

Polynomial and Complex Number Practice (Algebra II)

Instructions

Solve each problem. Show clear work. Aim to finish in 60 minutes or less. When applicable, leave answers in simplest exact form (no rounded decimals).

Problems

1. List all possible rational roots of $f(x) = 6x^3 - x^2 - 25x + 10$.
2. Find every rational zero of $g(x) = 2x^3 - 7x^2 - 8x + 24$ and factor $g(x)$ completely over the reals.
3. Factor $h(x) = x^4 - 8x^2 + 16$ completely over the reals.
4. Divide $4x^4 - 3x^3 + 7x^2 - 5x + 11$ by $x - 3$, and write the result in the form (quotient) + (remainder)/(divisor).
5. Given $p(x) = x^4 - x^3 - 3x^2 + 55x - 52$ has a root at $x = 2 - 3i$, find all remaining roots and express $p(x)$ as a product of two real quadratic factors.
6. Simplify $\frac{5 - 7i}{-2 + 3i}$ into $a + bi$ form.
7. For $y = -3x^2 + 18x + 41$, find the vertex, classify it as a maximum or minimum, and state that extreme value.
8. Factor $q(x) = 2x^3 - x^2 - 22x - 24$ completely over the rationals.
9. Determine whether $(x - 3)$ is a factor of $r(x) = 2x^4 - 13x^3 + 26x^2 - 3x - 36$. If it is, divide to factor once; if not, give the remainder.
10. For $s(x) = 5x^3 - 9x^2 - 11x + 18$, (a) list possible rational roots, (b) find one actual rational root, (c) use it to factor $s(x)$ completely over the reals.
11. A theater charges \$42 per ticket and sells 180 tickets. For each \$3.50 decrease in price, it sells 30 more tickets. (a) Write revenue $R(p)$ as a function of ticket price p . (b) Find the price that maximizes revenue and the corresponding maximum revenue.
12. (a) Divide $x^4 - 5x^3 + 6x^2 + 9x - 18$ by $x + 1$. (b) Using the result, factor the polynomial completely over the reals if possible. (c) State all real and complex roots in $a + bi$ form.