4 EXPRESSIONS USING LETTER-NUMBERS

4.1 The Notion of Letter-Numbers

In this chapter we shall look at a concise way of expressing mathematical relations and patterns. We shall see how this helps us in thinking about these relationships and patterns, and in explaining why they may hold true.

Example 1: Shabnam is 3 years older than Aftab. When Aftab's age 10 years, Shabnam's age will be 13 years. Now Aftab's age is 18 years, what will Shabnam's age be? _______

Given Aftab's age, how will you find out Shabnam's age?

Easy: We add 3 to Aftab's age to get Shabnam's age.

Can we write this as an expression?

Shabnam's age is 3 years more than Aftab's. In short, this can be written as:

Shabnam's age = Aftab's age + 3.

Such mathematical relations are generally represented in a shorthand form. In the relation above, instead of writing the phrase 'Aftab's Age', the convention is to use a convenient symbol. Usually, letters or short phrases are used for this purpose.

Let us say we use the letter a to denote Aftab's age (we could have used any other letter), and s to denote Shabnam's age. Then the expression to find Shabnam's age will be a + 3, which can be written as

Aftab's age	Expression for Shabnam's age
4	4 + 3
10	10 + 3
23	23 + 3
?	? + 3
a	a + 3

Fig. 4.1

$$s = a + 3.$$

If a is 23 (Aftab's age in years), then what is Shabnam's age?

Replacing a by 23 in the expression $a + 3$, we get, $s = 23 + 3 = 26$ years.

Letters such as a and s that are used to represent numbers are called **letter-numbers**. Mathematical expressions containing letter-numbers, such as the expression $a + 3$, are called **algebraic expressions**.

? Given the age of Shabnam, write an expression to find Aftab's age.

We know that Aftab is 3 years younger than Shabnam. So, Aftab's age will be 3 less than Shabnam's. This can be described as

Aftab's age = Shabnam's age – 3.

If we again use the letter a to denote Aftab's age and the letter s to denote Shabnam's age, then the algebraic expression would be: $a = s - 3$, meaning 3 less than s.

? Use this expression to find Aftab's age if Shabnam's age is 20.

? **Example 2:** Parthiv is making matchstick patterns. He repeatedly places Ls next to each other. Each L has two matchsticks as shown in Figure 4.2.

Fig. 4.2

How many matchsticks are needed to make 5 Ls? It will be 5×2.

How many matchsticks are needed to make 7 Ls? It will be 7×2.

How many matchsticks are needed to make 45 Ls? It will be 45×2.

Now, what is the relation between the number of Ls and the number of sticks?

First, let us describe the relationship or the pattern here. Every L needs 2 matchsticks. So **the number of matchsticks needed will be 2 times the number of L's**. This can be written as:

Number of matchsticks = 2 × Number of L's

Now, we can use any letter to denote the number of L's. Let's use n. The algebraic expression for the number of matchsticks will be:

$$2 \times n.$$

This expression tells us how many matchsticks are needed to make n L's. To find the number of matchsticks, we just replace n by the number of L.

? **Example 3:** Ketaki prepares and supplies coconut-jaggery laddus. The price of a coconut is ₹35 and the price of 1 kg jaggery is ₹60.

? How much should she pay if she buys 10 coconuts and 5 kg jaggery?

Cost of 10 coconuts = 10 × ₹35
Cost of 5 kg jaggery = 5 × ₹60
Total cost = 10 × ₹35 + 5 × ₹60 = ₹350 + ₹300 = ₹650.

? How much should she pay if she buys 8 coconuts and 9 kg jaggery?

? Write an algebraic expression to find the total amount to be paid for a given number of coconuts and quantity of jaggery.

Let us identify the relationships and then write the expressions.

Quantity needed	**Relationship**	**Expression**
Cost of coconuts	Number of coconuts × 35	$c \times 35$
Cost of jaggery	Number of kgs of jaggery × 60	$j \times 60$

Here, 'c' represents the number of coconuts and 'j' represents the number of kgs of jaggery. The total amount to be paid will be:

Cost of coconuts + Cost of jaggery.

The corresponding algebraic expression can be written as:

$$c \times 35 + j \times 60$$

? Use this expression (or formula) to find the total amount to be paid for 7 coconuts and 4 kg jaggery.

Notice that for different values of 'c' and 'j', the value of the expression also changes.

Writing this expression as a sum of terms we get:

$$\boxed{c \times 35} + \boxed{j \times 60}$$

? **Example 4:** We are familiar with calculating the perimeters of simple shapes. Write expressions for perimeters.

The perimeter of a square is **4 times the length of its side**. This can be written as the expression: $\mathbf{4 \times q}$, where q stands for the sidelength.

? What is the perimeter of a square with sidelength 7 cm? Use the expression to find out.

You must have realised how the use of letter-numbers and algebraic expressions allows us to express general mathematical relations in a concise way. Mathematical relations expressed this way are often called formulas.

Figure it Out

1. Write formulas for the perimeter of:
   (a) triangle with all sides equal.
   (b) a regular pentagon (as we have learnt last year, we use the word 'regular' to say that all sidelengths and angle measures are equal)
   (c) a regular hexagon
2. Munirathna has a 20 m long pipe. However, he wants a longer watering pipe for his garden. He joins another pipe of some length to this one. Give the expression for the combined length of the pipe. Use the letter-number '*k*' to denote the length in meters of the other pipe.
3. What is the total amount Krithika has, if she has the following numbers of notes of ₹100, ₹20 and ₹5? Complete the following table:

No. of ₹100 notes	No. of ₹20 notes	No. of ₹5 notes	Expression and total amount
3	5	6	
			$6 \times 100 + 4 \times 20 + 3 \times 5 = 695$
8	4	z	
x	y	z	

4. Venkatalakshmi owns a flour mill. It takes 10 seconds for the roller mill to start running. Once it is running, each kg of grain takes 8 seconds to grind into powder. Which of the expressions below describes the time taken to complete grind '*y*' kg of grain, assuming the machine is off initially?
   (a) $10 + 8 + y$
   (b) $(10 + 8) \times y$
   (c) $10 \times 8 \times y$
   (d) $10 + 8 \times y$
   (e) $10 \times y + 8$
5. Write algebraic expressions using letters of your choice.
   (a) 5 more than a number
   (b) 4 less than a number

(c) 2 less than 13 times a number

(d) 13 less than 2 times a number

6. Describe situations corresponding to the following algebraic expressions:

   (a) $8 \times x + 3 \times y$

   (b) $15 \times j - 2 \times k$

7. In a calendar month, if any 2 × 3 grid full of dates is chosen as shown in the picture, write expressions for the dates in the blank cells if the bottom middle cell has date 'w'.

