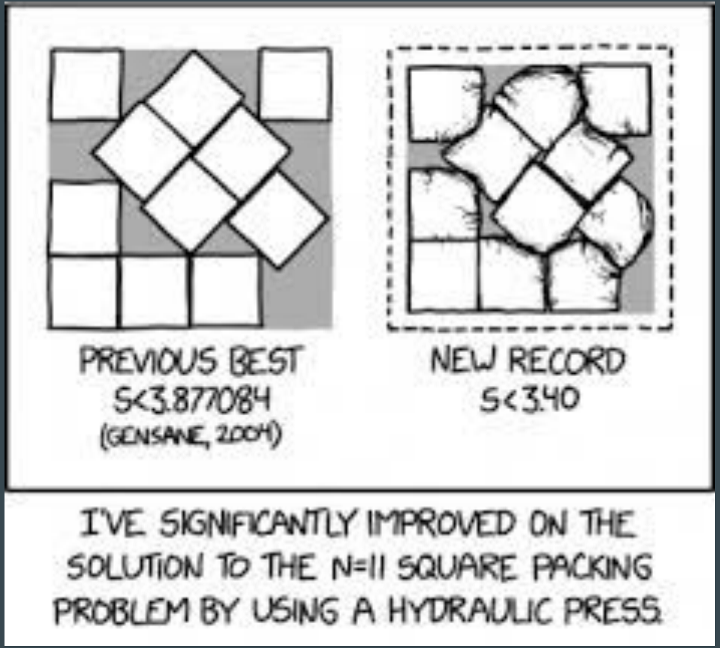
Calculus Final Project (copy)

(Skip slide 6 for visuals of the circle generation)

Introduction to Packing Problems:

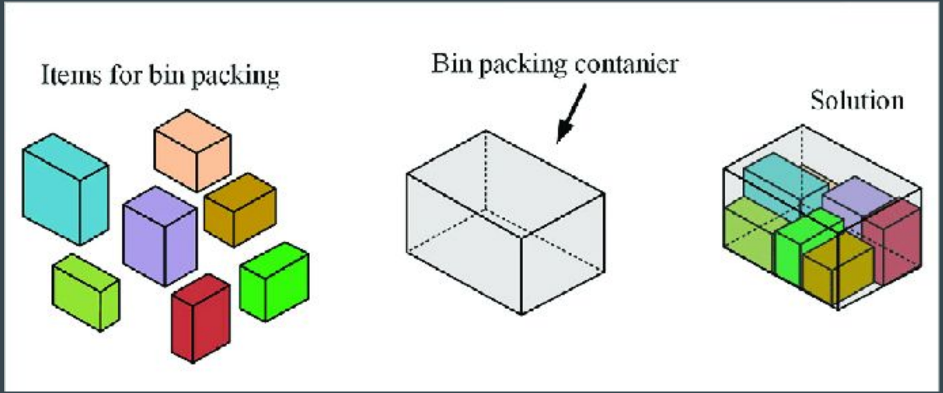
- Class of optimization problem in mathematics
- What is optimization?
  - *"Finding the best solution from all feasible solutions"*
  - Two types (**Discrete** or Continuous)

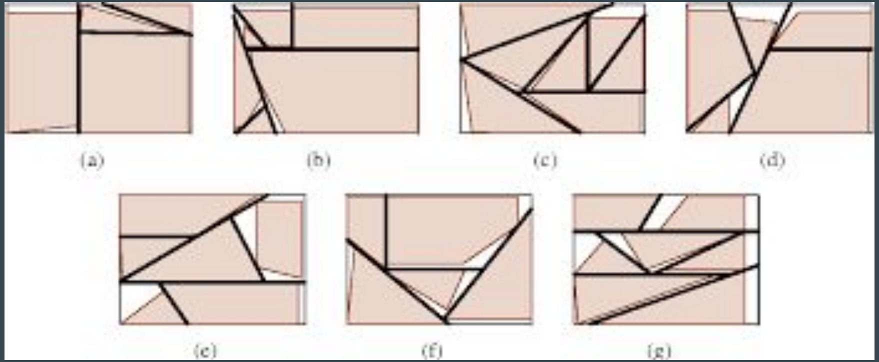
- Packing **OBJECTS** into **CONTAINERS**
- These types of problems are very important for real life applications
- Most famous type: **Bin Packing**



