Team2 BloodGlucoseProject Part1

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1 Prove $K_x = CF$

Assumptions :

- $X_1(0) = 0$
- $X(0) = 0$
- $U_1(s) = 1$

Consider the following system of equations from Bock et al.

$$\begin{cases} \dot{X} = -a_x X + a_x X_1 \\ \dot{X}_1 = -a_x X_1 + K_x a_x U_1 \end{cases} \tag{1}$$

Taking the Laplacian of each respective equation yields :

$$\begin{cases} sX(s) - X(0) = -a_x X(s) + a_x X_1(s) \\ sX_1(s) - X_1(0) = -a_x X_1(s) + K_x a_x U_1(s) \end{cases} \tag{2}$$

From the assumptions apriori :

$$\begin{cases} sX(s) = -a_x X(s) + a_x X_1(s) \\ sX_1(s) = -a_x X_1(s) + K_x a_x \end{cases} \tag{3}$$

Rearrangement gives :

$$\begin{cases} (s + a_x)X - a_x X_1 = 0 \\ (s + a_x)X_1 = K_x a_x \end{cases} \tag{4}$$

solving for Eqn 2 :

$$X_1 = K_x \frac{a_x}{s + a_x}$$

Plugging this solution into Eqn 1 yields :

$$(s + a_x)X - K_x \frac{{a_x}^2}{s + a_x} = 0$$

$$X = K_x \frac{{a_x}^2}{(s + ax)^2}$$
$$X = K_x \frac{{a_x}^2}{s^2 + 2sa_x + {a_x}^2}$$

From Bock et al., we also know the following :

$$\dot{G} = -X + U_g$$

Taking the Laplacian and plugging solutions yields :

$$\begin{aligned} sG(s) - G(0) &= -X(s) + U_g(s) && (5)\\ &= \lim_{s \to 0}(-K_x \frac{{a_x}^2}{(s + a_x)^2}) + U_g(s) && (6)\\ &= -K_x + U_g(s) && (7)\end{aligned}$$

By final value theorem, $\lim_{s \to 0} sG(s) = G(\infty)$. It is also reasonable to assume that $U_g(s)$, which represents the gut glucose absorption, reaches zero as the input s diminishes (ie. $\lim_{s \to 0} U_g(s) = 0$)

$$\begin{aligned} G(\infty) - G(0) &= -K_x + 0 && (8)\\ K_x &= -(G(\infty) - G(0)) && (9)\end{aligned}$$

By definition, $CF \equiv -(G(\infty) - G(0))$ and thus

$$K_x = CF$$

2 Prove $K_g = MS$

Assumptions

- $X(0) = 0$
- $U_g(0) = 0$
- $\dot{U}_g(0) = 0$
- $U_{CHO}(s) = 1$

Suppose the following system of equations : $\begin{cases} \dot{U}_g = U_g \\ \ddot{U}_g = -2a_g\dot{U}_g - a_g^2 U_g + K_g a_g^2 U_{CHO} \end{cases}$

Taking the Laplacian yields $\begin{cases} & = U_g(s) \\ s^2U_g(s) - sU_g(0) - \dot{U}_g(0) & = -2a_g\dot{U}_g(s) - a_g^2U_g(s) + K_g a_g^2 U_{CHO}(s) \end{cases}$

From our assumptions, we get the following : $\begin{cases} & = U_g(s) \\ s^2U_g(s) & = -2sa_gU_g(s) - a_g^2U_g(s) + K_ga_g^2 \end{cases}$

Isolating U_g in the second equation provides

$$(s^2 + 2sa_g + a_g^2)U_g(s) = K_ga_g^2 \tag{10}$$

$$U_g(s) = \frac{K_ga_g^2}{s^2 + 2sa_g + a_g^2} \tag{11}$$

From Bock et al., we have

$$\dot{G} = -X + U_g$$

Using the same logic as before :

$$G(\infty) - G(0) = K_g$$

By definition, $MS \equiv G(\infty) - G(0)$ and so

$$K_g = MS$$

3 Issues in Bock et al.

In section 2.2.3 of Bock et al., an algebraic mistake seems to have been made in solving $X(s)$. The paper provides the following

$$G(s) = -\frac{1}{s}\frac{K_x}{(1 + \frac{s}{a_x})^2}U_I(s) + \frac{1}{s}G(0)$$

However, the paper itself mentions that G is a function of X and U_g, so we should expect the final answer to be

$$G(s) = -\frac{1}{s}X(s) + \frac{1}{s}U_g(s) + \frac{1}{s}G(0) \tag{12}$$

$$\tag{13}$$

Since we assume only an insulin bolus, the component from the U_g term should be removed and after plugging our solutions, yields:

$$G(s) = -\frac{1}{s}K_x(\frac{a_x}{s + a_x})^2 + \frac{1}{s}G(0)$$

Only then does the $\lim_{s\to 0}$ provide a result where $K_x = CF$.