Semester 1 Final Review (Algebra II)

Instructions

Complete within 60 minutes. Default to exact forms (fractions, radicals, π, i) unless a decimal is requested. Circle one choice for each item.

Problems

1. All possible rational roots of $f(x) = 18x^4 - 7x^3 - 76x^2 + 41x + 30$ are:
   (A) $\{\pm1, \pm2, \pm3, \pm6, \pm9, \pm18\}$ (B) add halves only (C) $\{\pm1, \pm2, \pm3, \pm6, \pm9, \pm18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{5},$
   (D) missing fifths (E) missing tenths

2. Zeros of $g(x) = 3x^3 - 11x^2 - 14x + 48$ and its factorization:
   (A) $x = -1,\ 3,\ \frac{5}{2}$; $(x+1)(x-3)(2x-5)$ (B) $x = 1, 3, \frac{5}{2}$ (C) $x = -1, -3, \frac{5}{2}$ (D) $x = -1, 3, -\frac{5}{2}$ (E) $x = 1, -3, -\frac{5}{2}$

3. Factor $h(x) = 2x^4 - 7x^3 - 19x^2 + 56x - 24$ completely:
   (A) $(x-3)(x-2)(2x^2+x-4)$ (B) $(x-2)(x+2)(2x^2-5x+6)$ (C) $(x-3)(x+1)(2x^2-3x-8)$ (D) $(x-3)(x-1)(2x^2-3x-8)$ (E) $(x-2)^2(2x^2-3x-6)$

4. $\dfrac{7x^5 - 13x^4 + 9x^3 - 5x^2 + 16x - 20}{x-3} =$
   (A) $5x^4 - 12x^3 + 31x^2 - 76x + 177 + \dfrac{-365}{x+2}$ (B) $5x^4 - 12x^3 + 29x^2 - 80x + 169 + \dfrac{-341}{x+2}$
   (C) $5x^4 - 10x^3 + 31x^2 - 76x + 177 + \dfrac{-341}{x+2}$ (D) $5x^4 - 12x^3 + 31x^2 - 70x + 149 + \dfrac{-365}{x+2}$
   (E) $5x^4 - 14x^3 + 35x^2 - 82x + 181 + \dfrac{-365}{x+2}$

5. Factor $p(x) = x^4 - 6x^3 + 29x^2 - 66x + 65$ given root $2 + 3i$:
   (A) $(x^2-6x+13)(x^2-4x+5)$ (B) $(x^2-6x+13)(x^2+4x+5)$ (C) $(x^2+6x+13)(x^2-4x+5)$
   (D) $(x^2-6x+5)(x^2-4x+13)$ (E) $(x^2-6x+13)(x^2-2x+1)$

6. Simplify $\dfrac{7-5i}{2+i}$:
   (A) $2+i$ (B) $-2+i$ (C) $2-i$ (D) $-2-i$ (E) $1+2i$

7. Vertex of $y=-5x^2+30x-11$:
   (A) $(3,34)$ max (B) $(3,-34)$ min (C) $(-3,34)$ max (D) $(3,-11)$ max (E) $(-3,-11)$ min

8. Factor $q(x)=8x^3-26x^2-7x+30$:
   (A) $(x-3)(2x+3)(4x-5)$ (B) $(x-3)(2x-3)(4x+5)$ (C) $(x+3)(2x-3)(4x-5)$
   (D) $(x+3)(2x+3)(4x-5)$ (E) $(x-3)(2x-3)(4x+5)$

9. Factor $r(x)=x^3-7x^2-6x+72$:
   (A) $(x-3)(x-6)(x+4)$ (B) $(x-3)(x-6)(x-4)$ (C) $(x+3)(x-6)(x-4)$
   (D) $(x-3)(x+6)(x-4)$ (E) $(x+3)(x+6)(x-4)$

10. Solve $x^{7/3}=81$ for $x\geq 0$:
    (A) $x=9$ (B) $x=27$ (C) $x=3\sqrt[3]{9}$ (D) $x=\sqrt[3]{81}$ (E) $x=243$

11. Revenue with \$2.50 drops (variable k):
    (A) $R(k)=-45k^2+600k+6000$; max $k=\dfrac{20}{3}$; price \$33.33; revenue \$8,000
    (B) $R(k)=-45k^2+540k+6000$; max $k=6$; price \$35.00; revenue \$8,160
    (C) $R(k)=-50k^2+600k+6000$; max $k=6$; price \$35.00; revenue \$8,400
    (D) $R(k)=-45k^2+600k+6000$; max $k=6$; price \$35.00; revenue \$8,100
    (E) $R(k)=-40k^2+500k+6000$; max $k=6.25$; price \$34.38; revenue \$8,125

12. For $s(x)=x^4-7x^3-2x^2+56x-72$, dividing by $x-4$ then factoring gives:
    (A) $(x-4)(x-2)(x^2-9)$ (B) $(x-4)(x+2)(x^2-9)$ (C) $(x-4)(x-2)(x^2+9)$
    (D) $(x+4)(x-2)(x^2+9)$ (E) $(x+4)(x+2)(x^2-9)$

Solutions (Multiple Choice)

1. C
2. D
3. C
4. A
5. A
6. B
7. A
8. A
9. A
10. D
11. A
12. A

Formulas and Theorems

- Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; discriminant $b^2 - 4ac$ sets root type.
- Vertex of $y = ax^2 + bx + c$: $x = -\dfrac{b}{2a}$, $y = f\left(-\dfrac{b}{2a}\right)$; $a < 0$ gives a maximum, $a > 0$ gives a minimum.
- Rational Root Theorem: any rational root of $a_n x^n + \cdots + a_0$ is $\pm\dfrac{\text{factor of } a_0}{\text{factor of } a_n}$.
- Conjugate Root Theorem: non-real complex roots of polynomials with real coefficients occur in conjugate pairs $a \pm bi$.
- Remainder/Factor connection: dividing $f(x)$ by $(x-c)$ leaves remainder $f(c)$; if $f(c) = 0$, $(x - c)$ is a factor.
- Polynomial long/synthetic division: $f(x) = (x - c)q(x) + r$ where $r = f(c)$.
- Factoring patterns: $A^2 - B^2 = (A - B)(A + B)$; $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$.
- Exponent rules (for $a \neq 0$): $a^m a^n = a^{m+n}$, $\dfrac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$, $a^{-m} = \dfrac{1}{a^m}$.
- Absolute values with even roots: $\sqrt{x^2} = |x|$; include $|\cdot|$ unless the variable is restricted positive.
- Revenue/optimization: for quadratic $R(k) = ak^2 + bk + c$ with $a < 0$, maximum at $k = -\dfrac{b}{2a}$.