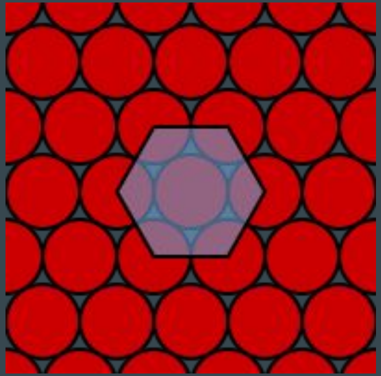
Bin Packing Problems:

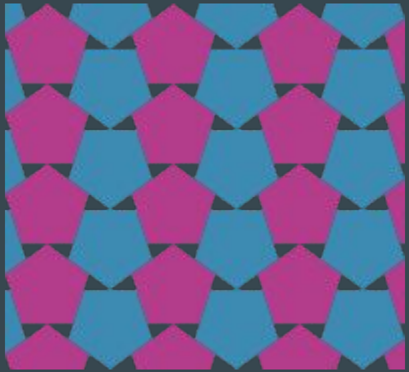
- Container
  - 2D or 3D space, possibly infinite
  - One or more containers
- Objects
  - Could be one shape used repeatedly or different shapes
- Two main goals
  - Optimal packing density
  - Pack all of the objects in the fewest containers as possible

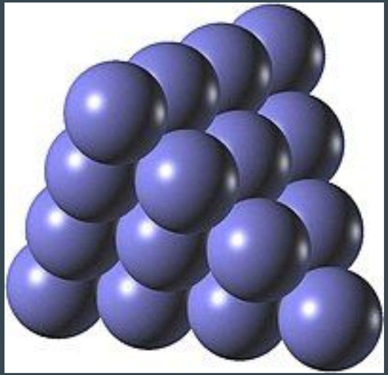




Packing Density





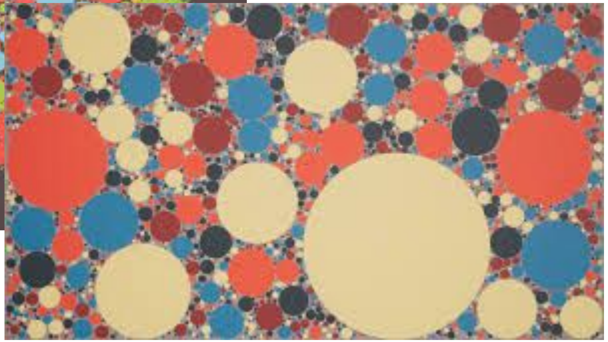


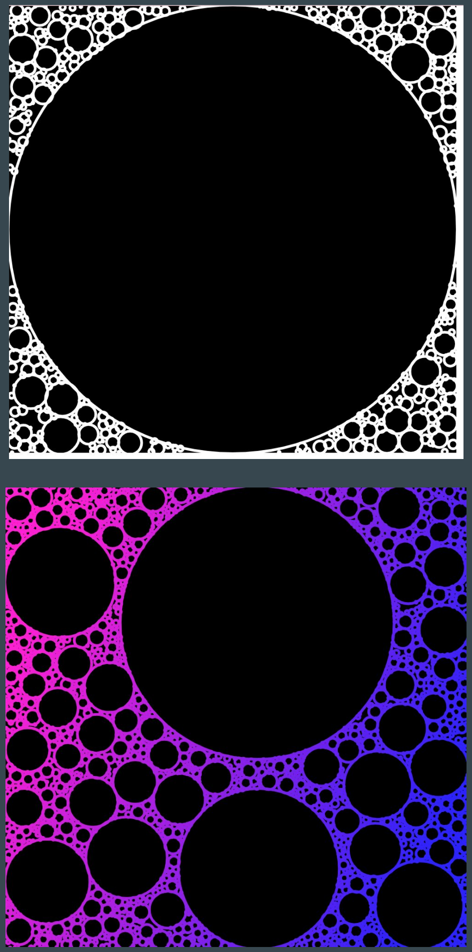
- Also known as packing fraction
- Ratio of the volume of the bodies in a space to the volume of the space itself
- Many problems seek to find the packing density or optimal packing density
- Many applications
  - Large Scale: Packing objects effectively within shipping containers
  - Smaller Scale: The packing of atoms within crystal structures
- Some shapes have known packing constants

