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Wormhole in 5D Kaluza–Klein cosmology

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We present wormhole as a solution of Euclidean field equations as well as the solution of the Wheeler–deWitt (WD) equation satisfying Hawking–Page wormhole boundary conditions in (4+1)-dimensional Kaluza–Klein cosmology. The wormholes are considered in the cases of pure gravity, minimally coupled scalar (imaginary) field and with a positive cosmological constant assuming dynamical extra-dimensional space. In above cases, wormholes are allowed both from Euclidean field equations and WD equation. The dimensional reduction is possible.

Keywords: Classical and quantum wormholes; 5D Kaluza–Klein spacetime.

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1. Introduction

The quantum fluctuation of spacetime geometry in the early universe may give rise to the wormhole configuration. A wormhole is an Euclidean field configuration and as such, it is the solution of the Euclidean field equation in some field theory containing gravity, consisting of two asymptotic regions connected by a throat. Giddings and Strominger[1] first presented the wormhole solution introducing axion field minimally coupled to Einstein's gravity. Henceforth, wormholes were discussed in many theories,[2–12] namely in the theory of complex scalar field coupled to Einstein's gravity,[6,7] the scalar–tensor gravity theory,[8,9] the higher-dimensional theory.[11,13] Further consequences of wormhole in the evolution of the universe were discussed extensively.[12–25]

It appears that quantum coherence is not at all lost as the wormholes connect two asymptotic regions by a throat and the usual wormhole solution is accepted as representing tunneling between spaces with different topologies. The wormholes might provide information in understanding the evaporation of black holes.[12]

Further, the microscopic wormholes could provide a mechanism in vanishing the cosmological constant problem.[15,16,23] It appears that the wormholes regularize the values of the important parameters, hence to study the important parameters, the wormhole solution should admit not only with arbitrary matter field, but also in pure gravity. In view of the existence of wormholes, Cheeger and Glommol[26] state that a classical solution allows a wormhole when the eigenvalues of the Ricci tensor be negative somewhere on the manifold; hence pure gravity or a minimally coupled real scalar field does not allow wormhole solution. Existence of wormhole condition in Ref. 26 is necessary but not sufficient, since minimally coupled scalar field, conformal coupled scalar field and some other matter sources admit wormholes.[1,3–5]

Instead of considering wormhole simply as a solution of Euclidean field equation, Hawking and Page[24] assumed wormholes as a quantum mechanical object governed by Wheeler–deWitt (WD) equation. In fact, Hawking and Page proposal[24] is in perfect harmony with the Euclidean configuration allowed by classical field equation. Hawking and Page[24] proposed wormhole as the solution of the WD equation satisfying boundary conditions that the wave function Ψ is exponentially damped for large three geometries and is regular when three geometries collapse to zero. Thus, we may explore wormholes not only as a solution of the classical Euclidean field equations, but also as a solution of the WD equation satisfying above boundary condition.[27–39] Above results are true in $(1 + 3)$-dimensional spacetime. Assuming validity of above results in higher-dimensional spacetime (greater than $1 + 3$) we may explore wormhole configuration introducing wormhole boundary conditions of Hawking–Page[24] in higher-dimensional spacetime.

Shen You Gen *et al.*[39] presented an analytical solution of wormhole in Kaluza–Klein (KK) theory with spacetime topology $R^1 \otimes S^3 \otimes M^d$ in spacetime dimension $D = 1+3+d$, where d is the dimension of internal space. They assumed that the internal space is compact, static and the usual four-dimensional spacetime is evolving with time. Idea of static internal space in Ref. 39 is a mathematical simplification for solution of the field equations.

In our work, we explore possible wormhole configuration in five-dimensional (5D) KK cosmology assuming time-dependent extra-dimensional scale factor $b(\tau)$ with the usual scale factor $a(\tau)$ of external space. The viability of wormholes is considered both from the solution of Euclidean field equations and the solution of wave function Ψ from the WD equation satisfying Hawking–Page[24] boundary conditions for wormholes in three different cases. The extension of Hawking–Page boundary condition for wormholes in four-dimensional spacetime to 5D KK cosmology (with topology $R^1 \otimes S^3 \otimes S^1$) also yields wormhole configuration under certain restriction on operator ordering in higher dimension assuming minimally coupled scaler field. In Sec. 2, we present the field equations, while in Sec. 3, the solutions of the field equations are presented in three different cases. Further in Sec. 4, we present the solution of the WD equation in above cases.

