BACHELOR OF TECHNOLOGY PROJECT

SYNTHESIS OF THE PROPULSIVE TAIL DRIVING SYSTEM FOR ROBOTIC FISH

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CERTIFICATE

This is to certify that the Dissertation Report entitled, **Synthesis of the propulsive tail driving System for robotic fish**"submitted by Mr. Vivek Singh to Indian Institute of Technology, Kharagpur, India, is a record of bonafide project work carried out by them under my supervision and guidance and is worthy of consideration for the award of the degree of Bachelor of Technology in Mechanical Engineering of the Institute.

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Supervisor
Prof. C.S Kumar
Date:

ABSTRACT

This report presents Kinematics analysis of the two mechanism that can generate undulatory and oscillatory mode of swimming. The mechanism is based on hypocycloid principle. Relationship between tail fin swing angle and input angular velocity of motor is carried out and further optimization of structural parameter is done. Simulation of mathematical model is done in Matlab & Solidworks. This can be used for underwater vehicles thruster design and its mechanism.

1. INTRODUCTION

Nature made mechanism has maximum efficiency and smooth. There has been increasing interest in the development and application of bio-inspires robots. In case of underwater navigation, researches say that bio-inspired robots has astonishing swimming ability and maneuverability and that motivates to replace the propulsion system that are being used such as in Autonomous Underwater Vehicles (AUVs). In marine protection and military defense, autonomous underwater vehicles require fast propulsion and multi-directional maneuverability. Robotic fish will be more competent than current AUV propelled by rotary propellers. [5] It was found that robotic fish increase the efficiency up-to 90 % while AUVs has the efficiency of around 40%. Besides advancing the understanding of fish swimming mechanisms, as an aquatic mobile platform, robotic fish will play an important role in fulfilling real-world tasks such as underwater exploration, mobile sensing, patrol, wreck surveying, search and rescue, and environmental monitoring.
Fish Robot can be propelled through undulatory or oscillatory motion of either body or fin. The first fish robot was built by MIT in 1994 [6]. It was designed to mimic the structure and dynamical properties of tuna.

Existing robot exhibit some deficiency in the structure and that affects their swimming behaviour. On the basis of their design and structure, it can either do undulatory or oscillatory motion. Undulation is for cruising larger distances while oscillation is for hovering and slow swimming. But for navigation, robot fish must exhibit both type of motion.

Most of existing models of fish swimming depends on Lighthill's work. [1] Harper developed a tail design whose dynamics rely on an optimal spring constant for actuation of oscillating fin. [2] Lighthill's description had been used by Barrett to develop a form of travelling wave as

$$y_{body}(x,t) = (c_1 x + c_2 x^2)\sin(kx + \omega t)$$

where y_{body} is the lateral displacement of the body, x is the body displacement along the main axis, $c1$ and $c2$ are linear and quadratic coefficients of wave amplitude envelopes, and k and ω are the body-wave number and frequency.

Undulation and oscillation

In undulatory swimming modes, thrust is produced by wave-like movements of the propulsive structure (usually a fin or the whole body) [7].
Oscillatory modes are characterized by thrust produced by swivelling of the propulsive structure on an attachment point without any wave-like motion [8].

Figure1 shows how the modes of swimming vary. In undulation, almost whole part of body does wavy like motion while in oscillation, only some part oscillates. Part that undulates or oscillate is shown by using black color on fish body.

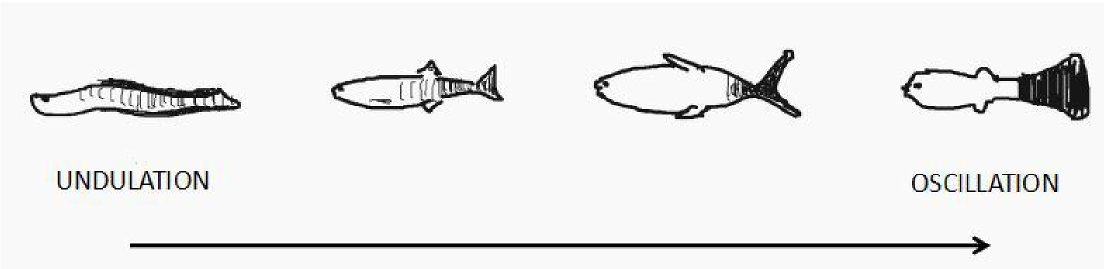
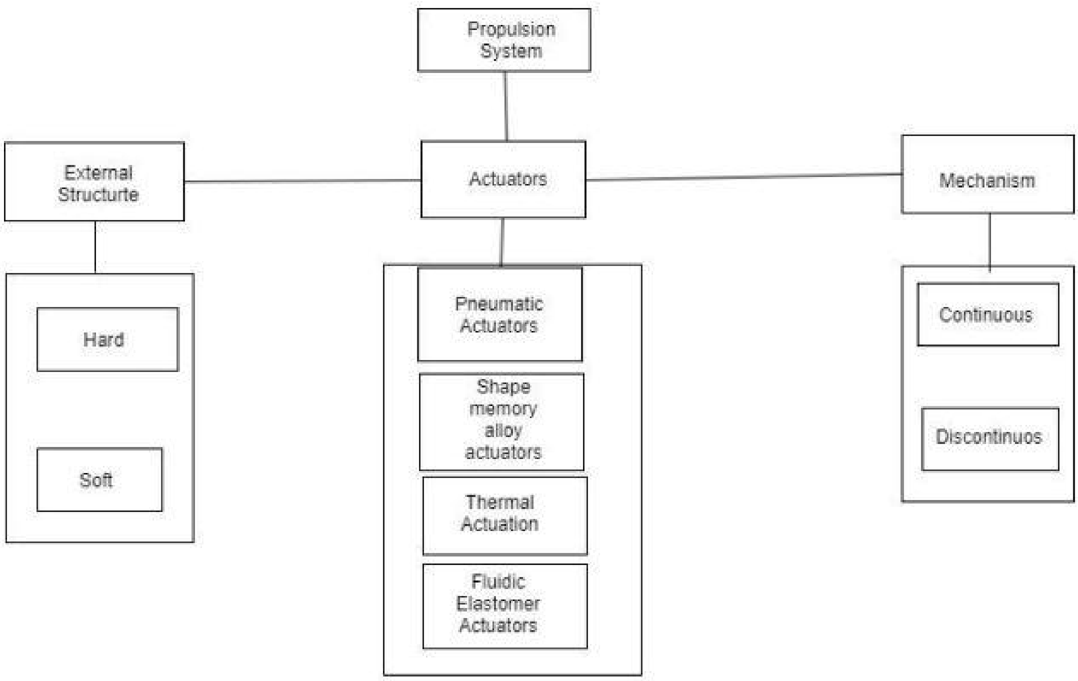


Figure1. Modes of swimming

Propulsive mechanism

Fish are propelled through the water by fins, body movements, or both. Fish swim by exerting force against the surrounding water. This is normally achieved by the fish contracting muscles on either side of its body in order to generate waves of flexion that travel the length of the body from nose to tail. The forces exerted on the water by such motion cancel out laterally, but generate a net force backwards which in turn pushes the fish forward through the water. Most fishes generate thrust using lateral movements of their body and caudal fin. While swimming, the fins are driven by muscles attached to the base of the fin spines and the rays. In particular, fish with fairly rigid bodies depend mostly on active fin action for propulsion.
Propulsion System can be made by different type of external structure, actuators and exhibit continuous or discontinuous mechanism. This is shown below.