Art of Circle Packing

https://viv511.github.io/CirclePacking/

- Packing in 2D is also commonly used in art
- “Circle Packing” is a technique used in computational art





Kepler Conjecture

A conjecture to a ***theorem***

74.05%

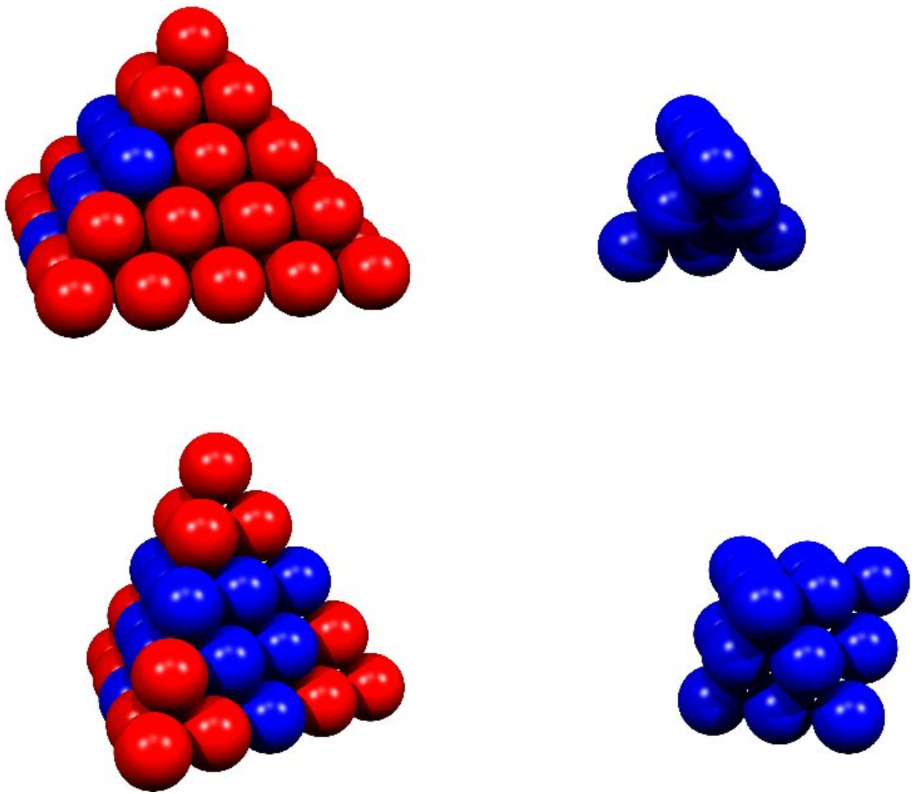
Don’t know what that means? That’s alright!

Introduction to Kepler Conjecture

- 400 year old problem
- Sir Walter Raleigh ⇒ Famous in Britain
- He wanted to know the best way of packing cannonballs on his ship
    - What is the best way of packing spheres?
- Intuitive approach
    - Three methods!
- Square, Triangle, & Hexagon

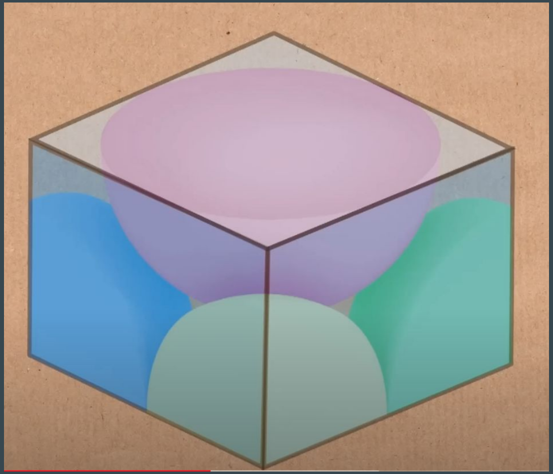
Guess what?

