Teaser: Divisibility by 3

Context

Divisibility by 3: An integer is divisible by 3 if and only if the sum of its digits is divisible by 3.

Problems

1. Is 3471 divisible by 3?
2. Find the smallest positive integer n such that $52n$ (digits $5, 2, n$) is divisible by 3.
3. Replace the blank with a single digit to make the number divisible by 3: $8\,4\,5\,\square\,2$.
4. Among the numbers 10,204, 10,207, 10,212, and 10,218, which are divisible by 3?
5. A five-digit number has digits that sum to 21. What can you say about its divisibility by 3? Give an example of one such number and state whether it is divisible by 9.

Solutions

1. $3 + 4 + 7 + 1 = 15$, which is divisible by 3, so 3471 is divisible by 3.

2. Sum of digits $5 + 2 + n = 7 + n$ must be divisible by 3. The smallest $n \in \{0, \ldots, 9\}$ that works is $n = 2$ (sum 9). So 522 is divisible by 3.

3. Sum $8 + 4 + 5 + \square + 2 = 19 + \square$ must be divisible by 3. The valid digits are $2, 5, 8$ (making sums $21, 24, 27$). Any of those works; e.g., 84582 is divisible by 3.

4. Sums: $1 + 0 + 2 + 0 + 4 = 7$ (no), $1 + 0 + 2 + 0 + 7 = 10$ (no), $1 + 0 + 2 + 1 + 2 = 6$ (yes), $1 + 0 + 2 + 1 + 8 = 12$ (yes). So 10,212 and 10,218 are divisible by 3.

5. Sum 21 is divisible by 3, so any five-digit number with digit-sum 21 is divisible by 3. Example: 65235 ($6 + 5 + 2 + 3 + 5 = 21$) is divisible by 3 but not by 9 (since 21 is not a multiple of 9). No five-digit number with digit-sum 21 is divisible by 9.