Introduction to Neural and Cognitive Modelling
Assignment 3

Deadline: 25 October 2025, 11:59 PM

1 General Instructions

1. All assignments should be implemented in Python, preferably using a Jupyter Notebook for clarity and reproducibility.
2. Submit a single PDF report along with your `.ipynb` notebook. Bundle them in a `.zip` file and upload to Moodle.
3. Academic honesty is expected at all times. Plagiarism will result in a score of **0**.
4. Start early. **The deadline is strict; no extensions will be granted.**

2 Dendrites and the Passive Cable Equation (20 marks)

2.1 Objective

Use a Brian2 implementation of a passive cable to study voltage propagation along a dendrite in space and time. Simulate pulse inputs, multi-site spike patterns, parameter changes, and compare steady states with analytical results.

2.2 Setup and Notes

- Use `neurodynex3.cable_equation.passive_cable` and `neurodynex3.tools.input_factory`.
- Example: `passive_cable.getting_started()` runs a demo pulse.
- Use default cable parameters unless specified.
- When plotting, scale Brian2 quantities to physical units, e.g., `voltage_monitor.t / b2.ms` and `voltage_monitor[0].v / b2.mV`.

2.3 Tasks

2.3.1 Spatial and temporal evolution of a pulse input (5 marks)

- Cable length: 800 µm
- Step current: 0.8 nA, duration 0.1 ms, applied at $t = 1.0\,\text{ms}$, $x = 200\,\mu\text{m}$
- Simulation time: 3 ms

Deliverables:

- Report maximum depolarization (value, location, and time)
- Plot $V_m(t)$ at $x = 0, 100, \ldots, 600$ µm for $t \in [0, 3]$ ms
- Plot $V_m(x)$ for $x \in [0, 800]$ µm at $t = 1.0, 1.1, \ldots, 1.6$ ms (all curves on one plot)
- Discuss and interpret results

2.3.2 Spatio-temporal input pattern (5 marks)

Three short pulses (100 µs, 0.8 nA) at:

$$\begin{aligned} &\text{A: } (t = 1.0 \text{ ms}, x = 100\ \mu\text{m}) \\ &\text{B: } (t = 1.5 \text{ ms}, x = 200\ \mu\text{m}) \\ &\text{C: } (t = 2.0 \text{ ms}, x = 300\ \mu\text{m}) \end{aligned}$$

(a) Plot soma voltage ($x = 0$) for $t \in [0, 5]$ ms, report maximal depolarization. (3 marks)

(b) Reverse order (C, B, A), overlay with original trace, report maxima and discuss effects. (2 marks)

2.3.3 Effect of cable parameters (4 marks)

Compare two parameter sets:

- Set 1 (default): $R_m = 1.25\,\text{M}\Omega \cdot \text{mm}^2$, $C_m = 0.8\,\mu\text{F/cm}^2$
- Set 2 (myelinated-like): $R_m = 5.0\,\text{M}\Omega \cdot \text{mm}^2$, $C_m = 0.2\,\mu\text{F/cm}^2$

Inject a brief pulse at $t = 0.05$ ms, $x = 400\,\mu\text{m}$, simulate for 0.2 ms with `b2.defaultclock.dt = 0.005*ms`.

- Plot $V_m(t)$ at $x = 500\,\mu\text{m}$ for both parameter sets on the same axes. Discuss effects on amplitude and kinetics.

2.3.4 Stationary solution and comparison with theory (6 marks)

- Cable length: $L = 500\,\mu\text{m}$, constant current $I_0 = 0.1\,\text{nA}$ at $x = 0$, simulate until steady state (100 ms), `b2.defaultclock.dt = 0.1*ms`

(a) Sketch expected $V_m(t)$ qualitatively at $x = 0$ and $x = L$, then simulate and plot. (2 marks)

(b) Compute characteristic length λ and compare with spatial decay. (2 marks)

(c) Bonus: Derive analytical steady-state solution for finite cable, overlay with simulation, discuss convergence. (2 marks)

2.4 Deliverables

- Jupyter Notebook with all simulation code, parameter settings, plots (labelled with axes and units), short answers, and discussions.
- Report `b2.defaultclock.dt` and number of compartments for each simulation.

2.5 Marks Distribution (20 total)

- Spatial and temporal evolution of a pulse input: 5 marks
- Spatio-temporal input pattern: 5 marks
- Effect of cable parameters: 4 marks
- Stationary solution and comparison with theory: 6 marks

3 Facilitation Effect - Synaptic Simulation Model (20 marks)

3.1 Objective

Simulate synaptic facilitation and how repeated presynaptic spikes affect postsynaptic potentials.

3.2 Setup and Notes

- Use a conductance- or current-based synaptic model in Brian2.
- Facilitation can be modeled by increasing synaptic efficacy:

$$u \to u + U(1-u) \quad \text{per spike,} \quad \tau_F \text{ decay back to baseline}$$

  where u is the utilization factor and U is the facilitation increment.

- Inject presynaptic spike trains of varying frequencies (10 Hz, 50 Hz, 100 Hz) and record postsynaptic voltage or current.
- Plot postsynaptic responses and measure peak amplitudes for each spike.

3.3 Tasks

1. Single presynaptic spike: plot Post-Synaptic Potential (PSP), report peak. (4 marks)
2. Train of 5 spikes at 50 Hz: plot PSPs, report facilitation. (6 marks)
3. Repeat for 10 Hz and 100 Hz: discuss frequency dependence. (6 marks)
4. Bonus: Vary τ_F to compare facilitation with/without recovery, discuss effect. (4 marks)

3.4 Deliverables

- Jupyter Notebook with simulation code, plots (labelled axes and units), and discussions.
- Report parameters used for facilitation (U, τ_F, number of spikes, frequency).

4 Discussion (10 marks)

1. In the passive cable experiments, voltage propagation shows exponential decay with distance. Suppose dendrites had no leak channels ($R_m \to \infty$).
   - Predict qualitatively how this would alter voltage spread, and
   - Discuss whether such a system could still function biologically.
2. In the facilitation model, repeated presynaptic spikes increase the utilization factor u, leading to stronger postsynaptic responses.
   - Why does facilitation show stronger effects at intermediate frequencies (e.g., 50 Hz) compared to very low (10 Hz) or very high (100 Hz) frequencies?
   - Relate this to the interaction between spike timing and τ_F.
3. Compare the role of the membrane time constant (τ_m in the cable equation) and the facilitation time constant (τ_F in synaptic plasticity).
   - How are these two constants similar in shaping temporal dynamics of neurons?
   - How do they differ in their biological interpretations?
   - What would happen if both were very small versus very large?
4. Imagine a neuron that integrates inputs from multiple dendritic cables, each with different R_m and C_m values, and synapses that exhibit facilitation.

- How would spatial filtering (cable properties) and temporal filtering (facilitation) together determine whether the soma fires?
- Propose one concrete example of how these combined effects might enhance or suppress information transfer in real neural circuits.

5 Grading Scheme

- Part A: Passive Cable — 20 marks
- Part B: Synaptic Facilitation — 20 marks
- Discussion — 10 marks
- Viva during Evaluation — 50 marks

Total Marks: 100

HAPPY DEEPAVALI!