2. LITERATURE REVIEW

Soft robotic fish

Soft Robotic fish ‘SoFi’ was built at MIT in 2018 [3]. Tail part is made up of soft material having the fluids in two cavities. A hydraulic pump attached to a soft body allows for water movement between two inner cavities, ultimately leading to a flexing actuation in a side-to-side manner. Undulating structures are one of the most diverse and successful forms of locomotion in nature, both on ground and in water. The inherent elasticity of the body forces it back into its neutral state after each pulse of actuation. A fluidic flow alternating into each lateral cavity structure leads to a complex undulating motion of the soft body and enables swimming.

SoFi can currently swim up to 0.51 body lengths per second, which is comparable to other robotic fish prototypes but still leaves room to improve toward real fish capabilities of 2 to 10 body lengths per second . Further optimizations of the pump system, the tail geometries, and the exterior profile of SoFi may improve swimming efficiency.

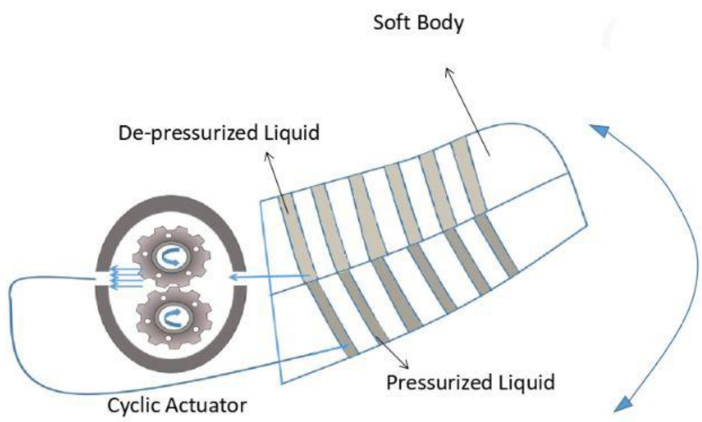


Figure2. Cyclic hydraulic actuation for SoFi

Tuna robotic fish

Tuna robotic fish was built at MIT [6]. This does undulatory motion having four degree of freedom. This model uses oscillatory type of motor which is desirable for small model. This has planer linkage and all linkage is covered with soft material Making large model of this robotic fish, it is not desirable because it requires larger torque-fluctuation.

3. PROBLEM DESCRIPTION

Underwater exploration, pollution search, military detection and mobile sensing needs the development of underwater robots. Remotely operated vehicles (ROVs) or autonomous underwater vehicles (AUVs) in ocean environments typically use propellers or jet-based propulsion systems. However, these propulsion systems generate substantial turbulence and have the potential to scare marine life and to prevent close-up observations. Further, the mere appearance of these vehicles, typically large and rigid like a submarine, does not integrate well into the marine environment. Since the best optimal solution is somewhere in nature. So we need tail driving propulsion system mimicking fish tail part motion.

Our model is of 1 meter in length, so if we use servo motor to do undulation or oscillation of tail fin, then it requires enormous amount of alternating torque. It increases the chances of mechanical and electrical failures. To reduce the chances of this failure, we need mechanism that convert uni-directional motion of motor into oscillatory or undulatory. Kinematics and dynamics analysis of such type of mechanism is proposed here with optimization design of structural parameter. There are two proposed model for propulsion system. First mechanical Fish tail system propels itself using oscillatory tail and other one using undulatory tail.

4. HYPOTHESIS

Direct transformation of rotary motion of motor into oscillatory motion is difficult and even if this transformation is made, it produces jerkiness in mechanism . Whenever necessary, to avoid such complexity we use servo motor. If we convert rotary motion of motor into linear motion somehow, then it is easy to convert that linear motion in oscillatory form. We use hypocycloid to generate linear motion and by connecting two planer linkage with re-volute joint we get oscillatory motion. This arrangement of converting rotary motion of motor into oscillatory form is given in figure7.

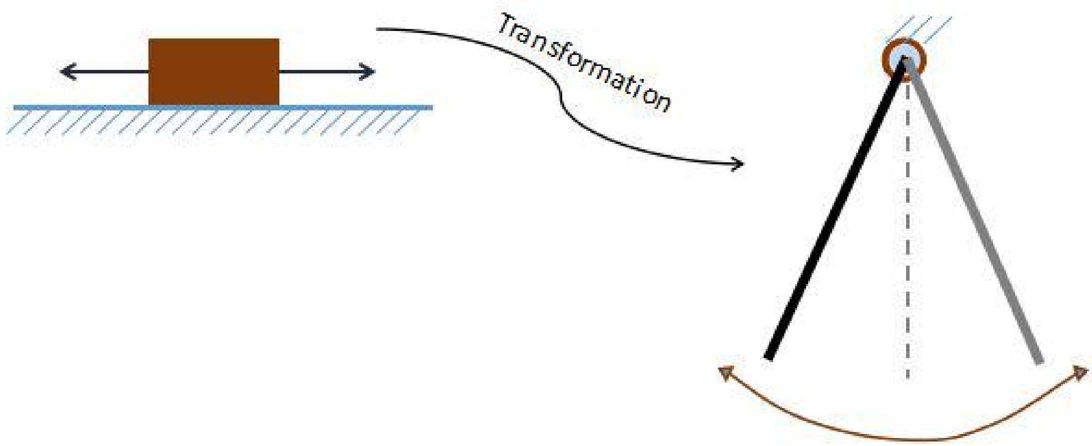


Figure3. Linear motion to Oscillatory motion

5. WORKING PRINCIPLE

Hypocycloid

In geometry, a hypocycloid is a special plane curve generated by the trace of a fixed point on a small circle that rolls within a larger circle. It is comparable to the cycloid but instead of the circle rolling along a line, it rolls within a circle [9].
One of special feature of this is that when inner circle's radius becomes equal to half of outer circle radius. The trajectory of any circumferential point on inner circle is straight line and performs simple harmonic motion when inner circle rolls with constant angular velocity.
r = Radius of inner circle
R = Radius of outer circle
ω = Angular velocity at which inner circle is rolling.
At any point, displacement of circumferential point P is
$y(t) = R\sin(\omega t + \beta)$

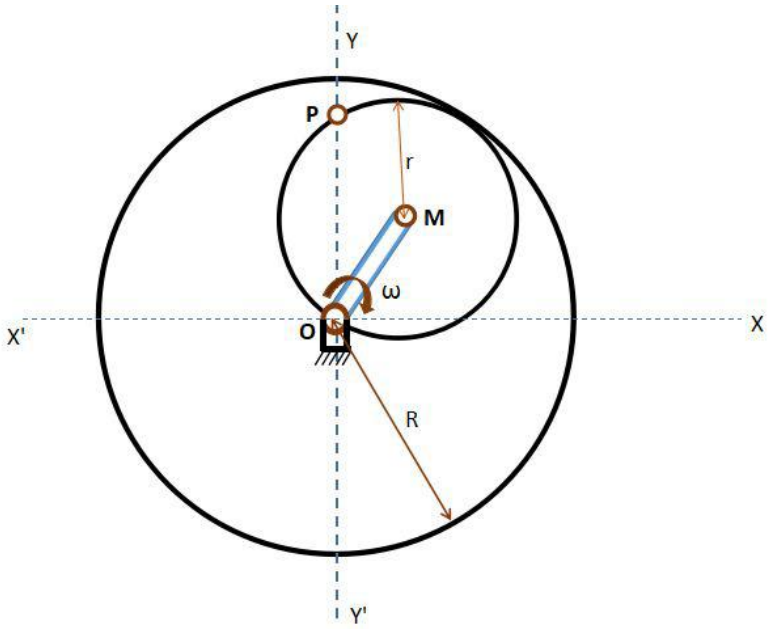


Figure4. Linear Hypocycloid

6. MECHANICAL DESIGN FOR OSCILLATORY MODE OF SWIMMING

The main factor that distinguish Fish robot from underwater vehicle is their propulsion system. This tail mechanism produces oscillatory motion.
We used hypocycloid principle in our mechanism, replacing inner circle by spur gear(Planetary Gear) and outer circle by ring gear. At the circumferential point P of

spur gear, attached a link PR with re-volute joint. Link RS is joint with link PR with re-volute and other end of link RS i.e. S is attached with body with re-volute joint. Link ST is rigidly attached with S so that it swipes same angle across x- axis. Planetary gear and sun gear are attached inside the main body.
Link PR and RS are also inside the main body. Tail ST is attached with main body through rod (oscillating in nature at point S). During oscillation of tail, average lateral forces by water on the tail is cancel out and there remains longitudinal forces that propels the main body forward. This system has 1 DOF. So it requires pectoral fin to move in 2D plane.