Wormholes are allowed in pure gravity, with cosmological constant (as source term) and minimally coupled complex scalar field in KK cosmology both from the

Euclidean field equations and WD equation. The configuration of the wormhole from the Euclidean equation in cosmological constant is quite different from the usual one. In this case, one universe is connected periodically in Euclidean time with another one by the throat of radius $a_{\min}$ and the maximum size of the radius of the universe is $a_{\max}$. Further, the dimensional reduction of internal space is possible unless the radius of the throat of wormhole is of order of Planck length. In Sec. 5, we present a brief discussion.

2. Field Equations

We consider the topology of spacetime as $R^1 \otimes S^3 \otimes S^1$ and assume the spacetime metric is

$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3^2 + b^2(\tau)dy^2\,, \tag{1}$$

where $a(\tau)$ and $b(\tau)$ are scale factors of the external and internal spaces, respectively, $d\Omega_3$ is the metric on three-sphere, y is the space-like coordinate of internal space and τ is the Euclidean time. We consider the Euclidean action

$$S_E = \frac{1}{16\pi G_5}\int_M d^5x\,\sqrt{g}R - \int_M d^5x\,\sqrt{g}\left(\frac{1}{2}\phi_{;\mu}\phi^{;\mu} + V(\phi)\right) + S_{\text{sur}}\,, \tag{2}$$

where ϕ is the scalar field, which is function of τ only and $V(\phi)$ is the potential of the scalar field. The Lagrangian for above action is

$$L = aba'^2 + a^2a'b' + ab - ba^3\frac{K^2}{3}\left(\frac{1}{2}\phi'^2 + V(\phi)\right), \tag{3}$$

where $\frac{K^2}{6} = \frac{4\pi G_5}{3}$. Now the field equations are

$$\frac{2a''}{a} + \frac{a'^2}{a^2} + 2\frac{a'b'}{ab} + \frac{b''}{b} - \frac{1}{a^2} = -K^2\left(\frac{\phi'^2}{2} + V\right), \tag{4}$$

$$\frac{a''}{a} + \frac{a'^2}{a^2} - \frac{1}{a^2} = -\frac{K^2}{3}\left(\frac{\phi'^2}{2} + V\right), \tag{5}$$

$$\phi'' + \left(3\frac{a'}{a} + \frac{b'}{b}\right)\phi' = \frac{\partial V}{\partial\phi}\,, \tag{6}$$

further, the constraint equation is

$$\frac{a'^2}{a^2} + \frac{a'b'}{ab} - \frac{1}{a^2} - \frac{K^2}{3}\left(\frac{\phi'^2}{2} - V\right) = 0\,. \tag{7}$$

The solution of the field equations is not simple in general. Thus, we introduce some simplifying assumption in solving the field equations.

3. Solution of the Field Equation

The solution of the field equations is not trivial, so we consider some simplifying assumptions to obtain wormhole solution.

3.1. *Solution for pure gravity*

A simple solution representing wormhole can be obtained in pure gravity. Thus, from (5) and (7), we have

$$a^2 = \tau^2 + a_0^2 \qquad \text{and} \qquad b(\tau) = b_0 \frac{\tau}{\sqrt{\tau^2 + a_0^2}}\,, \tag{8}$$

where a_0 and b_0 are constant. It is important to note that the internal scale factor $b(\tau)$ converges to a finite value b_0. The constant a_0 is the radius at the throat of the wormhole, which is the order of Planck length. The dimensional reduction is possible if b_0 is order of Planck length.

3.2. *Solution for $V(\phi) = V_0$ and $\phi' = 0$*

To obtain the solution of the field equation assuming a constant V (let V_0) in the absence of the scalar field, we have from (5)

$$a^2 = \frac{12}{V_2} + B \sin\left(\sqrt{\frac{V_2}{6}}\tau + c\right) \tag{9}$$

and the internal scale factor $b(\tau)$ from (7) gives

$$b(\tau) = \exp\left(-\frac{552}{BV_2}\tanh^{-1}\left[\tan\left(\sqrt{\frac{V_2}{6}}\tau + c\right)\right]\right)$$

$$\times \cos^{24}\left(\sqrt{\frac{V_2}{6}}\tau + c\right) \Bigg/ \sqrt{\frac{12}{V_2} + B\sin\left(\sqrt{\frac{V_2}{6}}\tau + c\right)}, \tag{10}$$

