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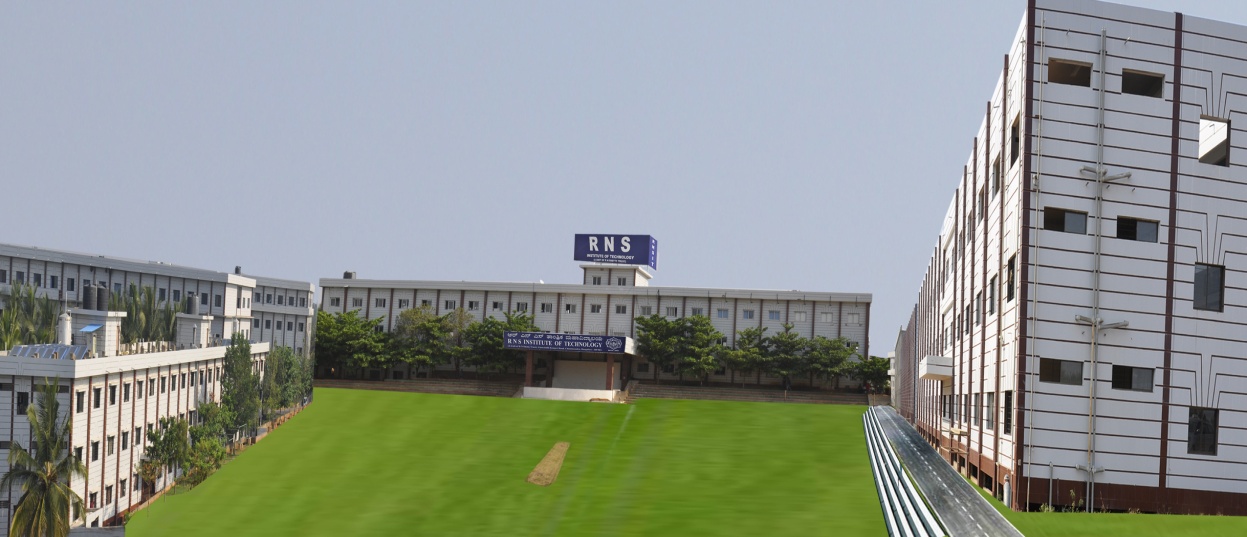
STUDY MATERIALS

FOR

MATHEMATICS

VTU NEW SYLLABUS

MODULE-2



Partial Differentiation and Infinite Series Taylor and Maclaurin

MODULE-2

* Partial Derivatives

1. 

 are second order derivatives of *Z* w r t *x*.

 are second order derivatives of *Z* w r t *y* and *x*.

 are second order derivatives of z w r t x and y.

5. Homogeneous function of *x* and *y* of degree ‘*n*’ if it can be expressed in the form

 or 

6. 

* Euler’s Theorem

 (or) 

* Total Derivative

1. 

2. 

* Composite Function



 and 

* Jacobian

1.  

2. 

3.  4.  5.  and  are not independent 

* Taylor Series





* Maclaurin Series



* L’Hopital Rule

1.  2.  3. 

4.  5. 

CHAPTER -1 Partial differentiation

Partial differentiation is applicable in many Engineering branches, where field quantities and physical parameters are functions of more than one independent variables. For example temperature  in a thin rod of finite width and of infinite extent is a function of y and t, i.e.,  where  is the width of the rod and is the time. Similarly in a simple beam of finite width and if infinite extent the stress is Further, the area of rectangle of length and breadth  is a function of two variables, i.e.,  Similarly the volume of parallelepiped i.e.,  depends on three variables length breadth  and height Also, water wave, acoustic on sonic wave, electromagnetic wave with an infinitesimal elevation is a function of and 

Partial differentiation is useful for Jacibian’s, total differentiation and errors and approximation.

2.1 Partial Derivatives

Let  be a function of two variables  and If we keep  as constant and very  alone, then  is a function of  only.

i.e.,  exists, then it is called the partial derivative of z w r t x

and denoted by or 

Similarly , if exists,

then it is called partial derivative of z w r t y and denoted by or 

Note :  partial differentiation of w r t *x* means, differentiation of *z* w r t *x* and

keeping *y* as constant. Similarly  means differentiation of *z* w r t *y* and keeping *x* as constant.