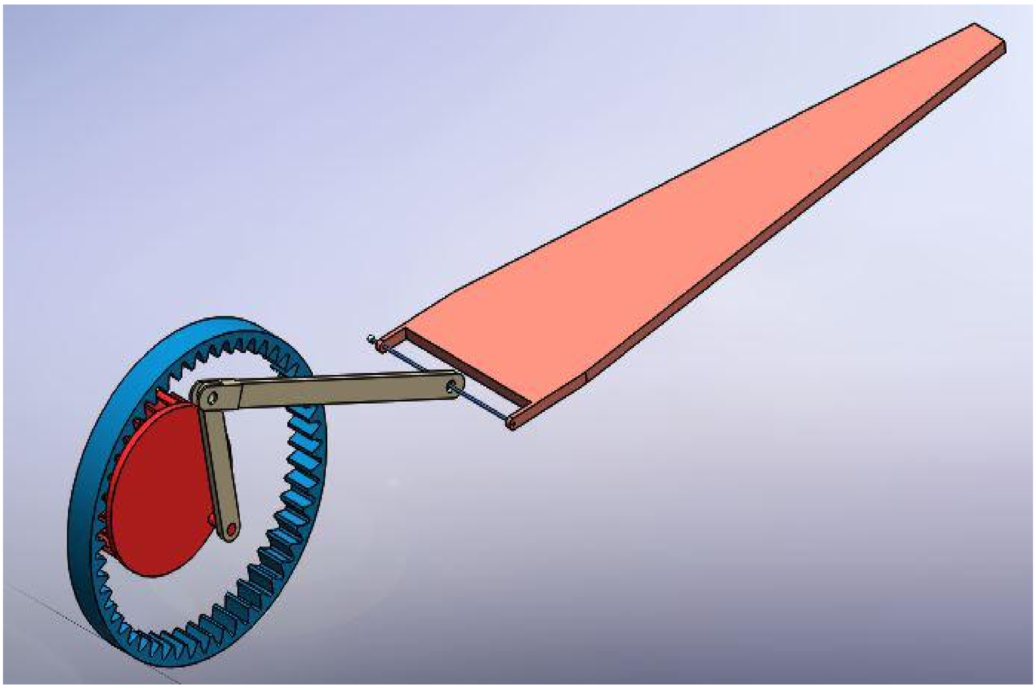


Figure5. 3D design of mechanism

7. KINEMATICS OF THE TAIL MECHANISM FOR OSCILLATORY MODE OF SWIMMING

Tail peduncle of this fish robot mechanism consists of 2 link and 2 gear and they are actuated by uni-directional motor. Once the motor starts actuating, all the links start moving and tail starts oscillating. Through kinematics analysis relationship between links and angle, tail sweeps with respect to relative frame of reference placed at O is provided with following equation.

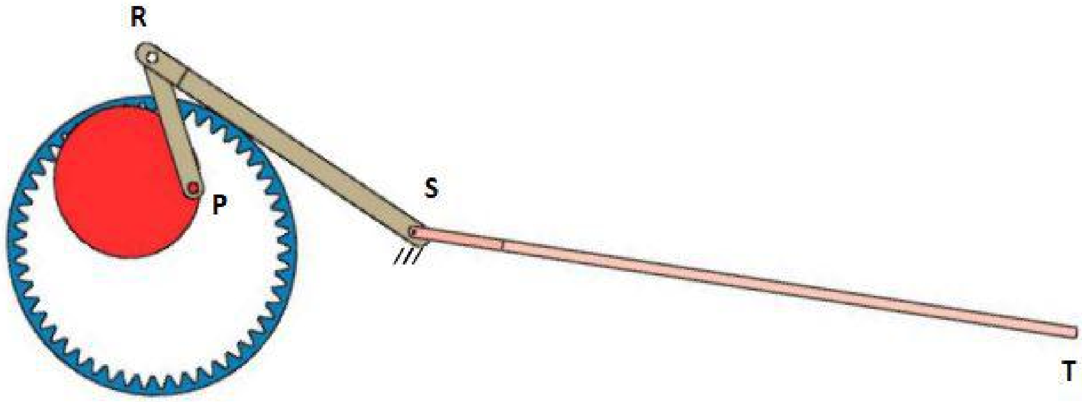


Figure6. Top View

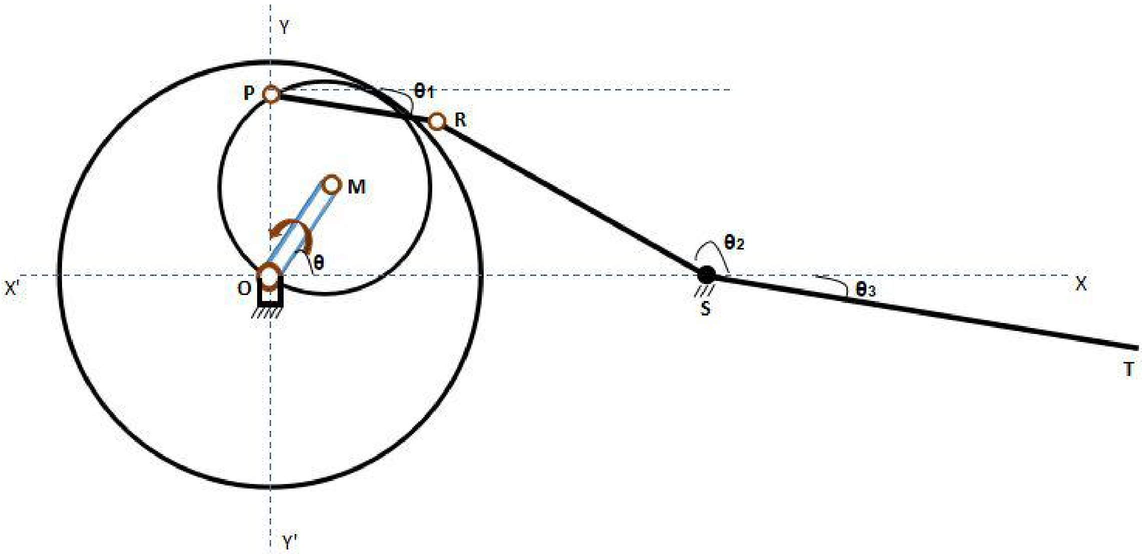


Figure7. A schematic sketch of mechanism

Angle between rigidly connected link RS and ST is $\frac{\theta 2_{\max} + \theta 2_{\min}}{2}$.