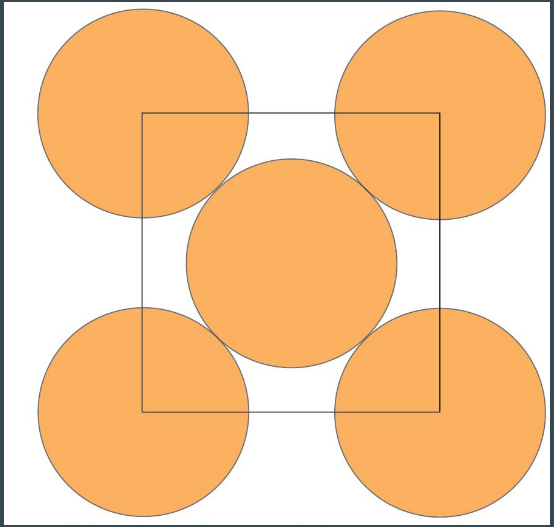
They are all the same!!

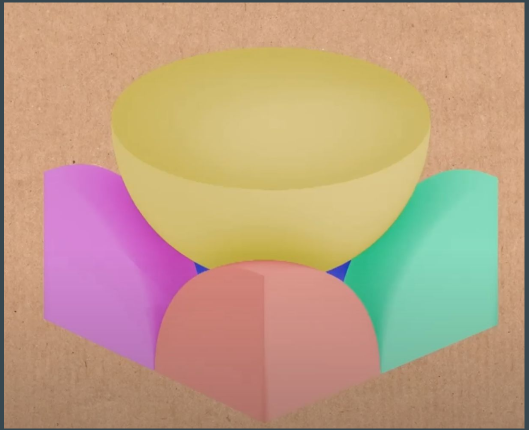


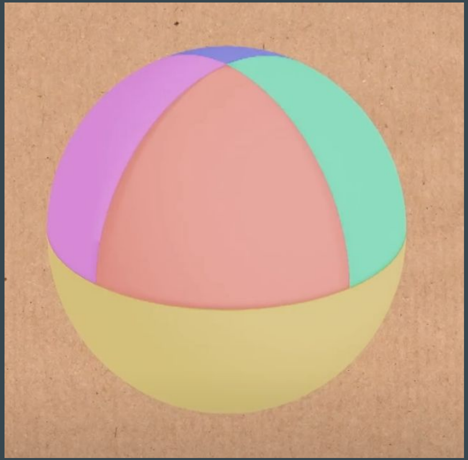
Key Insight:

