Calculus 1 Practice: Limits and Derivatives

Instructions

Select the best answer for each. Use exact values when possible. Unless stated otherwise, assume functions are real-valued.

Problems

1. Piecewise limit: $f(x) = \begin{cases} x^2 - 4, & x < 2 \\ \dfrac{\sin(ax)}{x-2}, & x \geq 2 \end{cases}$. Choose a so $\lim_{x \to 2} f(x)$ exists, and the limit value.

   (A) $a = 0$, limit 0
   (B) $a = \frac{\pi}{4}$, limit 0
   (C) $a = \frac{\pi}{2}$, limit -1
   (D) $a = \frac{\pi}{4}$, limit -2

2. Piecewise continuity/differentiability: $g(x) = \begin{cases} bx + 1, & x \leq 0 \\ x^2 + cx, & x > 0 \end{cases}$.

   (A) $b = 1, c = 0$
   (B) $b = 0, c = 1$
   (C) $b = 1, c = -1$
   (D) No real b, c give both continuity and differentiability at 0

3. Differentiability at 0 for $h(x) = \dfrac{x}{1 + |x|}$.

   (A) Continuous and differentiable; $h'(0) = 1$
   (B) Continuous but not differentiable
   (C) Not continuous at 0
   (D) Differentiable with $h'(0) = 0$

4. Evaluate $\lim_{x \to 0} \dfrac{1 - \cos(3x)}{x^2}$.

   (A) 0
   (B) $\frac{9}{2}$

(C) $\frac{3}{2}$
(D) 9

5. For $x^2 + xy + y^2 = 7$, find $\dfrac{dy}{dx}$ at $(1, 2)$.

(A) $-\frac{5}{4}$
(B) $-\frac{4}{5}$
(C) $-\frac{1}{2}$
(D) $-\frac{2}{5}$

6. Related rates (cone): $r = \frac{1}{3}h$, $\dfrac{dV}{dt} = 10$ at $h = 12$. Find $\dfrac{dh}{dt}$.

(A) $\frac{15}{\pi}$ cm/s
(B) $\frac{5}{3\pi}$ cm/s
(C) $\frac{15}{2\pi}$ cm/s
(D) $\frac{5}{\pi}$ cm/s

7. Related rates (ladder): 15 ft ladder, bottom sliding at 2 ft/s when $x = 9$. Rate of top sliding down?

(A) $-\frac{6}{5}$ ft/s
(B) $-\frac{9}{5}$ ft/s
(C) $-\frac{4}{3}$ ft/s
(D) $-\frac{10}{3}$ ft/s

8. Optimization (open cylinder): Express $S(r)$ for fixed V, minimize. What is optimal r?

(A) $r = \left(\dfrac{V}{2\pi}\right)^{1/3}$
(B) $r = \left(\dfrac{V}{\pi}\right)^{1/3}$
(C) $r = \left(\dfrac{V}{4\pi}\right)^{1/3}$
(D) $r = \left(\dfrac{3V}{2\pi}\right)^{1/3}$

9. Optimization (cone in cylinder): Cone in cylinder radius R, height H. Height maximizing cone volume?

(A) $h = \dfrac{H}{3}$
(B) $h = \dfrac{H}{2}$
(C) $h = \dfrac{2H}{3}$
(D) $h = H$

10. For $f(x) = x^4 - 4x^3 + 6x^2$, critical points and classifications?
   (A) $x = 0$ (min), $x = 2$ (min)
   (B) $x = 0$ (max), $x = 2$ (min)
   (C) $x = 0$ (inflection), $x = 2$ (min)
   (D) $x = 2$ only (min)

11. MVT for $p(x) = x + \dfrac{1}{x}$ on $[1, 4]$: choose a valid c.
   (A) $c = \sqrt{2}$
   (B) $c = \sqrt{3}$
   (C) $c = \sqrt{5}$
   (D) $c = 2$

12. Evaluate $\displaystyle\lim_{x \to 0^+} \left(\frac{\ln x}{x} \right)^x$.
   (A) 1
   (B) 0
   (C) $\frac{1}{e}$
   (D) e

13. Chain rule: $\dfrac{d}{dx} \left[\sin((3x^2 - 1)^4) \right]$ at $x = 1$.
   (A) $96 \cos(4)$
   (B) $48 \cos(4)$
   (C) $96 \sin(4)$
   (D) $48 \sin(4)$

14. Implicit second derivative for $x^2 + xy + y^2 = 7$ at $(1, 2)$.
   (A) $-\frac{41}{50}$
   (B) $-\frac{9}{25}$
   (C) $-\frac{23}{50}$
   (D) $-\frac{7}{10}$

15. Linearization of $\sqrt{1 + 3x}$ at 0; approximation for $\sqrt{1.06}$.
   (A) $\sqrt{1.06} \approx 1.03$
   (B) ≈ 1.09
   (C) ≈ 1.06
   (D) ≈ 1.01

16. MVT for $f(x) = x^{3/2}$ on $[1, 4]$: which c works?
   (A) $c = \dfrac{4}{\sqrt{3}}$

(B) $c = \dfrac{9}{4}$
(C) $c = \dfrac{25}{9}$
(D) $c = \dfrac{7}{3}$

17. Log differentiation: Find $\dfrac{dy}{dx}$ if $y = x^{x^2}(\sin x)^3$.
(A) $x^{x^2}(\sin x)^3\,(2x\ln x + x + 3\cot x)$
(B) $x^{x^2}(\sin x)^3\,(2x\ln x + 3\tan x)$
(C) $x^{x^2}(\sin x)^3\left(2x\ln x + \dfrac{3}{\sin x}\right)$
(D) $x^{x^2}(\sin x)^3\,(2x + 3\cos x)$

18. Rectangle under $y = 9 - x^2$ in first quadrant; dimensions maximizing area.
(A) $x = 2$, $y = 5$
(B) $x = 3$, $y = 0$
(C) $x = 3$, $y = 0$
(D) $x = 2$, $y = 9$

19. Curve analysis for $f(x) = \dfrac{x}{x^2+1}$: intervals of increase/decrease.
(A) Increasing on $(-\infty, -1) \cup (1, \infty)$
(B) Increasing on $(-1, 1)$
(C) Increasing on $(-\infty, -1)$ only
(D) Increasing on $(0, \infty)$ only

20. Newton's method: one iteration for $f(x) = x^3 - 5x + 2$ from $x_0 = 1$.
(A) $x_1 = \frac{7}{2}$
(B) $x_1 = 0$
(C) $x_1 = \frac{1}{2}$
(D) $x_1 = 1$