Point P moves on straight line YY'. ω is angular speed of motor.
All the kinematics equation are written below.

$$OP = R\cos(\omega t + \psi)$$

$$OS = PR\cos(\theta 1) - RS\cos(\theta 2)$$

$$OP = PR\sin(\theta_1) + RS\sin(\theta_2)$$

$$OP^2 - 2OP\,RS\sin(\theta_2) + RS^2 + OS^2 + 2OS\,RS\cos(\theta_2) - PR^2 = 0$$

$$OP\sin(\theta_2) - OS\cos(\theta 2) = \frac{OP^2 + RS^2 + OS^2 - PR^2}{2RS}$$

$$Here, \quad \sin(\phi) = \frac{OS}{\sqrt{OP^2 + OS^2}}$$

$$\sin(\theta 2 - \phi) = \frac{OP^2 + RS^2 + OS^2 - PR^2}{2RS\sqrt{OP^2 + OS^2}}$$

$$\theta 2 = \sin^{-1}\left(\frac{OP^2 + RS^2 + OS^2 - PR^2}{2RS\sqrt{OP^2 + OS^2}}\right) + \sin^{-1}\left(\frac{OS}{\sqrt{OP^2 + OS^2}}\right)$$

When OP is positive,

$$\theta 2 = \sin^{-1}\left(\frac{OP^2 + RS^2 + OS^2 - PR^2}{2RS\sqrt{OP^2 + OS^2}}\right) + \sin^{-1}\left(\frac{OS}{\sqrt{OP^2 + OS^2}}\right)$$

When OP is negative,

$$\theta 2 = \sin^{-1}\left(\frac{OP^2 + RS^2 + OS^2 - PR^2}{2RS\sqrt{OP^2 + OS^2}}\right) - \cos^{-1}\left(\frac{OP}{\sqrt{OP^2 + OS^2}}\right) + pi$$

Range of value of θ_2

$\theta 2$ is maximum when OP = -R

and it is minimum when OP = R

$$\theta_{range} = \theta 2_{\max} - \theta 2_{\min}$$

$$\theta_{range} = pi - 2\phi$$

Angular displacement and angular velocity of link ST is

$$\theta 3 = \theta 2 - \frac{\theta 2_{\max} + \theta 2_{\min}}{2}$$

$$\dot{\theta} 3 = \dot{\theta} 2$$

Displacement of T w.r.t O is written as

$$Tx = \overline{ST}\cos(\theta_3) + OS$$

$$Ty = \overline{ST}\sin(\theta_3)$$

Velocity of T w.r.t O is

$$\dot{T}x = -\overline{ST}\sin(\theta_3)\,\dot{\theta}_3$$

$$\dot{T}y = \overline{ST}\cos(\theta_3)\,\dot{\theta}_3$$

Optimal Design on Structural Parameters

$$\theta 2_{(OP=0)} - \theta 2_{\min} = \theta 2_{\max} - \theta 2_{(OP=0)}$$

This condition give:

$$RS^{2} = PR^{2} - OS^{2} - \frac{R^{2}}{1 - \sqrt{1 + \left(\frac{R}{OS^{2}}\right)}}$$

$$OS_{\max} \leq \sqrt{RS^{2} - R^{2}} + PR$$

$$OS_{\min} \geq PR$$

$$OS_{\min} \geq RS - PR$$

8. MECHANICAL DESIGN FOR UNDULATORY MODE OF SWIMMING

This tail driving propulsion system propels itself by using undulatory movement. This mechanism consists of four link. First link is connected to circumferential point of inner gear and to the link RS with re-volute joint. Link RS and SW are rigidly connected with each other with main body at S. Link SW performs oscillation about Z-axis and sweeps same angle across XX' axis. Link SW and AV are connected at U with re-volute-prismatic pair. Last link VT is joint with link SW at W and with link AV at V with re-volute joint.

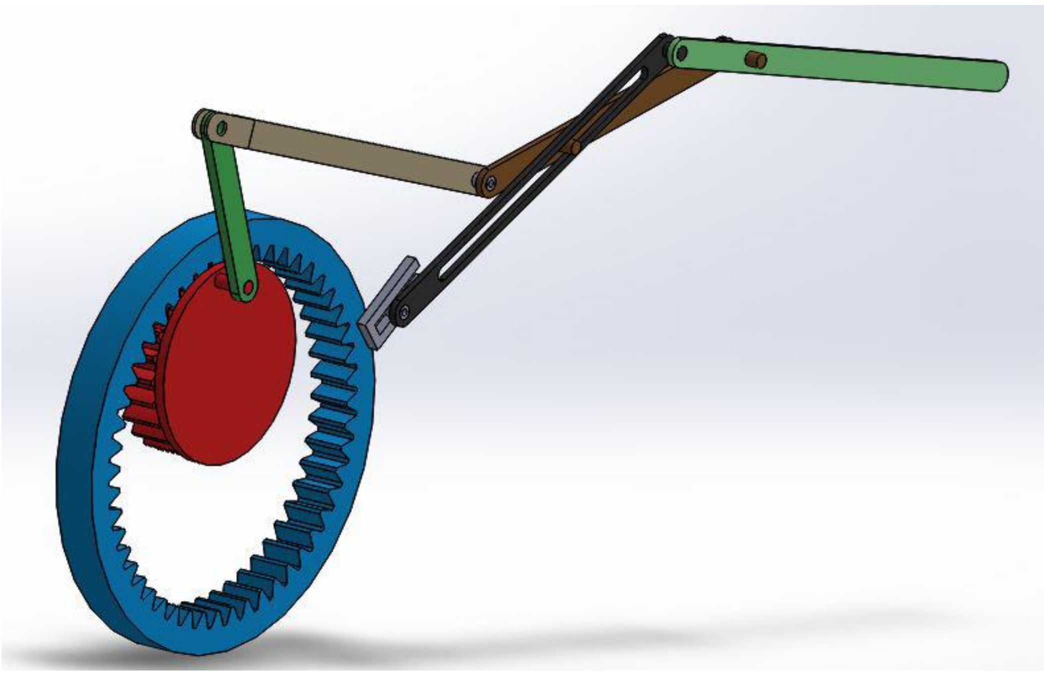


Figure8. 3D design of mechanism

9. KINEMATICS OF THE TAIL MECHANISM FOR UNDULATORY MODE OF SWIMMING

Kinematics analysis has been done and relation between link length and angular velocity is carried out.This yield four equation.