$w - 1$	w	

4.2 Revisiting Arithmetic Expressions

We learnt to write expressions as sums of terms and it became easy for us to read arithmetic expressions. Many times they could have been read in multiple ways and it was confusing. We used **swapping** (adding two numbers in any order) and **grouping** (adding numbers by grouping them conveniently) to find easy ways of evaluating expressions. Swapping and grouping terms does not change the value of the expression. We also learnt to use brackets in expressions, including brackets with a negative sign outside. We learnt the **distributive property** (multiple of a sum is the same as sum of multiples).

Let us revise these concepts and find the values of the following expressions:

1. $23 - 10 \times 2$
2. $83 + 28 - 13 + 32$
3. $34 - 14 + 20$
4. $42 + 15 - (8 - 7)$
5. $68 - (18 + 13)$
6. $7 \times 4 + 9 \times 6$
7. $20 + 8 \times (16 - 6)$

Let us evaluate the first expression, $23 - 10 \times 2$. First we shall write the terms of the expression. Notice that one of the terms is a number, while the other one has to be converted to a number before adding the two terms.

$$23 - 10 \times 2 = 23 + -10 \times 2 = 23 + -20 = 3$$

Let us now evaluate the second one. All the terms of this expression are numbers. If we notice the terms, we find that it will be easier to evaluate if we swap and group the terms.

$$83 + 28 - 13 + 32 =$$

$$\boxed{83} + \boxed{28} + \boxed{-13} + \boxed{32}$$

$$= \boxed{70} + \boxed{60} = \boxed{130}$$

Let us now look at the fifth expression. It has brackets with a negative sign outside. This can be evaluated in two ways — by solving the bracket first (like the solution on the left side) or by removing the brackets appropriately (as on the right side).

$$= \boxed{68} + \boxed{-(18 + 13)}$$

$$= \boxed{68} + \boxed{-31}$$

$$= \boxed{37}$$

OR

$$= \boxed{68} + \boxed{-(18 + 13)}$$

$$= \boxed{68} + \boxed{-18} + \boxed{-13}$$

$$= \boxed{50} + \boxed{-13} = \boxed{37}$$

Now, find the values of the other arithmetic expressions.

Algebraic expressions also take number values when the letter-numbers they contain are replaced by numbers. In Example 1, for finding Shabnam's age when Aftab is 23 years old, we replaced the letter-number a in the expression $a + 3$ by 23, and it took the value 26.

4.3 Omission of the Multiplication Symbol in Algebraic Expressions

Look at this number sequence:

4, 8, 12, 16, 20, 24, 28, ...

How can we describe this sequence or pattern? Easy: These are the numbers appearing in the multiplication table of 4 (multiples of 4 in an increasing order).

What is the third term of this sequence? It is 4×3.

What is the 29th term of this sequence? It is 4×29.

? Find an algebraic expression to get the nth term of this sequence.

Note that here 'n' is a letter-number that denotes a position in the sequence.

As it is the sequence of multiples of 4, it can be seen that the nth term will be 4 times n:

$$4 \times n$$

As a standard practice, we shorten $4 \times n$ to $4n$ by skipping the multiplication sign. We write the number first, followed by the letter(s). Find the value of the expression $7k$ when $k = 4$. The value is $7 \times 4 = 28$.

Find the value that the expression $5m + 3$ takes when $m = 2$.

As $5m$ stands for $5 \times m$, the value of the expression when $m = 2$ is $5 \times 2 + 3 = 13$.

Mind the Mistake, Mend the Mistake

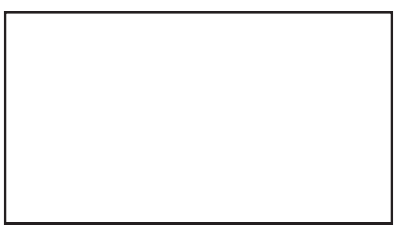
Some simplifications are shown below where the letter-numbers are replaced by numbers and the value of the expression is obtained.

1. Observe each of them and identify if there is a mistake.
2. If you think there is a mistake, try to explain what might have gone wrong.
3. Then, correct it and give the value of the expression.

1 If $a = -4$, then $10 - a = 6$.	**2** If $d = 6$, then $3d = 36$.	**3** If $s = 7$, then $3s - 2 = 15$.
4 If $r = 8$, then $2r + 1 = 29$.	**5** If $j = 5$, then $2j = 10$.	**6** If $m = -6$, then $3(m + 1) = 19$.
7 If $f = 3$, $g = 1$ then $2f - 2g = 2$.	**8** If $t = 4$, $b = 3$ then $2t + b = 24$.	**9** If $h = 5$, $n = 6$ then $h - (3 - n) = 4$.

4.4 Simplification of Algebraic Expressions

Earlier we found expressions to find perimeters of different regular figures in terms of their sides. Let us now find an expression to find the perimeter of a rectangle.



As in the previous cases, we will first describe how to get the perimeter when the length and the breadth of the rectangle are known:

Find the sum of length + breadth + length + breadth.

Let us use the letter-numbers l and b in place of length and breadth respectively. Let p denote the perimeter of the rectangle. Then we have

$$p = l + b + l + b$$

As we know, these represent numbers, and so the terms of an expression can be added in any order. Hence the above expression can be written as:

$$= l + l + b + b$$

As $l + l = 2 \times l = 2l$, and $b + b = 2 \times b = 2b$, we have

$$p = 2l + 2b.$$

Notice that the initial expression ($l + b + l + b$) and the final expression ($2l + 2b$) that we got for the perimeter look different. However, they are equal since the expression was obtained from the initial one by applying the same rules and operations we do for numbers; they are equal in the sense that they both take the same values when letter-numbers are replaced by numbers.

