# Calculus 1 Practice: Mixed Review

Instructions

Work without a calculator unless otherwise stated. Complete within 60 minutes. Unless a decimal is requested, give exact values (e.g., π, radicals, fractions). For free-response items, show setup and key steps.

Problems

1. A particle moves on the x-axis with velocity $v(t)$ shown below. At which times does the particle change direction?
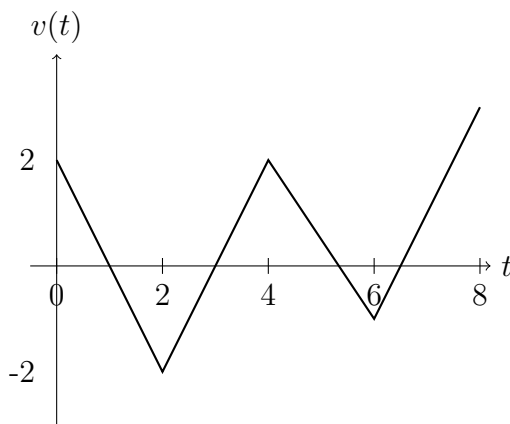
   (A) $t = 1, 3$ (B) $t = 1, 3, \frac{16}{3}$ (C) $t = 1, 3, \frac{16}{3}, \frac{13}{2}$ (D) $t = 3, \frac{16}{3}$ (E) never

2. If $f(x) = \ln(x^2 + 1)$ and $g(x) = \sin(e^x)$, find $\dfrac{d}{dx}\big(g(f(x))\big)$ at $x = 1$.
   (A) $\cos(2)$ (B) $2\cos(2)$ (C) $2\sin(2)$ (D) $e\cos(2)$ (E) $2e\cos(2)$

3. The graph of $y = f(x)$ passes through $(1, -2)$ with $f'(1) = 5$. Find the tangent line to $y = x^2 f(x)$ at $x = 1$.
   (A) $y = -x - 1$ (B) $y = x - 3$ (C) $y = 3x - 7$ (D) $y = 5x - 13$ (E) $y = 7x - 17$

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4. For $x^2 + xy + y^2 = 7$, find $\dfrac{d^2y}{dx^2}$ at the point $(1, 2)$.

(A) $-\dfrac{6}{25}$ (B) $-\dfrac{42}{125}$ (C) $-\dfrac{4}{25}$ (D) $\dfrac{6}{25}$ (E) $\dfrac{42}{125}$

5. If $f(1) = 4$, $f'(1) = \dfrac{1}{2}$, and $f^{-1}(4) = 1$, find $(f^{-1})'(4)$.

(A) $-\dfrac{1}{4}$ (B) $\dfrac{1}{8}$ (C) $\dfrac{1}{2}$ (D) 2 (E) 8

6. For $h(x) = x^4 - 4x^2$, at what x does the graph have a point of inflection?

(A) $x = 0$ (B) $x = \pm 1$ (C) $x = \pm\sqrt{\tfrac{2}{3}}$ (D) $x = \pm\sqrt{2}$ (E) None

7. Consider the piecewise function $p(x) = \begin{cases} -x + 1, & x < 1 \\ \dfrac{x^2 - 1}{x - 1}, & x > 1 \end{cases}$. What is the right-hand derivative at $x = 1$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) Does not exist

8. The graph of f has $f(2) = 1$, $f'(2) = -3$, $g(2) = -2$, and $g'(2) = 4$. Find $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right)$ at $x = 2$.