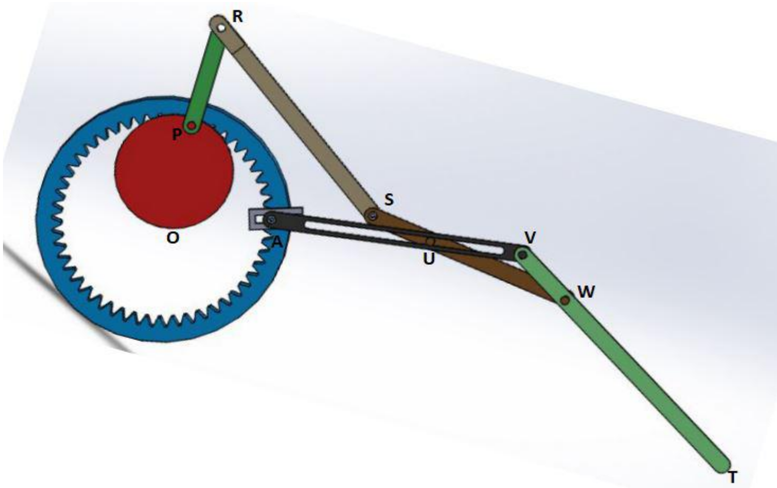


Figure9. Top view

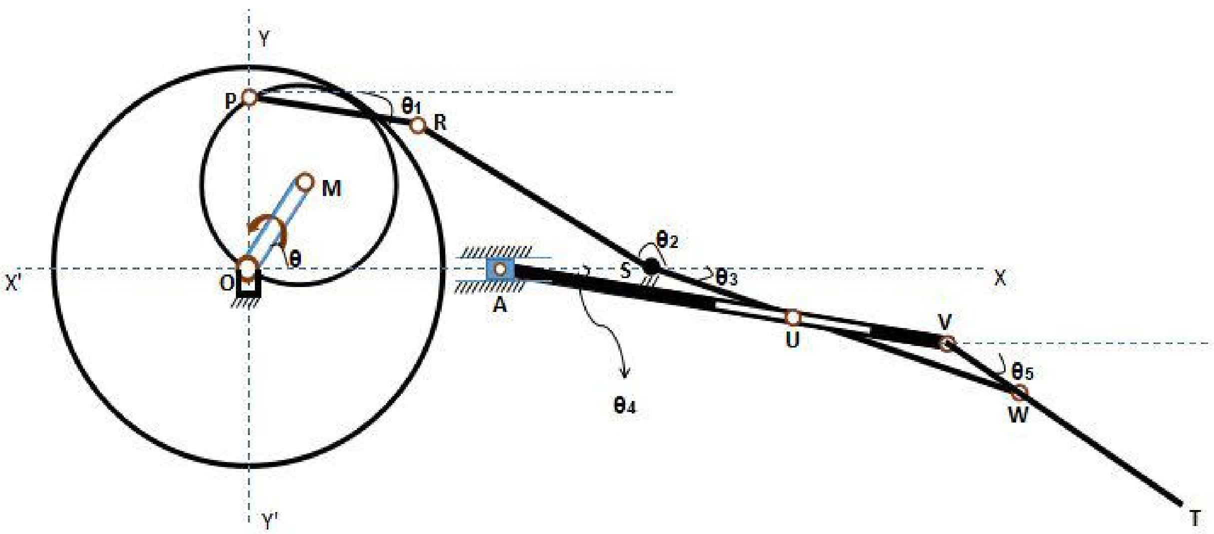


Figure10. A schematic sketch of Mechanism

$$\overline{AU}\cos(\theta_4) = \overline{SU}\cos(\theta_3) + \overline{AS}$$
$$\overline{AU}\sin(\theta_4) = \overline{SU}\sin(\theta_3)$$
$$\overline{AS} + \overline{SW}\cos(\theta_3) = \overline{AV}\cos(\theta_4) + \overline{VW}\cos(\theta_5)$$
$$\overline{SW}\sin(\theta_3) = \overline{AV}\sin(\theta_4) + \overline{VW}\sin(\theta_5)$$

$\theta_4, \overline{AU}, \overline{AS}, \theta_5$ can be obtained in the term of θ_3.

After theanalysis,
θ_5 can be writtenas $\mu\, \theta_3$
μ is functionof $\overline{SW}, \overline{AV}, \overline{VT}$ *and* $\overline{VW}$

Displacement of T w.r.t S is written as

$$T_x = \overline{SW}\cos(\theta_3) + \overline{WT}\cos(\theta_5)$$
$$T_y = \overline{SW}\sin(\theta_3) + \overline{WT}\sin(\theta_5)$$

Velocity of end point T w.r.t S is

$$\dot{T}_x = -\overline{SW}\sin(\theta_3)\,\dot{\theta}_3 - \overline{WT}\sin(\theta_5)\,\dot{\theta}_5$$
$$\dot{T}_y = \overline{SW}\cos(\theta_3)\,\dot{\theta}_3 + \overline{WT}\cos(\theta_5)\,\dot{\theta}_5$$

10. RESULTS

Results for First Mechanism

In order to analyze the motion of Point T, angular displacement and velocity or computed. DC motor is operating at angular speed of 4 rad/s.

Part	Length [m]
Radius: Planetary Gear	0.05
Ring Gear	0.1
$\overline{PR}$	0.1
$\overline{RS}$	0.234
$\overline{ST}$	0.15
$\overline{OS}$	0.2

By substituting this parameter in equation, $\theta 1, \theta 2, \theta 3, \dot{\theta} 1, \dot{\theta} 2, and\ \dot{\theta} 3$ can be obtained. Displacement of point T can be obtained . Range of $\theta 3$ depends on size of planetary gear and distance between O and S. It is shown in fig.

