Intro Real Analysis

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3.1 Definition of Metric Spaces. Examples

Definition 3.1 (Metric Space)**.** A Metric Space is an ordered pair (E, d) where E is some set along with a metric function $d\colon E \times E \to \mathbb{R}$ such that:

1. **Identity of Indiscernibles:** $d(p, q) = 0 \Leftrightarrow p = q, \forall p, q \in E$
2. **Symmetry:** $d(p, q) = d(q, p), \forall p, q \in E$
3. **Subadditivity:** $d(p, q) \leq d(p, r) + d(r, q), \forall p, q, r \in E$

Theorem 3.1. *Metric functions are non-negative*

Proof. Let d be a a metric function with points p,q $\in$ E. So,

$$\begin{aligned}
d(p,p) &\leq d(p,q) + d(q,p). && \text{(Subadditivity)}\\
d(p,p) &\leq d(p,q) + d(p,q). && \text{(Symmetry)}\\
d(p,p) &\leq 2 \cdot d(p,q).\\
0 &\leq 2 \cdot d(p,q). && \text{(Identity of Indiscernibles)}\\
0 &\leq d(p,q). && \square
\end{aligned}$$