where $V_2 = \frac{K^2 V_0}{3}$. B and c are integration constants. The scale factor $a(\tau)$ in (9) is periodic in Euclidean time with the period $\frac{6\pi}{K}\sqrt{\frac{2}{V_0}}$. The maximum and minimum values of the scale factors are $a_{\max} = \sqrt{\frac{12}{V_2} + B}$ and $a_{\min} = \sqrt{\frac{12}{V_2} - B}$. The value of $V_2 \approx 10^{-63}$ NKg^{-2},[40,41] then B could be large, but we restrict $\frac{12}{V_2} > B$ to get a realistic solution. The configuration of the wormhole in this case is quite different from the usual wormhole. In this model, one universe is connected periodically in Euclidean time by the throat of radius $a_{\min}$ and the maximum size of the radius of the universe is $a_{\max}$. Further, the dimensional reduction is also possible.

3.3. *Field equations for $V(\phi) = 0$ and $\phi' \neq 0$*

Some simple solutions may be obtained in this case. Thus, for the vanishing value of V, we have from (6)

$$\phi' = \frac{im}{a^3 b}\,. \tag{11}$$

We assume the scalar field is imaginary in the Euclidean regime. Now using (4), (5) and (11), we have

$$\frac{3a''}{a} + \frac{b''}{b} = \frac{8\pi G_5 m^2}{a^6 b^2} \,. \tag{12}$$

Now from (5) and (7), we have

$$\frac{a''}{a} + 2\frac{a'^2}{a^2} + \frac{a'b'}{ab} - \frac{2}{a^2} = 0 \,. \tag{13}$$

Further addition of (4) and (7) and using (13), we have

$$b'' + 3\frac{a'b'}{a} = 0 \,, \tag{14}$$

an integration of (14) yields

$$b'a^3 = \lambda^{-1} \,, \tag{15}$$

where λ is a constant. Now from Eqs. (7) and (11), we have

$$\frac{a'^2}{a^2} + \frac{a'b'}{ab} + \frac{4\pi G_5 m^2}{3a^6 b^2} - \frac{1}{a^2} = 0 \,. \tag{16}$$

It seems difficult to obtain the solution of $a(\tau)$ and $b(\tau)$ in closed form (14)–(16). So we consider some particular solutions depending on the values of the constant λ and δ^2 [δ^2 given in Eq. (19)].

3.3.1. *Solution when* $\lambda^{-1} = 0$

We have $b = b_0 =$ const. from (15). Further, the external scale factor a from (16) is governed by

$$a'^2 = 1 - \frac{4\pi G_5 m^2}{3b_0^2 a^4} \,. \tag{17}$$

In this case, the internal space is static, whereas the external space represents a wormhole solution, where two asymptotic Euclidean flat regions are connected by throat of radius $\left(\frac{4\pi G_5 m^2}{3b_0^2}\right)^{\frac{1}{4}}$.

3.3.2. *Equation and the solution for* $\lambda^{-1} \neq 0$

The internal space as well as external space are dynamical. Now, for mathematical simplicity in solving the field equations, we assume that the scale factor a is function of b, i.e. $a = a(b)$, then $a' = \frac{da}{db} b'$, thus from (15) and (16), we get

$$\left(\frac{da}{db}\right)^2 + \frac{a}{b}\left(\frac{da}{db}\right) + \frac{4\pi G_5 m^2 \lambda^2}{3} \frac{a^2}{b^2} - \lambda^2 a^6 = 0 \,. \tag{18}$$

Let us introduce a new constant δ related to m, λ and G_5 as

$$4\delta^2 = 1 - \frac{16\pi G_5 m^2 \lambda^2}{3} \,. \tag{19}$$

Equation (18) has the following solutions depending upon δ^2.

Solution for $\lambda^{-1} \neq 0$ and $\delta = 0$

In this case, we have

$$a^2 = b^{-1}[\bar{c}_0 \pm 2\lambda \ln b]^{-1}\,, \tag{20}$$

where $\bar{c}_0$ is a constant of integration. Now using (20) in (15), we can find a and b in terms of Euclidean time τ and the solution is unphysical in this case ($\delta^2 = 0$). Further, the solution for $\lambda^{-1} \neq 0$ and $\delta^2 < 0$ is not physically acceptable as the scale factor a becomes a function of a complex variable.