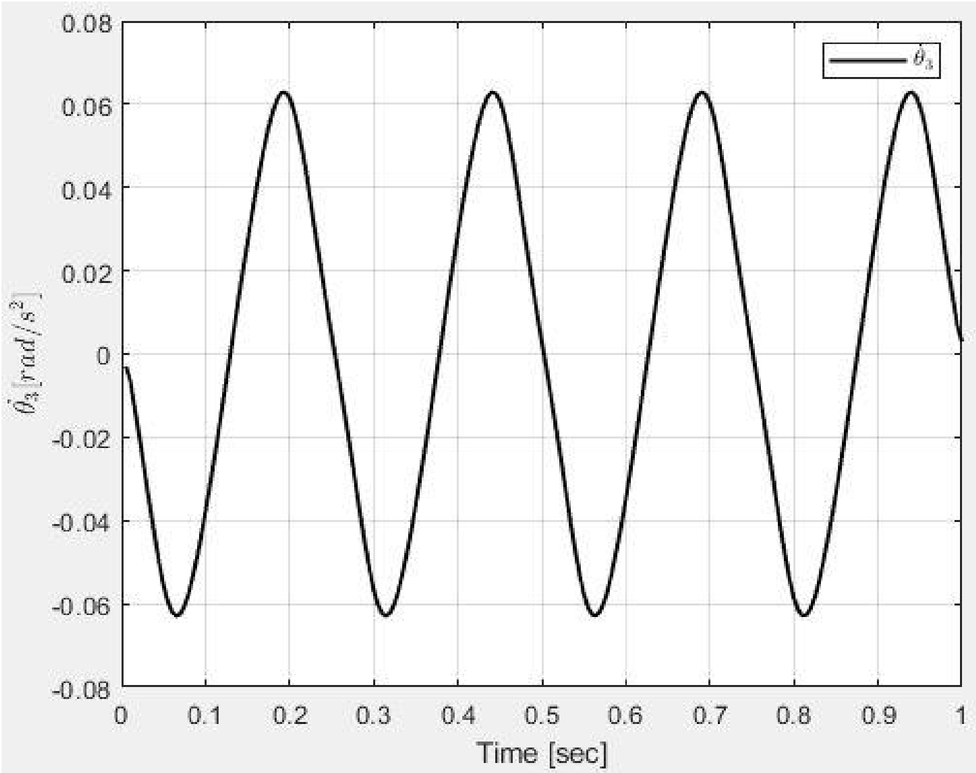


Figure11. Angular speed of link ST

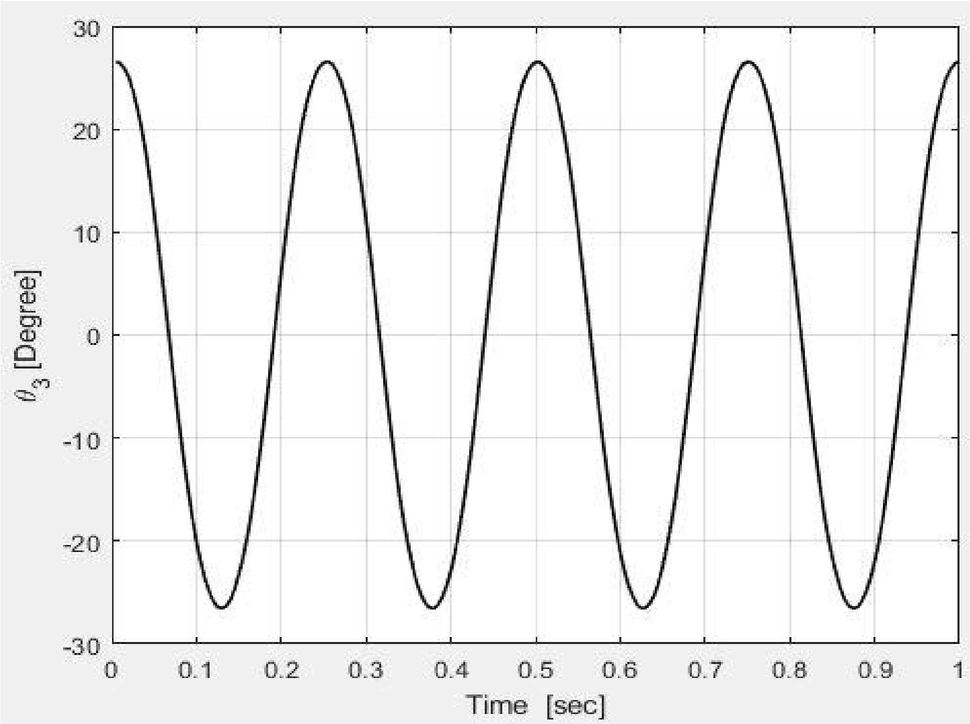


Figure12. Angular motion of link ST

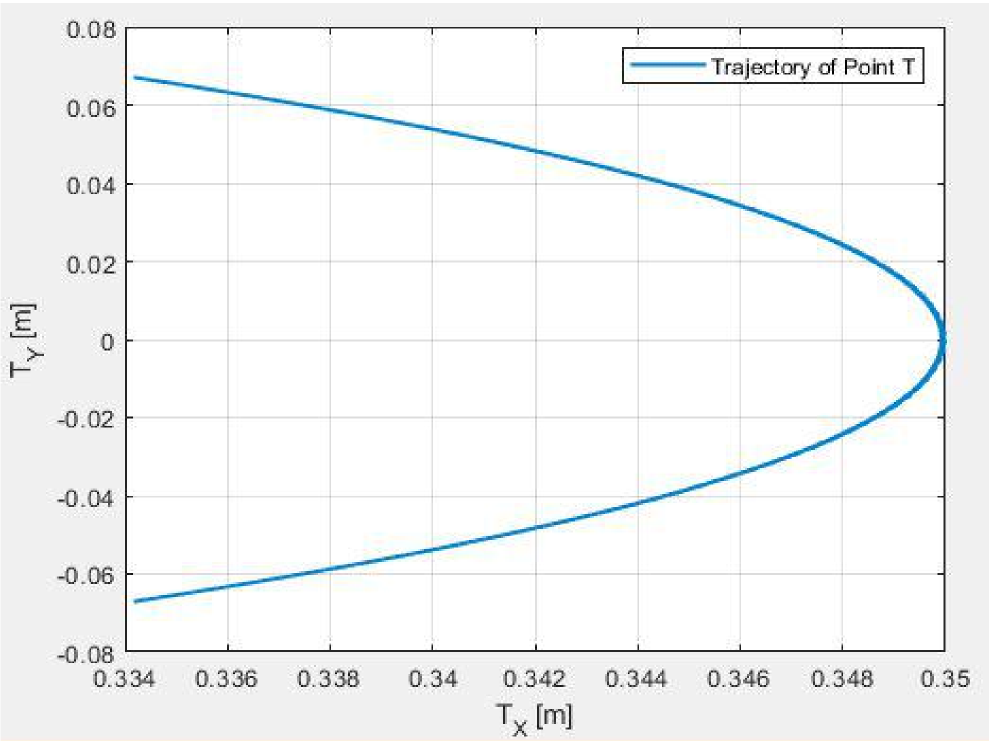


Figure13. Displacement of point T

How the swing angle of ST varies with distance between O and S is plotted below.

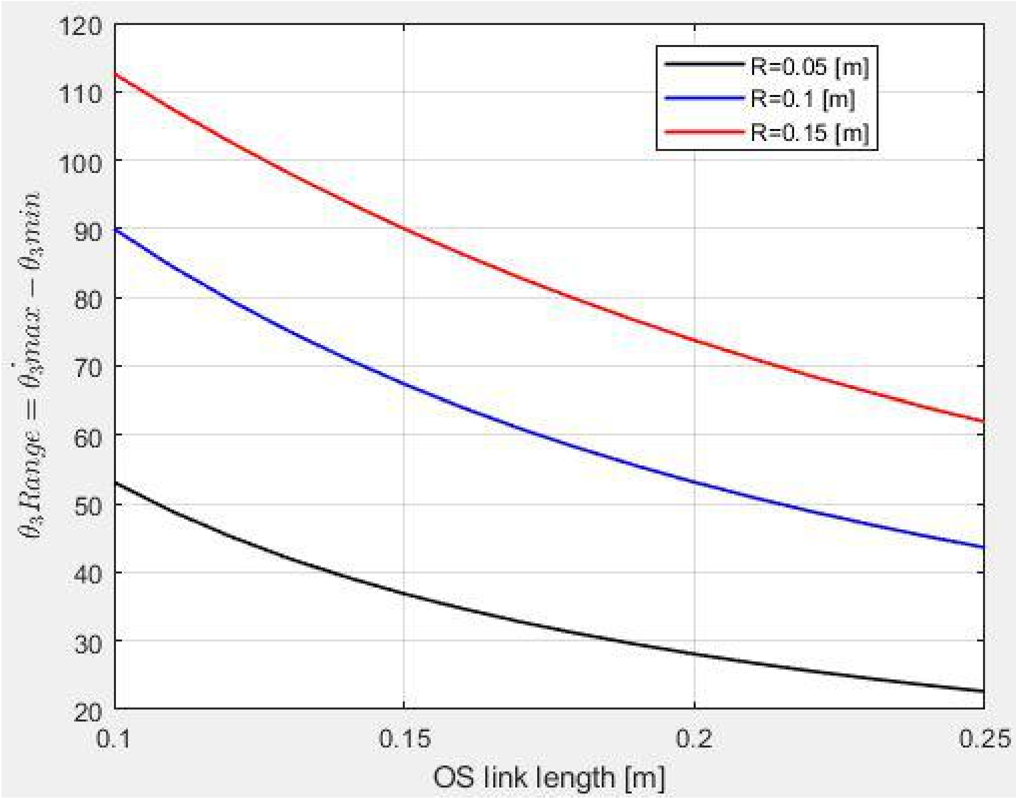


Figure14. Range of swing angle of link ST

Results for Second Mechanism

DC motor is operating at angular speed of 4 rad/s.

Part	Length [m]
Radius: Planetary Gear	0.05
Ring Gear	0.1
$\overline{PR}$	0.1
$\overline{RS}$	0.234
$\overline{SW}$	0.16
$\overline{AV}$	0.24
$\overline{VT}$	0.2
$\overline{VW}$	0.06

To know the velocity and displacement of point T, Kinematics analysis has been done. There is four equation with four unknown and all these four unknown can be found in terms of θ_3 . One of the interesting results is that θ_5 is proportional to θ_3 .

Relationship between angular speed of motor and θ_3 . Range of swing angle θ_3 depends on distance between point O ans point S. Displacement of point T is plotted w.r.t reference frame O. θ_3 and θ_5 is plotted on same graph showing relation between them.