Higher Order Partial Derivatives

 are second order derivatives of *Z* w r t *x*.

 are second order derivatives of *Z* w r t *y* and *x*.

 are second order derivatives of z w r t x and y.

 are second order derivatives of *z* w r t *y*.

Thus if, Example : 

then 

treat  as variable and as constant.

Similarly 

treat  as variable and as constant.

Note : In general, 

Example.1: If show that 

Solution : We have 

diff. (1) Partially w.r.t. (treating  as a constant ) we get,



Once again diff. (2) Partially w.r.t. 



then diff. (1) Partially w.r.t.



diff. the above Partially w.r.t. , again we get,



Form (3) and (5), we have



Example . 2: Verify that for the function 

Solution : Consider 

diff. (1) Partially w.r.t. 

diff. (2) Partially w.r.t., 

 log 

diff. (1) Partially w.r.t., 

diff. (4) Partially w.r.t., 





From (3) and (5), 

1. Find and  form the following

1)  (2)  (3) 

(4)  (5) 

Solution: (1) 

Differentiating partially w r t *x*, keeping *y* constant we get,





Differentiating partially w r t *y* keeping *x* constant we get,





2. 

Differentiating partially w r t *x*, 

Differentiating partially w r t *y*, 

3. 

Differentiating partially w r t *x*, 

Differentiating partially w r t *y*, 

4. 

Differentiating partially w r t *x*, 

Differentiating partially w r t *y*, 

5. 

Differentiating partially w r t *x*, 

Differentiating partially w r t *y*, 

Verify that  in the following cases:

(6)  (7)  (8) 

Solutions: (6)  ......(1)

Differentiating (1) partially w r t *x*, 

 .........(2)

Differentiating (2) partially w r t *y*, 



 ......(3)

differentiating (1) partially w r t *y*, 

 .....(4)

Differentiating (4) partially w r t *x*, 



 .......(5)

From (4) and (5) we find that 

(7)  ........(1)

Differentiating (1) w r t *x*, partially  ......(2)



Differentiating (2) w r t *y*, partially 

 ......(3)



Differentiating (1) w r t *y*. partially,  ......(4)

Differentiating (4) w r t *x*, partially, 



 .....(5)

From (3) and (5) we get, 

(8) ...... (1)

Differentiate (1) w r t *x*, partially,  .....(2)

Differentiate (2) w r t *y*, partially



 ......(3)

Differentiation (1) w r t *y*, partially,  ......(4)

Differentiation (4) w r t *x*, partially,



 .....(5)

From (3) and (5) we find that 

Example: 14. If  where 

Show that 

Solution: We have 



Differentiating  partially w r t *x*, we get

 using (1)



 ..... (2)

Similarly  ...... (3)





Example: 15. If  show that 

Delhi 2013, UTV 2011, PTU 2010, Anna 2009, Bhopal 2008 (Grewal)







Now 







Example: 16. If  show that at  (VTU 2012, 2013)

Solution: Given  taking logarithm both sides, we have 

Differentiating partially w r t *x*, we get 

i. e., 

Similarly, we have 







Example : 18. If  Show that  and 

(UTV 2012 Mumbai 2008 -Grewal)

Solution: 















Similarly 















Thus we have shown that 

Example: 19. If  Prove that 

(VTU 2009) Similar to VTU 2017

Solution: 





 ...... (1)

Again 

 ....... (2)

From (1) and (2) it follows that 

Example: 20. If Prove that  (This equation is known as Laplace equation)

Solution: We have  (VTU 2006)





 ......... (1)

Similarly 





Thus we have proved the result.

Example: 21. If 

Prove that  (Grewal)

Solution: We have



Differentiating partially w r t *x*, we get.