Solution for $\lambda^{-1} \neq 0$ and $\delta^2 \geq 0$

The solution of (18) with $\delta^2 \geq 0$ is

$$a = \frac{a_0 b^{\delta - \frac{1}{2}}}{(c_0 - b^{4\delta})^{\frac{1}{2}}}\,, \tag{21}$$

where $c_0 = \frac{a_0^4 \lambda^2}{4\delta^2}$ and a_0 are constants. It is observed that the solution of the field equations becomes unphysical for $\delta^2 \leq 0$, so we may expect a reasonable solution only in the range $0 \leq 4\delta^2 \leq 1$ in view of (19). An explicit solution can be obtained from

$$b' = (\lambda a_0^3)^{-1}[c_0 b^{1-2\delta} - b^{1+2\delta}]^{\frac{3}{2}}\,, \tag{22}$$

which is obtained from (15) and (21). Now in order that an Euclidean solution to be a wormhole solution, one should have a minimum value of a at some finite time τ and at the minimum, where $a' = 0$; further, $a'' \leq 0$ at all epoch and asymptotically $\lim_{\tau \to \pm\infty} \frac{a}{\tau} = 1$. This restricts the values of δ from (19) and (21), further, we have

$$a'' = \lambda^{-2} a_0^{-5} (c_0 - b^{4\delta})^{\frac{3}{2}} b^{-3\delta + \frac{1}{2}} 2\delta \left[\left(\delta + \frac{1}{2}\right) b^{2\delta} + c_0 \left(\frac{1}{2} - \delta\right) b^{-2\delta}\right], \tag{23}$$

so, we may expect wormhole solution in the range $0 \leq \delta \leq \frac{1}{2}$.

It is noted from (21) or (22) that $b(\tau)$ has an upper bound, which is equal to $(c_0)^{\frac{1}{4\delta}}$. It seems that the solution of (22) is not possible in closed form with arbitrary δ, however, wormhole solution in closed form is possible with specific choice of δ in the allowed range $0 \leq \delta \leq \frac{1}{2}$. Now we present wormhole solution from (22) in closed form with two particular choices on δ.

Solution when $\delta \approx \frac{1}{2}$

This choice is acceptable when $\frac{16\pi G_5 m^2 \lambda^2}{3} \ll 1$ (from (19)), the solution from (21) and (22) then becomes

$$b^2 = \frac{c_0^2 \tau^2}{c_0 \tau^2 + a_0^2} \qquad \text{and} \qquad a^2 = \tau^2 + \frac{a_0^2}{c_0}\,, \tag{24}$$

where $c_0^2 = \lambda^2 a_0^4$. The solution (24) represents a wormhole solution. The throat of the wormhole is at $\tau = 0$ and the radius of wormhole is $\frac{a_0}{\sqrt{c_0}} = \frac{1}{a_0 \lambda}$ and the radius of internal space is $b = 0$ at the same epoch. It is to be noted that both the scale factors $a(\tau)$ and $b(\tau)$ increase with simultaneous increase of τ and the asymptotic Euclidean regions are described by $a^2 \approx \tau^2$ and $b^2 \approx \lambda^2 a_0^4$ (i.e. static internal space) for $\tau \gg \pm\frac{a_0}{\sqrt{c_0}}$. This case is identical with pure gravity when $m = 0$ or $\delta = \frac{1}{2}$ in (19).

Solution when $\delta = \frac{1}{10}$

Now from (19), we get $G_5^2 m^2 \lambda^2 = \frac{9}{50\pi}$ and the solution from (21) and (22) is

$$a_\pm^2 = \frac{a_0^2}{4c_0^3} \frac{\left[4 + \frac{\tau^2}{\tau_0^2} \pm \left(4\frac{\tau^2}{\tau_0^2} + \frac{\tau^4}{\tau_0^4}\right)^{\frac{1}{2}}\right]^3}{\left[2 + \frac{\tau^2}{\tau_0^2} \pm \left(4\frac{\tau^2}{\tau_0^2} + \frac{\tau^4}{\tau_0^4}\right)^{\frac{1}{2}}\right]} \tag{25}$$

and

$$b_\pm^2 = \frac{(2c_0)^5}{\left[4 + \frac{\tau^2}{\tau_0^2} \pm \left(4\frac{\tau^2}{\tau_0^2} + \frac{\tau^4}{\tau_0^4}\right)^{\frac{1}{2}}\right]^5}\,, \tag{26}$$

where $a_\pm$ and $b_\pm$ are the scale factors of external and internal spaces, respectively, for both the positive and negative signs in (25) and (26); $\tau_0 = \frac{5\lambda a_0^3}{c_0^2}$ and $c_0 = 25\lambda^2 a_0^4$.