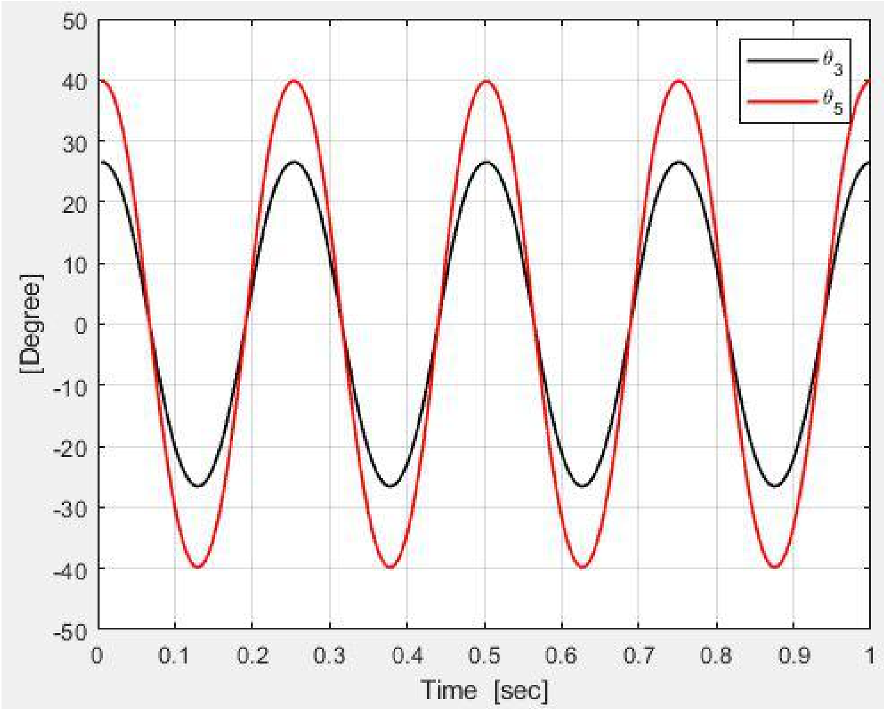


Figure15. Angular motion of link SW and VT

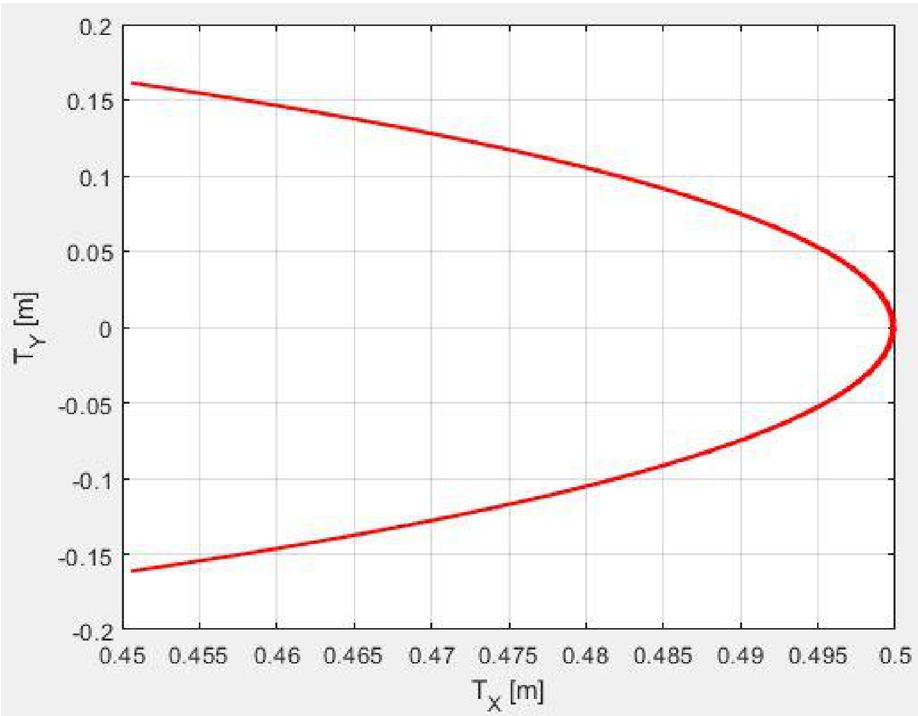


Figure16. Displacement of point T

Comparative study

Angular motion of tail fin (ST) is shown in Figure12 and Figure15. Motion can be perceived as sinusoidal and range of swing angle can be decreased and increased as per the desire and it resembles to the motion of tail fin of static live fish [11].
Angular displacement of tail fin of first and second mechanism is similar to that that tail fin motion of live static fish. So these two mechanism are able to generate desired thrust to propels itself and produce fish like motion.

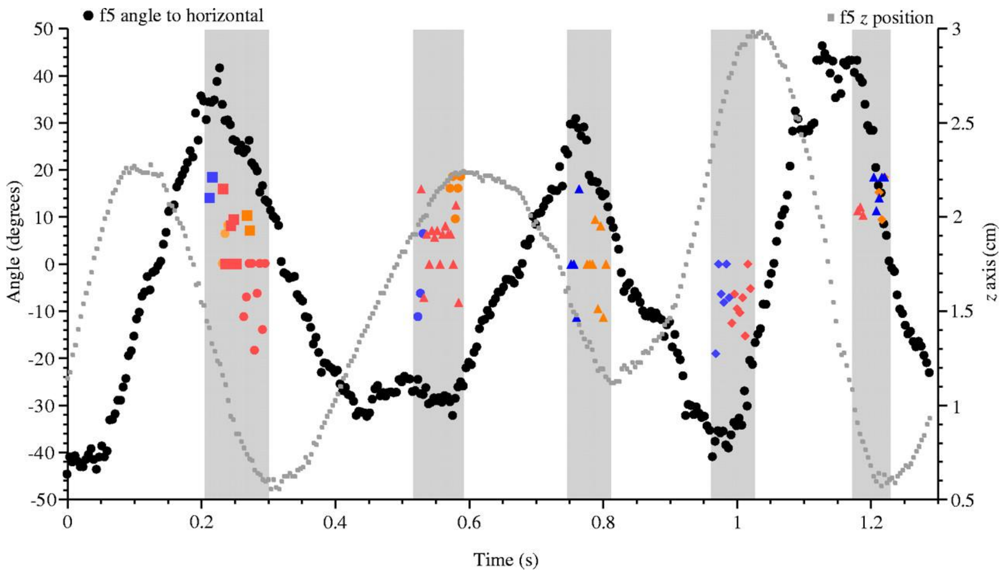


Figure17. Motion of end point of tail fin of static live fish [11]

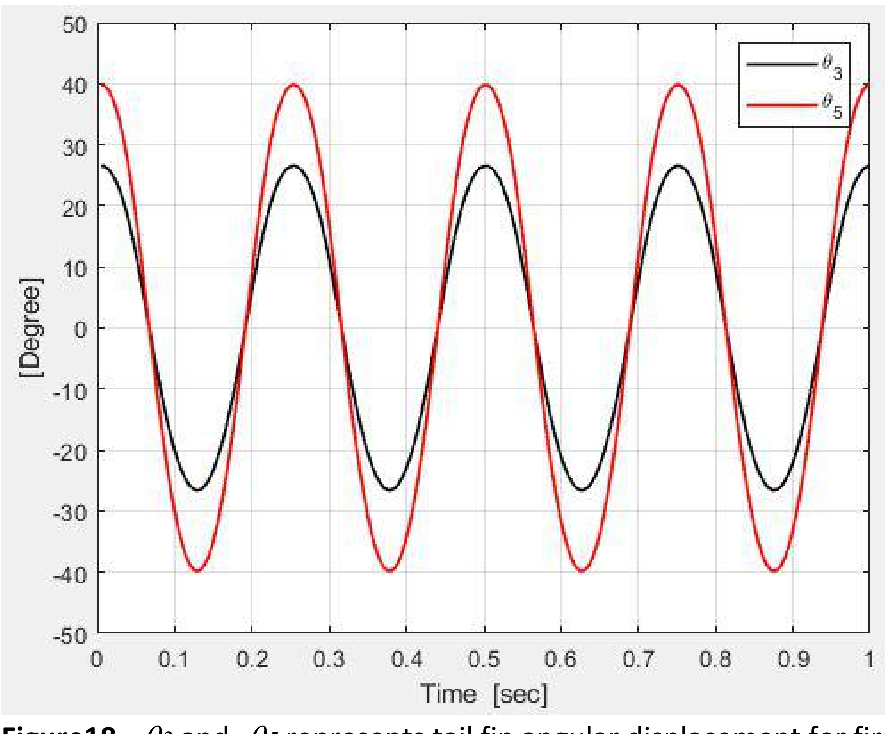


Figure18. θ_3 and θ_5 represents tail fin angular displacement for first and second mechanism respectively.