For example, if we assign $l = 3$, $b = 4$, we get

$$l + b + l + b = 3 + 4 + 3 + 4 = 14, \text{ and}$$

$$2l + 2b = 2 \times 3 + 2 \times 4 = 14.$$

We call the expression $2l + 2b$ the **simplified form** of $l + b + l + b$.
Let us see some more examples of simplification.

Example 5: Here is a table showing the number of pencils and erasers sold in a shop. The price per pencil is c, and the price per eraser is d. Find the total money earned by the shopkeeper during these three days.

	Day 1	**Day 2**	**Day 3**
Pencils (Price 'c')	5	3	10
Erasers (Price 'd')	4	6	1

Let us first find the money earned by the sale of pencils.

The money earned by selling pencils on Day 1 is $5c$. Similarly, the money earned by selling pencils on Day 2 is _____, and Day 3 is ______.

The total money earned by the sale of pencils is $5c + 3c + 10c$. Can we simplify this expression further and reduce the number of terms?

The expression means 5 times c is added to 3 times c is added to 10 times c. So in total, the letter-number c is added (5 + 3 + 10) times. This is what we have seen as the distributive property of numbers. Thus,

$$5 \times c + 3 \times c + 10 \times c = (5 + 3 + 10) \times c$$

$(5 + 3 + 10) \times c$ can be simplified to $18 \times c = 18c$.

? If c = ₹50, find the total amount earned by the scale of pencils.

? Write the expression for the total money earned by selling erasers. Then, simplify the expression.

The expression for the total money earned by selling pencils and erasers during these three days is $18c + 11d$.

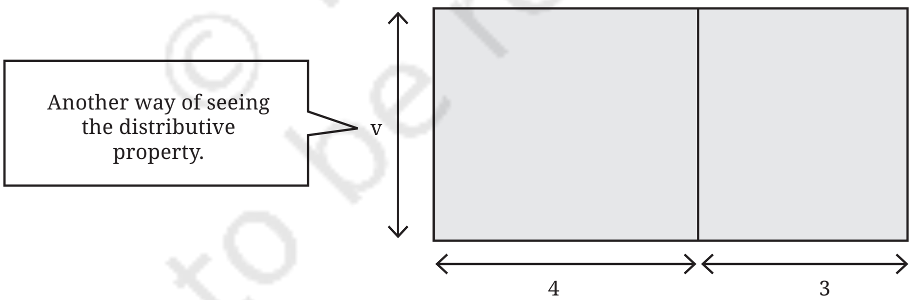
? Can the expression $18c + 11d$ be simplified further?

There is no way of further simplifying this expression as it contains different letter-numbers. It is in its simplest form.

In this problem, we saw the expression $5c + 3c + 10c$ getting simplified to the expression $18c$.

? Check that both expressions take the same value when c is replaced by different numbers.

? **Example 6:** A big rectangle is split into two smaller rectangles as shown. Write an expression describing the area of the bigger rectangle.



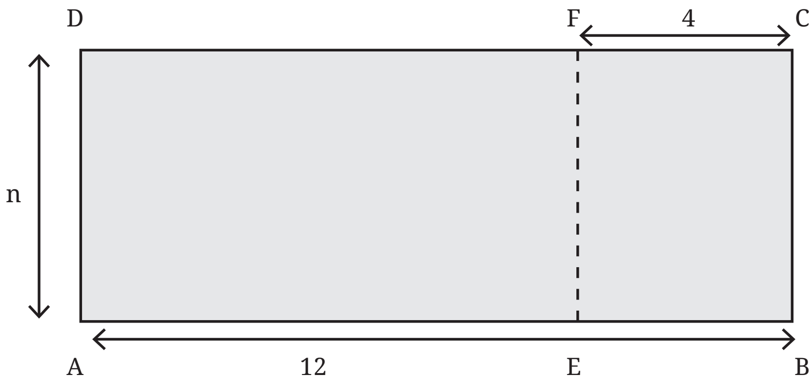
The areas of the smaller rectangles are $4v$ sq. units and $3v$ sq. units.

The area of the bigger rectangle can be found in two ways: (i) by directly using its side lengths v and (4 + 3), or (ii) by adding the areas of the smaller rectangles.

The first way gives $7v$, and the second way gives $4v + 3v$. We know that they are equal: $4v + 3v = 7v$, and this is the required expression for the area of the bigger rectangle.

As earlier, a big rectangle is split into two smaller rectangles as shown below. Write an expression to find the area of the rectangle AEFD.

Even in this case, the area of rectangle AEFD can be found in two ways: (i) by directly using the side lengths n and (12 – 4), or (ii) subtracting the area of the rectangle EBCF from that of ABCD.



The first method gives us 8n, and the second method gives us $12n - 4n$, and they are equal, since $12n - 4n = 8n$. This is the expression for the area of the rectangle AEFD.

Sets of terms such as ($5c$, c, $10c$), ($12n$, $-4n$) that involve the same letter-numbers are called **like terms**. Sets of terms such as {$18c$, $11d$} are called **unlike terms** as they have different letter-numbers.

As we have seen, like terms can be added together and simplified into a single term.

Example 7: A shop rents out chairs and tables for a day's use. To rent them, one has to first pay the following amount per piece.

Item	Amount
Chair	₹40
Table	₹75

When the furniture is returned, the shopkeeper pays back some amount as follows.

	Amount returned
Chair	₹6
Table	₹10

Write an expression for the total number of rupees paid if x chairs and y tables are rented.

For x chairs and y tables, let us find the total amount paid at the beginning and the amount one gets back after returning the furniture.

Describe the procedure to get these amounts.

The total amount in rupees paid at the beginning is $40x + 75y$, and the total amount returned is $6x + 10y$.

So, the total amount paid = $(40x + 75y) - (6x + 10y)$.

Can we simplify this expression? If yes, how? If not, why not?