Or 

 where 

Then 

Similarly 













We observe that 

Example : 22. If  what value of *n* will make  (VTU 2011)

Solution: We have, 















Since 







Example : 24. If   show that 

Solution : We have  









Now 



Example : 25. If  show that 

(VTU2012)(VTU 2017)

Solution : We have 





Similarly   




Now 



 .......(1)

RHS: 



 .........(2)

Form (1) and (2) we find that



Example : 26. If prove that  (VTU 2010)

Solution : We have 





 ........(1)





 ..........(2)

Adding (1) and (2) we get







Example : 27. Given 

Prove that  and 

Solution : We have 





 ........(1)





 .........(2)

Form (1) and (2) we observe that 

Similarly we can prove that  *(Left as an exercise)*

Example 28: If show that 

Solution : 







 ..........(1)









Form (1) and (2), 

Example : 29 Prove that 

If i)  ii) 

Solution : i) 

















Now 

ii) 

















Now 





Example : 30 If  show that 

Solution : 







Now 

Example: 30. If  Prove that 

Solution : 







Similarly 



Now 

 Which is the required result.

Example 32: If  prove that 

Solution : We have 



Differentiating *v* partially w r t *x*





 ....... (2)

Differentiating *v* partially w r t *y*



 ........ (3)

From (2) and (3) we get



Which is the required result.

Example: 35. If  and  prove that 

Solution:  ..... (1)

Differentiating (1) w r t *x,*  we get

 Or  Similarly 

Now 

Similarly 

LHS 

RHS 

Hence LHS = RHS

Homogeneous Function

A function  is said to be Homogeneous function of *x* and *y* of degree ‘*n*’ if it can be expressed in the form or 

Similarly, a function  is said to be Homogeneous function of *x, y* and *z* of degree *‘n’* if it expressed in the form

 or  or 

Illustrative Examples:

1. If  then 

 is a Homogeneous function of degree 3

2. If  then 

 is a Homogeneous function of degree 

3. If  then  and then

 is a Homogeneous function of degree 0

4. If  then 



5. If  then  and then

 is a Homogeneous function of degree 1

EULER’S THEOREM

Statement: If  is a Homogeneous function of *x* and *y* of degree ‘*n’* then

 (or) 

Proof : Since,  is a Homogeneous function of *x* and *y* of degree *‘n’*, it can be expressed in the form



Differentiating (1) partially w r t *‘x’*



Multiply both side by *‘x’*



Differentiating (1) partially w r t *‘y’*



Multiply both side by *‘y’*



Adding (2) & (3) gives,





 [by using (1)]



Note : If  is a Homogeneous function of  and  of degree  then

 (or) 

Remarks :

(1) If  is not a Homogeneous function and  is a Homogeneous function of degree  then 

(2) If  is not a Homogeneous function and  is a Homogeneous function of degree  then  where 

Verify Euler’s theorem for the function 

Solution: Given  ..... (1)



 is a Homogeneous function of  and  of degree 3.

By Euler’s theorem, we have 



Now, Differentiating (1) Partially w r t ‘x’ and w r t ‘y’, we get



 ......(2)





 ......(3)

Adding (2) & (3), we get







If  than show that 

Solution :

Given 



 is a Homogeneous function of  and  of degree .

By Euler’s theorem, we have 

If Then show that 

Solution: Given 



 it a Homogeneous function of and  of degree zero.

By Euler’s theorem, we have 

(1) If  then show that 

Solution: Given, 



Let  where 



 is a Homogeneous function of  and  of degree .

By Euler’s theorem, we have 









(2) If  then show that (i)  (VTU 2017)

(ii) 

Solution: Given 



Let  where 



 is a Homogeneous function of  and  of degree .

By Euler’s theorem, we have 











Also, by Euler’s theorem Remark, we have

If  is not a Homogeneous function and  is a Homogeneous function of degree

 then  where 

Now , 





(3) If then show that 

Solution:

Given 



Let where 



 is a Homogeneous function of  and  of degree .

By Euler’s theorem, we have









(4) If  Then show that 

Solution:

Given, 



Let where 



 is a Homogeneous function of  and  of degree .

By Euler’s theorem, we have

 (





(5) If  then show that  and

(ii) 

Solution:

Given , 



Let where 



 is a Homogeneous function of of degree .