11. CONCLUSION

In this report, two mechanism for tail driving propulsion system is mathematically modeled. Both mechanism are based on similar hypocycloid principle, that converts rotary motion of motor into linear motion, further this linear motion is converted into oscillatory motion of the link as link RS in the case of first mechanism and second mechanism. First mechanism propels itself by using oscillatory motion of the tail fin and second mechanism propels itself using undulatory motion of the tail fin as it generates wavy like motion by using link SW, AV and VT. Both mechanism are being actuated by using DC motor. At first, kinematics analysis of both mechanism have been done. Optimization for structural part has also been carried out that ensured that both tail part generating undulatory and oscillatory motion sweeps same angle about XX' axis so that lateral forces cancel out and longitudinal forces add up to help the system to move forward.

Range of angle that tail part sweeps can be increased or decreased as per the requirement by tuning distance between O and S.

We have not consider the computational fluid dynamics and drag and lift forces, so dynamics analysis remains to complete yet. Future work is to make a real model of this mechanism and to establish the control part for 2D motion generation.

12. APPENDIX

Matlab code for analysis and plot :

```
clc
clear;
num_points=200;
Time=1;
const = 2;
R=5*const;
omega=2*pi;
t = linspace(0,Time,numpoints);
OP = R*cos(omega*4*t);
x1=0; y1 =0;

x2=10*const; y2 =OP;
r2 = 5*const;
OS=x2-x1;

PR = r2;
h=1;
RS = sqrt((PR^2-OS^2) -(( R^2)/(1-sqrt(1+((R/OS)^2)))));

for i = 1:length(OP)
[xout,yout] = circcirc(x1,y1,RS,x2,y2(i),r2);
if yout(1)>=yout(2)
s(h) = xout(1);
p(h) = yout(1) ;
else
s(h) = xout(2);
p(h) =yout(2) ;
end

h=h+1;
end
save data.mat s p OP
```

```
DELAY = 0.01;
numPoints = 600;
x = s;
y = p;

%# plot graph
figure('DoubleBuffer','on')                    %# no flickering
plot(x,y, 'LineWidth',2), grid on
xlabel('x'), ylabel('y'), title('angle')
xlim([min(x)-0.5 max(x)+.5])
ylim([min(y)-0.5 max(y)+0.5])


hLine = line('XData',x(1), 'YData',y(1), 'Color','r', ...
    'Marker','o', 'MarkerSize',6, 'LineWidth',2);
hTxt = text(x(1), y(1), sprintf('(%.3f,%.3f)',x(1),y(1)), ...
    'Color',[0.2 0.2 0.2], 'FontSize',8, ...
    'HorizontalAlignment','left', 'VerticalAlignment','top');


i = 1;
while true
    %# update point & text
    set(hLine, 'XData',x(i), 'YData',y(i))
    set(hTxt, 'Position',[x(i) y(i)], ...
        'String',sprintf('(%.3f,%.3f)',[x(i) y(i)]))
    drawnow                                    %# force refresh
    pause(.1)                             %# slow down animation

    i = rem(i+1,numPoints)+1;                  %# circular increment
    if ~ishandle(hLine), break; end            %# in case you close the figure
end
w1 = max(s);
w2 = min(p);
w3 = atan((p-y1)./(s-x1))*180/pi;
w4 = min(w3);
w5 = max(w3);
w6 = w5-w4;
avg_angle = (w4+w5)/2;
tail_angle = w3-avg_angle;
figure ;
c = gradient(tail_angle)*pi/180;

t1 = 1:length(c);
plot(t1/200,c,'Linewidth',1.5,'color',[0 0 0])

ylabel('$\dot{\theta_3 }[rad/s^2]$', 'Interpreter','latex')
xlabel('Time [sec]')
  hLeg = legend('$$\dot{\theta}_{3}$$');
 set(hLeg,'Interpreter','latex');
grid on;
figure ;
ylim([-90 90])

t2 = 1:length(tail_angle);
plot(t2/200,tail_angle,'Linewidth',1.25,'color',[0 0 0])
ylabel('\theta_3 [Degree]')
xlabel('Time  [sec]')
grid on;
title('')

figure;
angle = tail_angle*pi/180;
```

```
plot((((15)/100)*cos(angle))+(OS/100),(15/100)*sin(angle),'Linewidth',1.5,'color',[0 .5 .8]);
xlabel('T_X [m]')
ylabel('T_Y [m]')
grid on;
R = R/100;
w3_rad = w3*pi/180;
OS = OS/100;
RS = RS/100;
ST = 15/100;
OP =OP/100;
% co-ordinate of M
M = [(OS+RS*cos(w3_rad))/2; (OP+RS*sin(w3_rad))/2];
% co-ordinate of N
N = [OS+(RS*cos(w3_rad)/2); RS*sin(w3_rad)];
% co-ordinate of N
K = [OS+(ST*cos(angle)/2); ST*sin(angle)];

Z1= [(R/2)*cos(omega*4*t) ; (R/2)*sin(omega*4*t)];
Z2 = Z1/2;
 %   Masses of link and gear
m_bgear = 80.46/1000; %mass of ring gear
m_sgear = 67.76/1000;  %mass of spur gear
m_tail = 89.1/1000;      %mass of tail fin
m_slink = 2.6/1000;      %mass of link PR
m_blink =  6.1/1000;     %mass of link RS
m_mgearlink = 2.6/1000;  %mass of link that join motor with spur gear center

center_mass =0*m_bgear+Z1*m_sgear+Z2*m_mgearlink +M*m_slink+N*m_blink....
                  +K*m_tail;

centermass = center_mass/(m_mgearlink +m_blink+m_slink+m_tail+m_sgear+m_bgear);
```


```
center_mass =0*m_bgear+Z1*m_sgear+Z2*m_mgearlink +M*m_slink+N*m_blink....
                  +K*m_tail;

centermass = center_mass/(m_mgearlink +m_blink+m_slink+m_tail+m_sgear+m_bgear);
```


```
figure;
plot(centermass(1,:),centermass(2,:));
xlabel('X center mass [m]')
ylabel('Y center mass  [m]')
title('movement of center of mass')


function [] = range_angle(a_max,k)
e = (10:1:25)/100; %meter

theta = asin(e./sqrt(((e.^2)+a_max^2)));
range = pi-2*theta;
range =range*180/pi;
plot(e,range,'color',k,'Linewidth',1.25);
ylabel('$\dot{\theta_3 Range=\theta_3max- \theta_3min}$', 'Interpreter','latex')
xlabel('OS link length [m]');

grid on;
```

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