The solutions corresponding to a_+ and b_+, respectively, in (25) and (26); as well as the solutions for a_- and b_-, respectively in (25) and (26) represent two set of wormhole solutions. The throat of the wormhole is at $\tau = \frac{\tau_0}{\sqrt{2}}$ and the radius of wormhole is $a_+(\tau = \frac{\tau_0}{\sqrt{2}}) = (\frac{27a_0^2}{2c_0^3})^{\frac{1}{2}}$ and at the same epoch the radius of internal space is $b_+\left(\tau = \frac{\tau_0}{\sqrt{2}}\right) = \left(\frac{c_0}{3}\right)^{\frac{5}{2}}$. The asymptotic Euclidean regions are described by $a_+(\tau \gg \pm\tau_0) \approx \tau^2$, $b_+(\tau \gg \pm\tau_0) \approx 0$. So dimensional reduction is possible.

It is to be noted that the scale factor of external space increases with increase of Euclidean time τ, and asymptotically $a_-(\tau \gg \tau_0) \approx \tau^2$, whereas the scale factor of internal space decreases with increase of τ and asymptotically $b_-(\tau \gg \tau_0) \approx 0$, so we have dimensional reduction. The radius of the wormhole is $a_-\left(\tau = \frac{\tau_0}{\sqrt{2}}\right) = \left(\frac{27a_0^2}{4c_0^3}\right)^{\frac{1}{2}}$ and at the same epoch the radius of internal space is $b_-\left(\tau = \frac{\tau_0}{\sqrt{2}}\right) = \left(\frac{2c_0}{3}\right)^{\frac{5}{2}}$.

4. Solution of Wheeler–deWitt Equation Obeying Wormhole Boundary Condition

The WD equation (from the action (2) or Lagrangian (3)) in the mini-superspace yields

$$b^2\left[\frac{\partial^2\Psi}{\partial b^2} + \frac{p}{b}\frac{\partial\Psi}{\partial b}\right] - ab\frac{\partial^2\Psi}{\partial a\partial b} + \frac{3}{2K^2}\frac{\partial^2\Psi}{\partial\phi^2} - \frac{b^2a^4}{\hbar^2}\Psi$$
$$+ \frac{K^2}{3\hbar^2}b^2a^6V(\phi)\Psi = 0\,, \tag{27}$$

where p is a parameter representing operator ordering ambiguity. The solution of WD equation is not possible in general with arbitrary potential. So we consider some simple cases.

Solution in pure gravity

The solution of (27) in pure gravity (i.e. the wave function $\Psi(a,b)$ without any scalar field) is

$$\Psi(a,b) = (ba^2)^{\frac{p-1}{2}}\left[C_1 J_{\frac{p-1}{2}}\left[\frac{ba^2}{\hbar}\right] + C_2 Y_{\frac{p-1}{2}}\left[\frac{ba^2}{\hbar}\right]\right], \tag{28}$$

where C_1 and C_2 are integration constants. The wave function (28) is not well behaved at all points in the mini-superspace with arbitrary parameter p and C_1 and C_2. The classical solution (8) in pure gravity allows wormhole solution. The solution of WD equation (28) in pure gravity also represents a wormhole solution provided $p \geq 1$ and $C_2 = 0$ to obtain conformity with the Hawking–Page wormhole condition both at large (four-space geometry) geometry and also at degenerate (four-space geometry) geometry.

4.1. *Solution when $V = V_0$: In absence of scalar field*

The wave function including a cosmological constant in pure KK gravity can be obtained from (27). In this case, $\frac{\partial^2\Psi}{\partial\phi^2}$ is absent in (27) but $V = V_0$. The solution of (27) is

$$\Psi(a,b) = (a^4b^2[a^2 - a_0^2])^{\frac{1-p}{2}}\left(C_1 J_{1-p}\left[\sqrt{\frac{2a^4b^2}{\hbar^2}\left(\frac{a^2}{a_0^2} - 1\right)}\right]\Gamma(2-p)\right.$$

$$\left. + C_2 J_{p-1}\left[\sqrt{\frac{2a^4b^2}{\hbar^2}\left(\frac{a^2}{a_0^2} - 1\right)}\right]\Gamma(p)\right), \tag{29}$$

where $a_0^2 = \frac{3}{V_0K^2}$. The wave function is exponentially damped at large four-space geometry in the range $a^2 \geq a_0^2$ and well-behaved when the four-space geometry collapses to zero provided $p \leq 0$ and $C_2 = 0$, thus satisfies the Hawking–Page boundary conditions for wormhole. The wave function with above restriction represents a wormhole solution in the range $a^2 \geq a_0^2$. So the radius of the wormhole at the throat is a_0^2 from the lower bound of a^2.