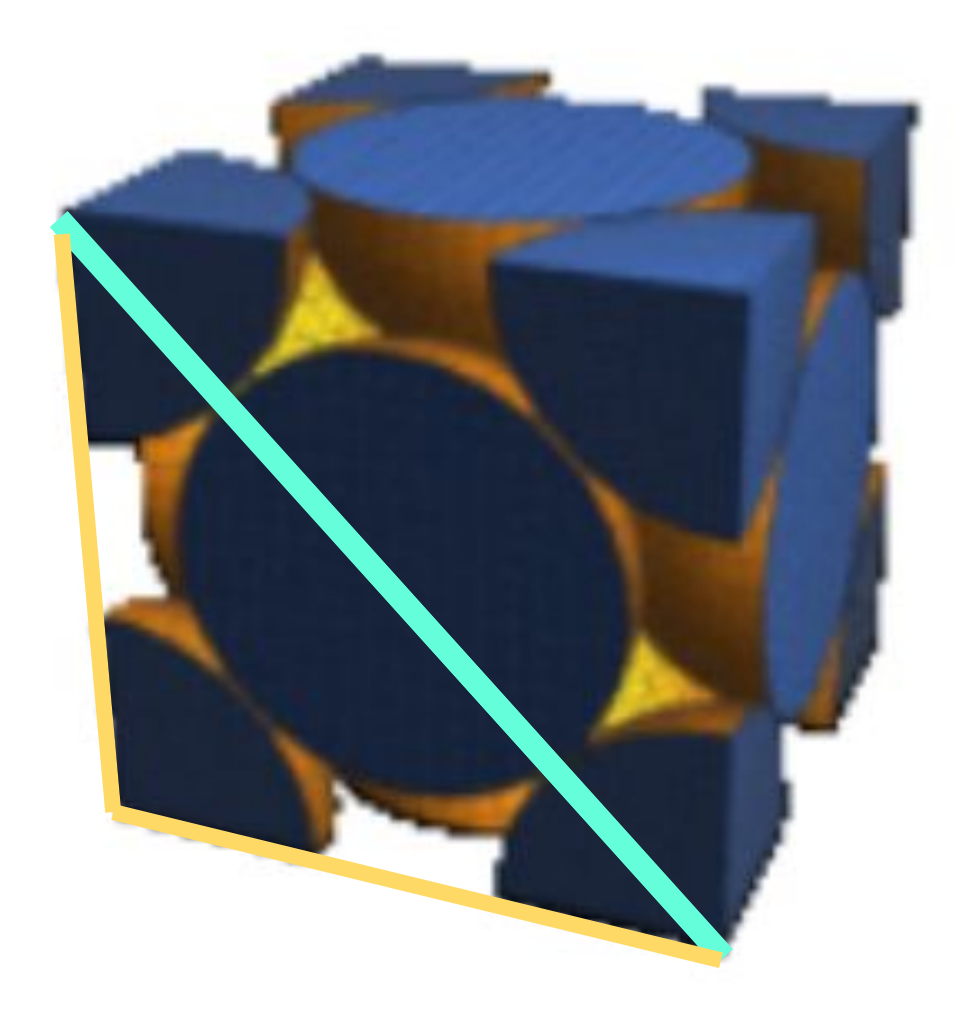
Representation by repeated units through slicing. Finding the density of this unit would then give us the optimal packing density.











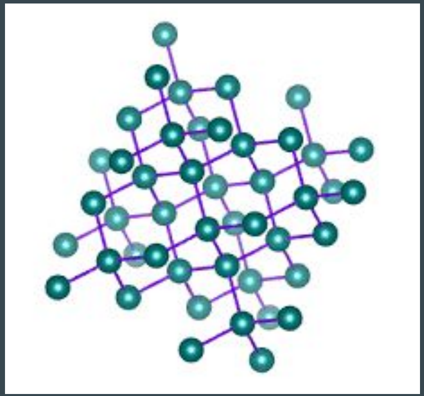
Putting it all together

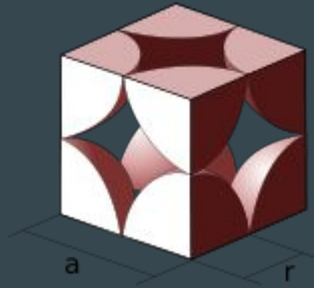
The optimal packing density for spheres is **~74.05%**

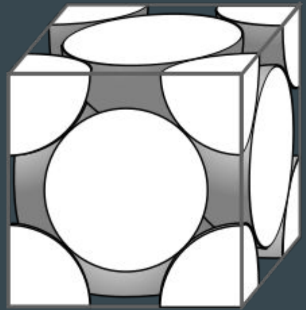
Applications in crystallography.