By Euler’s theorem, we have









Also, By Euler’s theorem remark, we have

If  is not a Homogeneous function and is a Homogeneous function of degree

‘*n*’ then where 

Now , 





(6) If then show that 

Solution :

Given , 





 is a Homogeneous function of  and  of degree .







(7) If then show that

Solution :

Given, 



Let  where 



 is a Homogeneous function of of degree .

By Euler’s theorem, we have









Exercise 2.4

1. Verify Euler’s Theorem of the following :

(i)  (ii) 

(iii)  (iv) 

(v)  (vi) 

(2) If  then show that 

(3) If  then show that 

(4) If  then show that 

(5) If  then show that 

(6) If  then show that 

(7) If  then verify that 

(8) If  then show that 

(9) If  then show that 

(10) If  then show that 

(11) If  Then show that (i)  and



(12) If then show that (i) and

(ii) 

(13)If  then show that and

(ii) 

(14)If  then show that 

TOTAL DIFFRENTIATION

Let and  then the total differential coefficient is denoted by and is defined by



SimilarLet and  then the total differential coefficient is denoted by



(2) If then find 

Solution:

We have 









(3) If  then find 

Solution :

We have, 















(4)  then find 

Solution: We have, 









(5) If  then find 

Solution: We have, 













(6) If then find 

Solution: We have, 















(7) If  then find  at 

Solution: We have, 















(8) If  then find  at 

Solution: We have, 









Exercise 2.5

1. Find  if 

2. Find  if 

3. Find  if 

4. If  then show that 

5. Find  if 

6. Find  if 

7. Find  if 

8. Find  if 

9. Find  if  where 

10. Find  if  where 

Answers

1.  2.  3. 

5.  6.  7.  8. 

9.  10. 

Partial differentiation of composite function

Let *z* be a function of *u* and *v,* and *u* and *v* are themselves functions of *x* and *y*

i.e.,  and  then *z* is called a composite function of 

Then,  and  are computed by using the following formulae:



 and 

Similarly, let *z* be a function of *x* and *y*, and *x* and *y* are themselves functions of *u* and *v*

i.e.,  and  then *z* is called a composite function of .

Then,  and  are composed by using the following formulae:



 and 

These formulae are called the chain rule for partial differentiation.

Worked examples

1. If  then prove that 

Solution: Given, 

Then, we have  and 

 and 

L. H. S 









2. If  then prove that 

Solution Given, 

Then, we have  and 

 and 

L. H. S 











3. If  then prove that 

Solution Given, 

Then, we have  and 

 and 

L. H. S 













4. If  then prove that 

Solution Given, 

Then, we have  and 

 and 

L. H. S 













5. . If then prove that 

Solution Given, 

Then, we have  and 

 and 

R. H. S 













6. If  then prove that 

Solution: Let 

Then, 

We have 

And 

Then, 

And 

L. H. S 







7. If  then prove that  (VTU 2017)

Solution: Let 

Then, 

We have 

And 

Then, 

And 

L. H. S 







8. If  then prove that 

Solution: Let 

Then, 

We have 

And 

Then, 

And L. H. S 







9. If  then prove that

Solution: Let 

Then, 

We have 

And 

Then,  and 

L. H. S 







10. If  then prove that 

Solution: Let 

Then, 

We have 

And 

Then, 

And 

L. H. S 







11. If  then prove that 

Solution: Let 

Then, 

We have 

And 

Then, 

And 

L. H. S 



 RHS

EXERCISE

1. If  and  then show that 

2. If  and  then show that 

3. If  and  then show that 

4. If  then show that 

5. If  and  then show that 

6. If  and  then show that 

7. If , then prove that 

8. If , then prove that 

9. If   then show that

10. If  is a function of  and , and  and  be two other variables such that  show that 

JACOBIANS

If  and  are functions of two independent variables  and  then the Jacobian  of  and  with respect to  and  denoted by  or  and is defined by  

Similarly, if  and  are functions of  and  then the Jacobian  of  and  with respect to  and  is denoted by  or  and is defined by



Properties of Jacobians

1. If  and  are functions of  and  where  and  are functions of  and  then



2. If  is the Jacobian of  and  with respect to  and  and  is the Jacobian of  and  with respect to  and .. then 

3. If functions  and  of two independent variables  and  are not independent, then 

NOTE: If  and  are not independent then the Jacobian 

WORKED EXAMPLES

1. If  then find the Jacobian 

Solution: We have 





2. If  and  then show that the Jacobian  (VTU 2017)

OR

If  and  then show that the Jacobian  is a constant.