4.2. *Solution when $V(\phi) = 0$*

In this case, we introduce separation of variables using $\Psi(a,b,\phi) = \psi(a,b)\Phi(\phi)$ in (27) and we have

$$\Phi(\phi) = Ae^{i\sqrt{\frac{2}{3}}\omega\phi K} + Be^{-i\sqrt{\frac{2}{3}}\omega\phi K} \tag{30}$$

and

$$b^2\frac{\partial^2\psi}{\partial b^2}+pb\frac{\partial\psi}{\partial b}-ab\frac{\partial^2\psi}{\partial a\partial b}-\omega^2\psi-\frac{b^2a^4}{\hbar^2}\psi=0\,, \tag{31}$$

where A, B are constants and ω is a separation constant. Further, a choice of $ba^2=x$ in (31) yields

$$\psi_{xx}+\frac{2-p}{x}\psi_x+\left(\frac{\omega^2}{x^2}+\frac{1}{\hbar^2}\right)\psi=0\,, \tag{32}$$

where $\psi=\psi(a,b)=\psi(x)$. The solution of (32) is

$$\psi(a,b)=(ba^2)^{\frac{p-1}{2}}\left[C_1J_m\left[\frac{ba^2}{\hbar}\right]+C_2Y_m\left[\frac{ba^2}{\hbar}\right]\right], \tag{33}$$

where C_1, C_2 are constants and $m=\sqrt{\left(\frac{p-1}{2}\right)^2-\omega^2}$. It is important to note that the wave function is regular under some restriction on p, m and C_1, C_2. The wave function $\Psi(a,b,\phi)$ is regular at degenerate four-space geometry and also damped at large four-space geometry provided $p\geq 1$ and $C_2=0$ with integral and non-integral value of m. Thus, the Hawking–Page wormhole boundary conditions allow wormhole in this case.

5. Discussion

Wormhole configuration appears due to quantum fluctuations of spacetime geometry at Planck length. In the work, we explore possible wormhole configuration in 5D KK-theory assuming time-dependent extra-dimensional scale factor $b(\tau)$ with the usual scale factor $a(\tau)$ (of external space). The extension of Hawking–Page boundary conditions for wormhole in four-dimensional spacetime to 5D KK-cosmology also yields wormhole configuration with the boundary conditions where the wave function Ψ in KK-cosmology is exponentially damped for large four-space geometry and is regular when four-space geometry collapses to zero. The viability of wormholes is considered both from the solution of the Euclidean field equations and from the solution of wave function Ψ from WD equation satisfying Hawking–Page boundary conditions for wormhole.

Wormholes are allowed in pure gravity and minimally coupled scalar field in 5D KK-cosmology both from the solution of Euclidean field equations and from the solution of the wave function Ψ satisfying Hawking–Page boundary conditions under some restriction on operating ordering parameter p and constants. In pure gravity case, dimensional reduction is possible with adjustment of integration constant, while minimally coupled scalar field case dimensional reduction does not require choice of constants. Wormholes are also allowed with cosmological constant both by the Euclidean field equation and WD equation.

The Euclidean field equations with cosmological constant give an unusual wormhole solution showing periodicity in Euclidean time. In this case, a universe is connected periodically in Euclidean time by the throat of radius $a_{\min}$ with another universe. Further, the wave function Ψ satisfies Hawking–Page conditions for wormhole under some restriction on the operator ordering parameter p and on the scale factor a. Interestingly, restriction on a yields lower bound of a at a_0, where $a_0^2 = \frac{3}{V_0 K^2}$, so a_0 acts as the radius at the throat of wormhole. Usually, the radius of the wormhole is obtained from the Euclidean field equations, whereas in this case it follows from the wormhole boundary conditions. In this case, dimensional reduction is also possible.

A general solution of spacetime including d-dimensional internal space may restrict the asymptotic value of the radius of internal space as well as the value of cosmological constant.

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