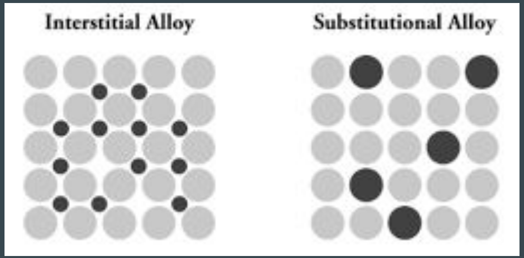
- How are atoms arranged around each other?
- When the arrangement is regular, the material is a crystal
- APF = Atomic Packing Factor
- Simple cubic, face-centered cubic, body-centered cubic
- Metal Alloys → Interstitial and Substitutional
- Covalent Network Solids

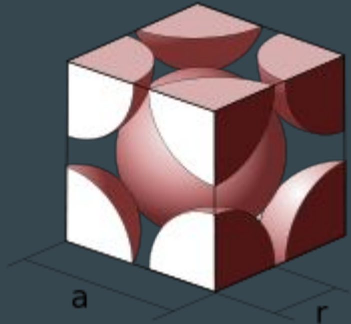
$$\mathrm{APF} = \frac{N_{\mathrm{particle}} V_{\mathrm{particle}}}{V_{\mathrm{unit\ cell}}}$$





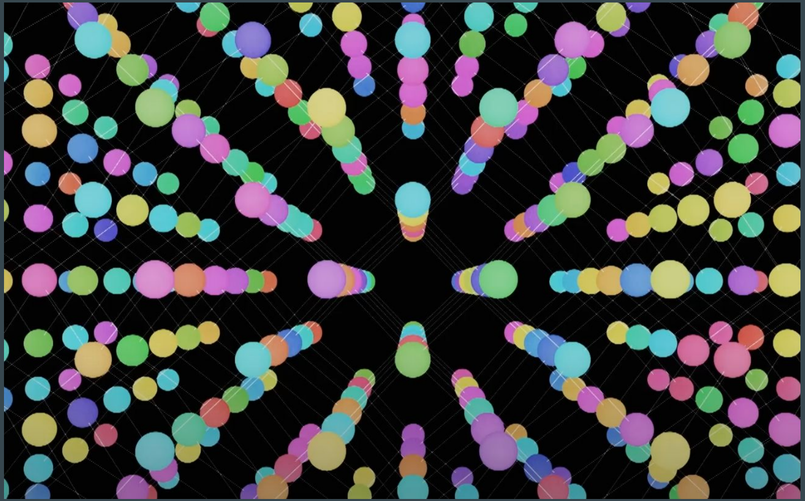


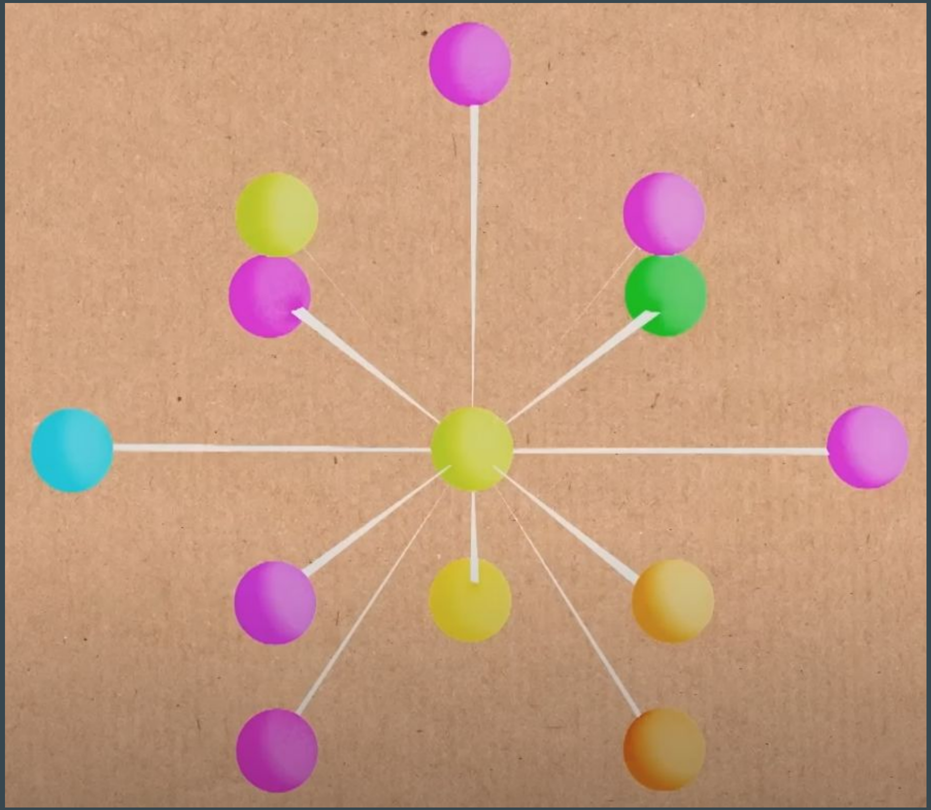




Why did this “intuitive” way take so long to prove?