(A) $-\dfrac{3}{2}$ (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) 1 (E) $\dfrac{3}{2}$

9. Evaluate $\lim\limits_{x \to 0} \dfrac{e^{3x} - 1 - 3x}{x^2}$.

(A) $\dfrac{9}{2}$ (B) 3 (C) $\dfrac{3}{2}$ (D) 0 (E) Diverges

10. Find $\dfrac{d}{dx}\left[\arccos\left(\dfrac{1}{x^2}\right)\right]$ for $x > 1$.

(A) $-\dfrac{2}{x^3\sqrt{1 - \frac{1}{x^4}}}$ (B) $\dfrac{2}{x^3\sqrt{1 - \frac{1}{x^4}}}$ (C) $\dfrac{1}{x^2\sqrt{1 - \frac{1}{x^4}}}$ (D) $-\dfrac{1}{x^2\sqrt{1 - \frac{1}{x^4}}}$ (E) $\dfrac{1}{\sqrt{1 - \frac{1}{x^4}}}$

11. In a right triangle with hypotenuse 13, side x is opposite angle θ. If θ increases at 0.02 rad/min when $x = 5$, what is $\dfrac{dx}{dt}$?

(A) $\dfrac{3}{25}$ (B) $\dfrac{6}{25}$ (C) $\dfrac{12}{25}$ (D) $\dfrac{18}{25}$ (E) $\dfrac{24}{25}$

12. A conical tank has radius $r = \dfrac{h}{3}$. If $\dfrac{dh}{dt} = 2$ ft/min when $h = 9$, what is $\dfrac{dV}{dt}$? (Volume $V = \tfrac{1}{3}\pi r^2 h$.)

(A) 12π (B) 18π (C) 24π (D) 30π (E) 36π

13. Solve for x: $\ln(x-1) + \ln(x+3) = \ln 9$.
    (A) 2   (B) 3   (C) 4   (D) 5   (E) No real solution

14. A curve is defined implicitly by $x^2 y + y^3 = 2$. At $(1,1)$, find $\dfrac{dy}{dx}$.
    (A) -1 (B) $-\dfrac{1}{2}$ (C) 0   (D) $\dfrac{1}{2}$ (E) 1

15. Use linearization of $f(x) = \sqrt{4+x}$ at $x = 0$ to approximate $\sqrt{4.09}$.
    (A) 2.00   (B) 2.015   (C) 2.0225   (D) 2.045   (E) 2.09

16. If $f(x) = x^3$ on $[0,2]$, which c satisfies the Mean Value Theorem?
    (A) $c = \dfrac{1}{\sqrt{3}}$ (B) $c = \dfrac{2}{\sqrt{3}}$ (C) $c = 1$ (D) $c = \sqrt{2}$ (E) $c = \dfrac{3}{2}$

17. A rectangle is inscribed under $y = 16 - x^4$ in the first quadrant with one vertex at the origin and one on the curve. What x gives maximum area?
    (A) $x = 1$ (B) $x = 2^{1/4}$ (C) $x = \left(\dfrac{16}{3}\right)^{1/4}$ (D) $x = \left(\dfrac{16}{5}\right)^{1/4}$ (E) $x = 3^{1/4}$

18. Given $f'(x) > 0$ and $f''(x) < 0$ for all x, which statement is true about f?
    (A) Increasing and concave up   (B) Increasing and concave down   (C) Decreasing and concave up   (D) Decreasing and concave down   (E) Not enough information

19. One Newton iteration for $F(x) = x^3 - 6x + 5$ starting at $x_0 = 2$ gives $x_1 = ?$
    (A) $\dfrac{11}{6}$ (B) 2   (C) $\dfrac{7}{3}$ (D) 3   (E) $\dfrac{11}{3}$

20. For $r(x) = \begin{cases} x^3 - 8, & x < 2 \\ 4x - 8, & x \geq 2 \end{cases}$, determine continuity and differentiability at $x = 2$.
    (A) Continuous and differentiable   (B) Continuous, not differentiable   (C) Differentiable, not continuous   (D) Neither   (E) Limit does not exist

Answers

1. C

2. B

3. B

4. B

5. D

6. C

7. B

8. C

9. A

10. B

11. B

12. B

13. E

14. B

15. C

16. B

17. D

18. B

19. A

20. B