Math Talk

Recalling how we open brackets in an arithmetic expression, we get

$$(40x + 75y) - (6x + 10y) = (40x + 75y) - 6x - 10y$$

Since the terms can be added in any order, the remaining bracket can be opened and the expression becomes $40x + 75y + -6x + -10y$

We can group the like terms together, This results in

$$40x + -6x + 75y + -10y$$

$$= (40 - 6)x + (75 - 10)y$$

$$= 34x + 65y.$$

The expression $(40x + 75y) - (6x + 10y)$ is simplified to $34x + 65y$, which is the total amount paid in rupees.

? Could we have written the initial expression as $(40x + 75y) + (-6x - 10y)$?

Math Talk

? **Example 8:** Charu has been through three rounds of a quiz. Her scores in the three rounds are $7p - 3q$, $8p - 4q$, and $6p - 2q$. Here, p represents the score for a correct answer and q represents the penalty for an incorrect answer.

? What do each of the expressions mean?

If the score for a correct answer is 4 ($p = 4$) and the penalty for a wrong answer is 1 ($q = 1$), find Charu's score in the first round.

Charu's score is $7 \times 4 - 3 \times 1$. We can evaluate this expression by writing it as a sum of terms.

$7 \times 4 - 3 \times 1 = 7 \times 4 + -3 \times 1 = 28 + -3 = 25$

What are her scores in the second and third rounds?

What if there is no penalty? What will be the value of q in that situation?

What is her final score after the three rounds?

Her final score will be the sum of the three scores: $(7p - 3q) + (8p - 4q) + (6p - 2q)$.

Since the terms can be added in any order, we can remove the brackets and write

$7p + -3q + 8p + -4q + 6p + -2q$

$= 7p + 8p + 6p + -(3q) + -(4q) + -(2q)$ (by swapping and grouping)

$= (7 + 8 + 6)p + -(3 + 4 + 2)q$

$= 21p + -9q$

$= 21p - 9q.$

Charu's total score after three rounds is $21p - 9q$. Her friend Krishita's score after three rounds is $23p - 7q$.

Give some possible scores for Krishita in the three rounds so that they add up to give $23p - 7q$.

Can we say who scored more? Can you explain why?

How much more has Krishita scored than Charu? This can be found by finding the difference between the two scores.

$23p - 7q - (21p - 9q)$

Simplify this expression further.

Example 9: Simplify the expression $4\,(x + y) - y$

Using the distributive property, this expression can be simplified to

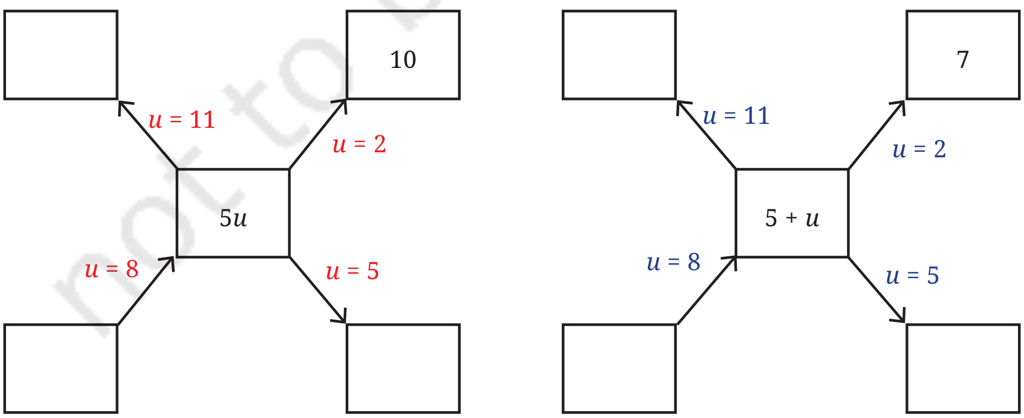
$$4\,(x + y) - y = 4x + 4y - y$$
$$= 4x + 4y + -y$$
$$= 4x + (4 - 1)y$$
$$= 4x + 3y.$$

Example 10: Are the expressions $5u$ and $5 + u$ equal to each other?

The expression $5u$ means 5 times the number u, and the expression $5 + u$ means 5 more than the number u. These two being different operations, they should give different values for most values of u.

Let us check this.

Fill the blanks below by replacing the letter-numbers by numbers; an example is shown. Then compare the values that $5u$ and $5 + u$ take.

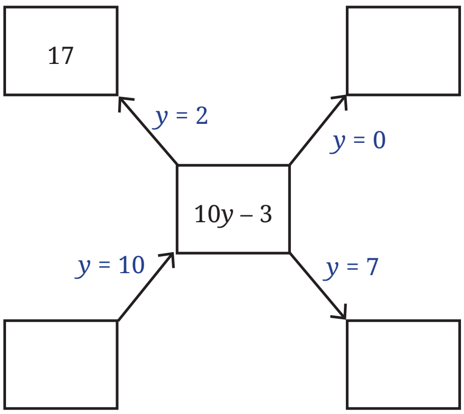


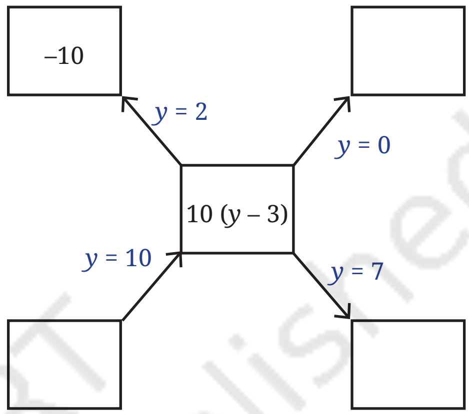
If the expressions $5u$ and $5 + u$ are equal, then they should take the same values for any given value of u. But we can see that they do not. So, these two expressions are not equal.

Are the expressions $10y - 3$ and $10(y - 3)$ equal?

$10y - 3$, short for $10 \times y - 3$, means 3 less than 10 times y,
$10(y - 3)$, short for $10 \times (y - 3)$, means 10 times (3 less than y).

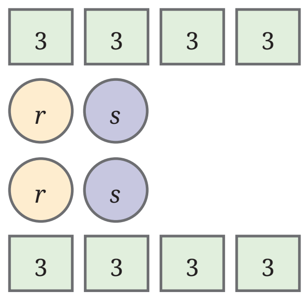
Let us compare the values that these expressions take for different values of y.