- Gauss
  - Showed it was best packing if the structure was regular
  - What about irregular?
- Hilbert's problems
  - 23 problems in mathematics published by David Hilbert in 1900
  - They were all unsolved at the time
  - 18th Problem == Proof of Densest Sphere Packing / “Kepler Conjecture”
- (1998) The Hales-Ferguson Proof
  - Essense of idea:
    - Instead of packing spheres, we are packing points (centers of spheres)
    - “Network of points”
    - Look at local structure around a given point
      - Gave each structure a “score”
      - By computational analysis, iterated through many counterexamples / structures
        - “proof by exhaustion”
      - Found that this was the best structure, therefore the optimal packing density was proven to be 74.05%





Validity by Computation

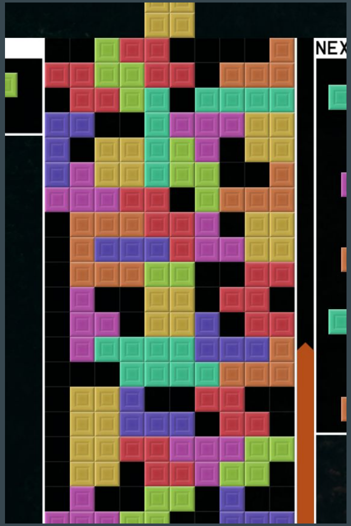
- Proof by exhaustion
  - Not a kind that most are used to, the paper was over 250 pages
  - It took mathematicians four years to work through it
- There was confidence, but not absolute confidence in the proof (99%)
- Hales worked extensively to translate his proof to “formal logic” (ProjectRhea)
  - Fifteen year project
- By doing so, a computer could automatically check the proof because they rewrote the proof in formal mathematical language
  - Eventually accepted fully in 2017
  - Finally proving this conjecture

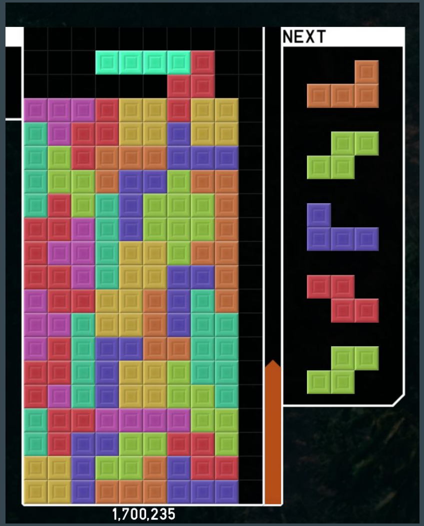
Higher Dimensional Geometry (poster reference!)

- 3D version of sphere packing problem is equivalent to the Kepler Conjecture
- 2D it is equivalent to packing circles on a plane
- 1D it is equivalent to packing line segments in a “linear universe”
- In higher dimensions higher than three, densest lattice packings are known up to 8 dimensions

Conclusion

- Everyday life → Groceries, Oranges
- Remember the elegance and art of packing everyday
- Some people may also choose to incorporate high density packing principles into their own hobbies :))





Thank You!

Credits

- Mr. Verner
- https://arxiv.org/abs/math/9811072
- https://www.youtube.com/watch?v=CROeIGfr3gs
- Wikipedia
- Encyclopedia Britannica
- https://www.projectrhea.org/rhea/index.php/Sphere_Packing_4:_Kepler%27s_Conjecture_Intuition
- https://blog.kleinproject.org/?p=742
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