After filling in the two diagrams, do you think the two expressions are equal?

Example 11: What is the sum of the numbers in the picture (unknown values are denoted by letter-numbers)?



There are many ways to go about it. Here, we show some of them.

1. Adding row wise gives:

$$(4 \times 3) + (r + s) + (r + s) + (4 \times 3)$$

2. Adding like terms together gives:

$$(8 \times 3) + (r + r) + (s + s)$$

3. Adding the upper half and doubling gives:

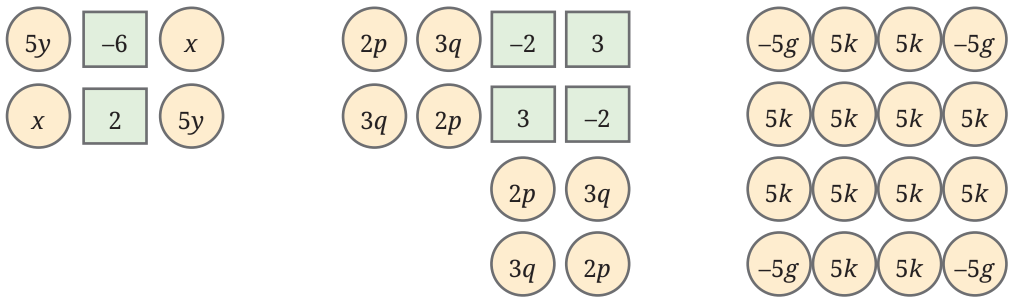
$$2 \times (4 \times 3 + r + s)$$

The three expressions might seem different. We can simplify each one and see that they all are the same: $2r + 2s + 24$.

Figure it Out

1. Add the numbers in each picture below. Write their corresponding expressions and simplify them. Try adding the numbers in each picture in a couple different ways and see

that you get the same thing.



2. Simplify each of the following expressions:

   (a) $p + p + p + p$, $p + p + p + q$, $p + q + p - q$,

   (b) $p - q + p - q$, $p + q - p + q$,

   (c) $p + q - (p + q)$, $p - q - p - q$

   (d) $2d - d - d - d$, $2d - d - d - c$,

   (e) $2d - d - (d - c)$, $2d - (d - d) - c$,

   (f) $2d - d - c - c$

Mind the Mistake, Mend the Mistake

Some simplifications of algebraic expressions are done below. The expression on the right-hand side should be in its simplest form.

- Observe each of them and see if there is a mistake.
- If you think there is a mistake, try to explain what might have gone wrong.
- Then, simplify it correctly.

Expression	Simplest Form	Correct Simplest Form
1. $3a + 2b$	5	
2. $3b - 2b - b$	0	
3. $6\ (p + 2)$	$6p + 8$	
4. $(4x + 3y) - (3x + 4y)$	$x + y$	
5. $5 - (2 - 6z)$	$3 - 6z$	
6. $2 + (x + 3)$	$2x - 6$	
7. $2y + (3y - 6)$	$-y + 6$	
8. $7p - p + 5q - 2q$	$7p + 3q$	
9. $5\ (2w + 3x + 4w)$	$10w + 15x + 20w$	

10. $3j + 6k + 9h + 12$ $\quad$ $3\,(j + 2k + 3h + 4)$

11. $4\,(2r + 3s + 5)$ $\quad$ $-\,20 - 8r - 12s$

? Take a look at all the corrected simplest forms (i.e. brackets are removed, like terms are added, and terms with only numbers are also added). Is there any relation between the number of terms and the number of letter-numbers these expressions have?

4.5 Pick Patterns and Reveal Relationships

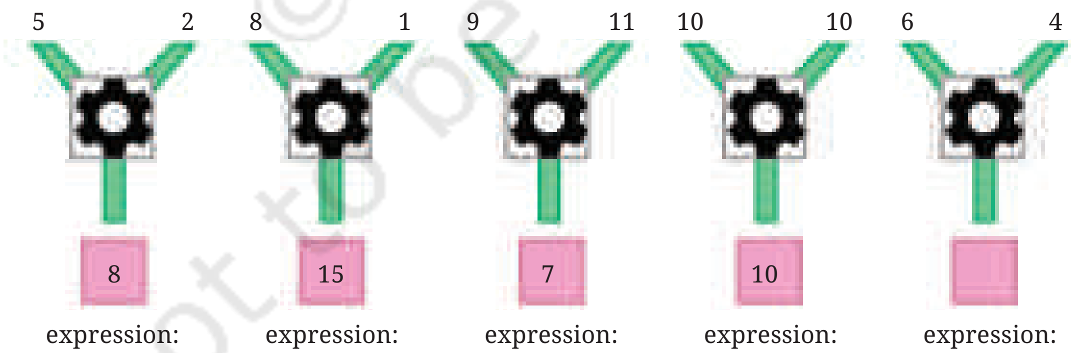
In the first section we got a glimpse of algebraic expressions and how to use them to describe simple patterns and relationships in a concise and elegant manner. Here, we continue to look for general relationships between quantities in different scenarios, find patterns and, interestingly, even explain why these patterns occur.

Remember the importance of describing in simple language, or visualising mathematical relationships, before trying to write them as expressions.

Formula Detective

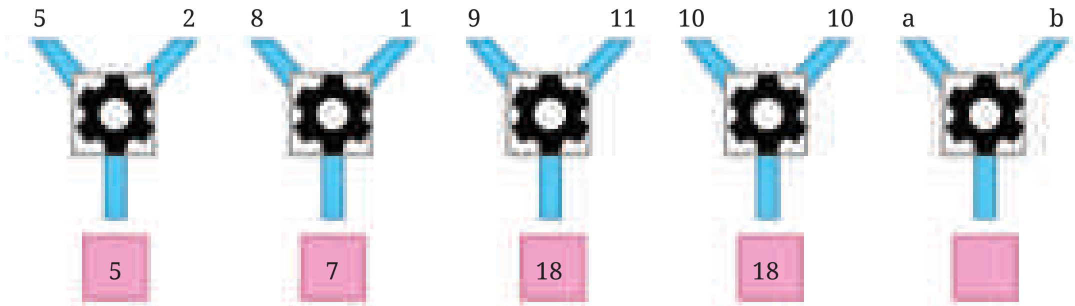
Look at the picture given. In each case, the number machine takes in the 2 numbers at the top of the 'Y' as inputs, performs some operations and produces the result at the bottom. The machine performs the same operations on its inputs in each case.

? Find out the formula of this number machine.

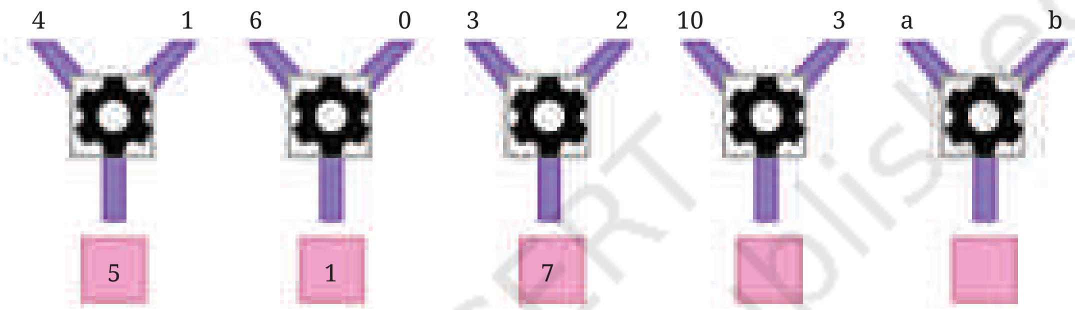


The formula for the number machine above is "two times the first number minus the second number". When written as an algebraic expression, the formula is $2a - b$. The expression for the first set of inputs is $2 \times 5 - 2 = 8$. Check that the formula holds true for each set of inputs.

Find the formulas of the number machines below and write the expression for each set of inputs.



expression: expression: expression: expression: expression:



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Now, make a formula on your own. Write a few number machines as examples using that formula. Challenge your classmates to figure it out!

> **Note to the Teacher:** Not just solving problems but creating new questions is also very much a part of learning and doing mathematics!

Algebraic Expressions to Describe Patterns

Example 12: Somjit noticed a repeating pattern along the border of a saree.

A B C D E F

Somjit wonders if there is a way to describe all the positions where the (i) Design A occurs, (ii) Design B occurs, and (iii) Design C occurs.

Let us start with design C. It appears for the first time at position 3, the second time at position 6.

? Where would design C appear for the n^{th} time?

We can see that this design appears in positions that are multiples of 3. So the n^{th} occurrence of Design C will be at position 3n.

? Similarly, find the formula that gives the position where the other Designs appear for the n^{th} time.

The positions where B occurs are 2, 5, 8, 11, 14, and so on.

We can see that the position of the nth appearance of Design B is one less than the position at which Design C appears for the n^{th} time. Thus, the nth occurrence of Design B is at position:

$$3n - 1$$

Similarly, the expression describing the position at which the design A appears for the nth time is: $3n - 2$.

? Given a position number can we find out the design that appears there? Which Design appears at Position 122?

If the position is a multiple of 3, then clearly we have Design C. As seen earlier, if the position is one less than a multiple of 3, it has Design B, and if it is 2 less than a multiple of 3, then it has Design A.

? Can the remainder obtained by dividing the position number by 3 be used for this? Observe the table below.

Position no.	**Quotient on division by 3**	**Remainder**
99	33	0
122	40	2
148	49	1

? Use this to find what design appears at positions 99, 122, and 148.

Patterns in a Calendar

Here is the calendar of November 2024. Consider 2 × 2 squares, as marked in the calendar. The numbers in this square show an interesting property.

November 2024

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

12	13
19	20

Let us take the marked 2 × 2 square, and consider the numbers lying on the diagonals; 12 and 20; 13 and 19. Find their sums; 12 + 20, 13 + 19. What do you observe?

They are equal.

Let us extend the numbers in the calendar beyond 30, creating endless rows.

November 2024

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31	32	33	34	35	36	37
38	39	40	41	42	43	44

? Will the diagonal sums be equal in every 2 × 2 square in this endless grid? How can we be sure?

To be sure of this we cannot check with all 2 × 2 squares as there are an unlimited number of them.

Let us consider a 2 × 2 square. Its top left number can be any number. Let us call it ‘a’.

a	?
?	?

? Given that we know the top left number, how do we find the other numbers in this 2 × 2 square?

As we have been doing, first let us describe the other numbers in words.

- the number to the right of 'a' will be **1 more than it**.
- the number below 'a' will be 7 **more than it**.
- the number diagonal to 'a' will be **8 more than it**.

a	$a + 1$
$a + 7$	$a + 8$

So the other numbers in the 2 × 2 square can be represented as shown in the grid. Let us find the diagonal sums; $a + (a + 8)$, and $(a + 1) + (a + 7)$.

Let us simplify them.

Since the terms can be added in any order, the brackets can be opened.

$a + (a + 8) = a + a + 8 = 2a + 8$

$(a + 1) + (a + 7) = a + 1 + a + 7 = a + a + 1 + 7 = 2a + 8$

We see that both diagonal sums are equal to $2a + 8$ (8 more than 2 times a).

? Verify this expression for diagonal sums by considering any 2 × 2 square and taking its top left number to be 'a'.

Thus, we have shown that diagonal sums are equal for any value of a, i.e., for any 2 × 2 square!

This problem is an example that shows the power of algebraic modelling in verifying whether a pattern will always hold.

Consider a set of numbers from the calendar (having endless rows) forming under the following shape:

	8	
14	15	16
	22	

? Find the sum of all the numbers. Compare it with the number in the centre: 15. Repeat this for another set of numbers that forms this shape. What do you observe?

We see that the total sum is always 5 times the number in the centre.

? Will this always happen? How do you show this?

Math Talk

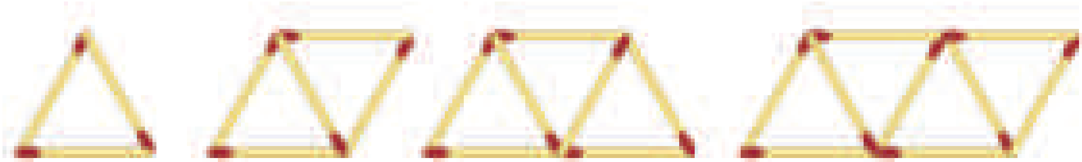
[**Hint:** Consider a general set of numbers that forms this shape. Take the number at the centre to be 'a'. Express the other numbers in terms of 'a'.]

Find other shapes for which the sum of the numbers within the figure is always a multiple of one of the numbers.

Math Talk

Matchstick Patterns

Look at the picture below. It is a pattern using matchsticks. Can you identify what the pattern is?



1 2 3 4

We can see that Step 1 has 1 triangle, Step 2 has 2 triangles, Step 3 has 3 triangles, and so on.

Can you tell how many matchsticks there will be in the next step, Step 5? It is 11. You can also draw this and see.

? How many matchsticks will there be in Step 33, Step 84, and Step 108? Of course, we can draw and count, but is there a quicker way to find the answers using the pattern present here?

What is the general rule to find the number of matchsticks in the next step? We can see that at each step 2 matchsticks are placed to get the next one, i.e., the number of matchsticks increases by 2 every time.

Step Number	1	2	3	4	5	6
No. of Matchsticks	3	5	7	9	11	13

Think of a way to use this to find out the number of matchsticks in Step 33 (without continuing to write the numbers).

As each time 2 matchsticks are being added, finding out how many 2s will be added in Step 33 will help. Look at the table below and try to find out.

Step Number	1	2	3	4	5	6
No. of Matchsticks	3	5	7	9	11	13
		3 + 2	3 + 2 + 2	3 + 2 + 2 + 2	3 + 2 + 2 + 2 + 2	

The number of matchsticks needed to make 33 triangles (Step 33) is __________. Similarly, find the number of matchsticks needed for Step 84 and Step 108.

What could be an expression describing the rule/formula to find out the number of matchsticks at any step?

The pattern is such that in Step 10, nine 2*s* and an added 3 ($3 + 2 \times 9$) gives the number of matchsticks; in Step 11, ten 2*s* and an added 3 ($3 + 2 \times 10$) gives the number of matchsticks. For step y, what is the expression?

It is: one less than y (i.e. $y - 1$) 2*s* and *a* 3.

Therefore, the expression is

$$3 + 2 \times (y - 1).$$

This expression gives the number of matchsticks in Step y. Now we can find the number of matchsticks at any step quickly.

You might have already noticed that there is a 2 in the first step also, $3 = 1 + 2$. Using this, the expression we get is

$$2y + 1.$$

? Does the above expression also give the number of matchsticks at each step correctly? Are these expressions the same?

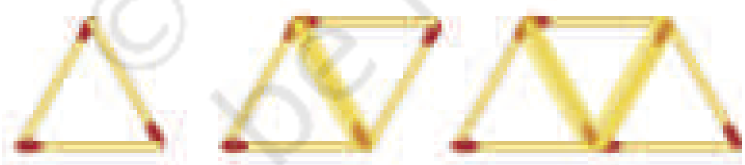
We can check by simplifying the expression $3 + 2 \times (y - 1)$.

$$3 + 2 \times (y - 1) = 3 + 2y - 2$$

$$= 2y + 1.$$

Both expressions are the same.

There is a different way to count, or see the pattern. Let us take a look at the picture again.



Matchsticks are placed in two orientations — (a) horizontal ones at the top and bottom, and (b) the ones placed diagonally in the middle. For example, in step 2 there are 2 matchsticks placed horizontally and 3 matchsticks placed diagonally.

? What are these numbers in Step 3 and Step 4?

? How does the number of matchsticks change in each orientation as the steps increase? Write an expression for the number of matchsticks at Step 'y' in each orientation. Do the two expressions add up to $2y + 1$?

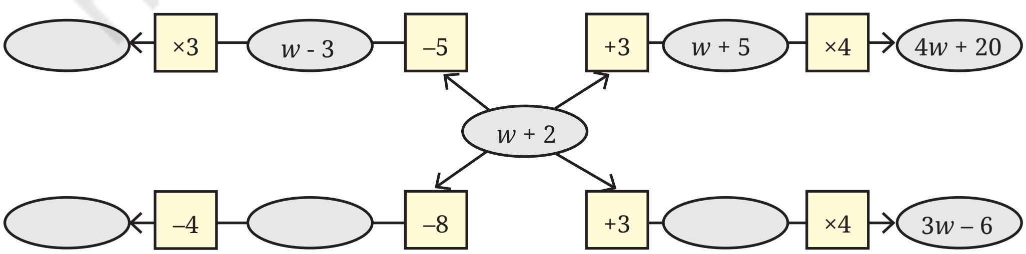
Figure it Out

For the problems asking you to find suitable expression(s), first try to understand the relationship between the different quantities in the situation described. If required, assume some values for the unknowns and try to find the relationship.

1. One plate of *Jowar roti* costs ₹30 and one plate of *Pulao* costs ₹20. If x plates of *Jowar roti* and y plates of *pulao* were ordered in a day, which expression(s) describe the total amount in rupees earned that day?
   (a) $30x + 20y$ (b) $(30 + 20) \times (x + y)$
   (c) $20x + 30y$ (d) $(30 + 20) \times x + y$
   (e) $30x - 20y$
2. Pushpita sells two types of flowers on Independence day: champak and marigold. '*p*' customers only bought champak, '*q*' customers only bought marigold, and '*r*' customers bought both. On the same day, she gave away a tiny national flag to every customer. How many flags did she give away that day?
   (a) $p + q + r$ (b) $p + q + 2r$
   (c) $2 \times (p + q + r)$ (d) $p + q + r + 2$
   (e) $p + q + r + 1$ (f) $2 \times (p + q)$
3. A snail is trying to climb along the wall of a deep well. During the day it climbs up '*u*' cm and during the night it slowly slips down '*d*' cm. This happens for 10 days and 10 nights.
   (a) Write an expression describing how far away the snail is from its starting position.
   (b) What can we say about the snail's movement if $d > u$?
4. Radha is preparing for a cycling race and practices daily. The first week she cycles 5 km every day. Every week she increases the daily distance cycled by '*z*' km. How many kilometers would Radha have cycled after 3 weeks?

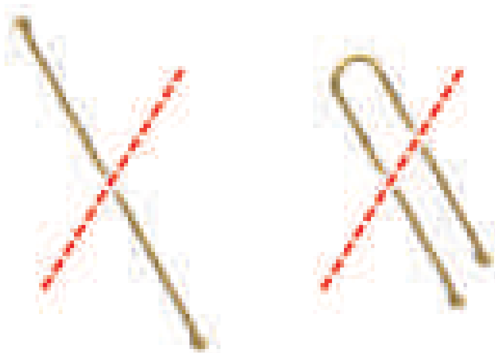
Try This

5. In the following figure, observe how the expression $w + 2$ becomes $4w + 20$ along one path. Fill in the missing blanks on the remaining paths. The ovals contain expressions and the boxes contain operations.

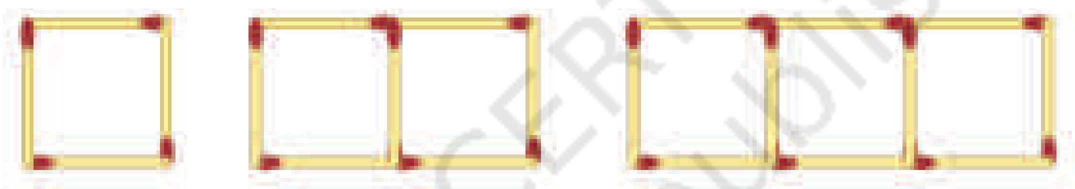


6. A local train from Yahapur to Vahapur stops at three stations at equal distances along the way. The time taken in minutes to travel from one station to the next station is the same and is denoted by t. The train stops for 2 minutes at each of the three stations.
   (a) If $t = 4$, what is the time taken to travel from Yahapur to Vahapur?
   (b) What is the algebraic expression for the time taken to travel from Yahapur to Vahapur? [Hint: Draw a rough diagram to visualise the situation]
7. Simplify the following expressions:
   (a) $3a + 9b - 6 + 8a - 4b - 7a + 16$
   (b) $3\,(3a - 3b) - 8a - 4b - 16$
   (c) $2\,(2x - 3) + 8x + 12$
   (d) $8x - (2x - 3) + 12$
   (e) $8h - (5 + 7h) + 9$
   (f) $23 + 4(6m - 3n) - 8n - 3m - 18$
8. Add the expressions given below:
   (a) $4d - 7c + 9$ and $8c - 11 + 9d$
   (b) $-6f + 19 - 8s$ and $-23 + 13f + 12s$
   (c) $8d - 14c + 9$ and $16c - (11 + 9d)$
   (d) $6f - 20 + 8s$ and $23 - 13f - 12s$
   (e) $13m - 12n$ and $12n - 13m$
   (f) $-26m + 24n$ and $26m - 24n$
9. Subtract the expressions given below:
   (a) $9a - 6b + 14$ from $6a + 9b - 18$
   (b) $-15x + 13 - 9y$ from $7y - 10 + 3x$
   (c) $17g + 9 - 7h$ from $11 - 10g + 3h$
   (d) $9a - 6b + 14$ from $6a - (9b + 18)$
   (e) $10x + 2 + 10y$ from $-3y + 8 - 3x$
   (f) $8g + 4h - 10$ from $7h - 8g + 20$
10. Describe situations corresponding to the following algebraic expressions:
   (a) $8x + 3y$
   (b) $15x - 2x$
11. Imagine a straight rope. If it is cut once as shown in the picture, we get 2 pieces. If the rope is folded once and then cut as shown, we

get 3 pieces. Observe the pattern and find the number of pieces if the rope is folded 10 times and cut. What is the expression for the number of pieces when the rope is folded *r* times and cut?

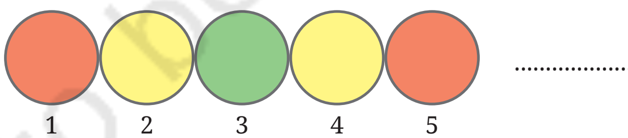


12. Look at the matchstick pattern below. Observe and identify the pattern. How many matchsticks are required to make 10 such squares. How many are required to make w squares?

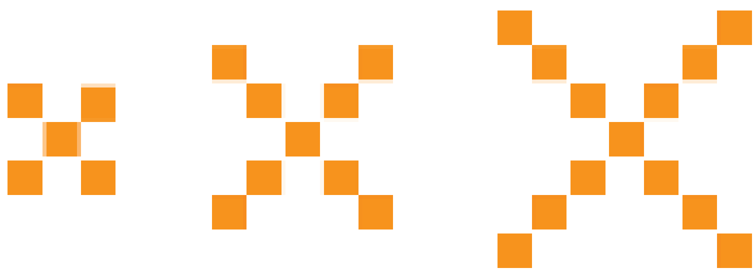


13. Have you noticed how the colours change in a traffic signal? The sequence of colour changes is shown below.

Find the colour at positions 90, 190, and 343. Write expressions to describe the positions for each colour.



14. Observe the pattern below. How many squares will be there in Step 4, Step 10, Step 50? Write a general formula. How would the formula change if we want to count the number of vertices of all the squares?



15. Numbers are written in a particular sequence in this endless 4-column grid.

1	2	3	4
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

   (a) Give expressions to generate all the numbers in a given column (1, 2, 3, 4).

   (b) In which row and column will the following numbers appear:

      (i) 124

      (ii) 147

      (iii) 201

   (c) What number appears in row *r* and column *c*?

   (d) Observe the positions of multiples of 3.

Do you see any pattern in it? List other patterns that you see.

SUMMARY

- Algebraic expressions are used in formulas to model patterns and mathematical relationships between quantities, and to make predictions.
- Algebraic expressions use not only numbers but also letter-numbers. The rules for manipulating arithmetic expressions also apply to algebraic expressions. These rules can be used to reduce algebraic expressions to their simplest forms.
- Algebraic expressions can be described in ordinary language, and vice versa. Patterns or relationships that are easily written using algebra can often be long and complex in ordinary language. This is one of the advantages of algebra.