Solution: We have,   








 which is constant

3. If then find the Jacobian   


Solution: We have, 

 (expanding along 3rd column)















4. If  and  then show that the Jacobian 

Solution: Given,  and 



And 

 and 

We have,   














5. If  then verify that 

Solution: We have, 





Given, 

Squaring and adding these we get, 

And by dividing these we get, 

We have,  







Now, 

Hence verified

6. If  then verify that 

Solution: We have, 





Given, 

Squaring and subtracting these we get,

And by dividing these we get, 

We have,  









Now, 

Hence verified

7. If  and  then find the Jacobian 

We have,





8. If and  then verify that 

We have, 







Given, and 

Squaring and subtracting these we get, 

And by dividing these we get, 

We have,  









Now, 

Hence verified

9. If and then verify that 

Solution :

We have, 





Given, and 

Squaring and adding these we get, 

and by dividing these we get, 

We have, 







Now, .

Hence verified

10. If  where  then show that



Solution: We have, 













11. If  where  then find 

Solution: We have, 













12. If  and  then show that the Jacobian

 (VTU 2017)

Solution: We have, 









 Two rows are identical



13. If  and  then show that the Jacobian   


solution: We have, 

 (Expanding along first column)





14. If  and  then show that the Jacobian 

Solution: We have, 



















15. If and  then verify that 

Solution: We have, 







Given, and 

Squaring and adding these we get, 



And by dividing these we get, 

We have, 













Now, .

Hence verified

16. If  and  then verify that 

solution: We have, 





Given,  and 

by adding these we get, 

And 

 and 

We have, 





Now, 

Hence verified

17. If  and  then find the jacobian of  and  with respect to  and 

solution: Given,  and 

By adding these, we get



And by subtracting these, we get



We have, 











18. Prove that the function  are functionally dependent.

Solution: Given functions are said to be functionally dependent, if they are not independent.

i.e., 

We have, 







 The given functions are functionally dependent.

CHAPTER-2 Taylor and Maclaurin Series of One variable

If  satisfies the following two conditions

(i)  and its first  derivatives be continuous in , and

(ii)  exists for every value of  in , then there exists at least one number , such that

 ...... (1)

Which is called Taylor’s theorem with Lagrange’s form of remainder, where remainder is?



Putting and  in (1), we get

 ...... (2)

Which is known as Maclaurin’s theorem with Lagrange’s form of remainder.

Also, by putting  in (1), we get

 ...... (3)

Which is called Taylor’s theorem in powers of  with Lagrange’s form of remainder.

If  possesses derivatives of all orders and the remainder  tends to zero as , then the Taylors theorem becomes the Taylor’s series and the Maclaurin’s theorem becomes Maclaurin’s series

i.e., The Taylor’s series expansions of  in power series about  is

 ... ...... (4)

And the Maclaurin’s series expansions of  in power series about  is

 .... (5)

* It can be used to approximate solutions of differential equations
* It is used for determining limits
* It has various apllications in physics in system under conservative force,
* simple harmonic oscillators.
* They are also used to approxiamate values of function to very high accuracy.



Brooke Taylor

EXAMPLES

(1) If  using Taylor’s theorem, show that for   Deduce that  for .

Solution: By Maclaurin’s theorem with remainder , we have

 ...... (1)

Given, 







Substituting these in (1), we get





Since  and 





Hence,  for 

(2)Expand  in a series of powers of ) up to the term containing  Hence find an approximate value of 

Solution:

The Taylor’s series expansion of in power series about  is



It is required to expand in powers of This expansion is



Given, 

Substituting these in (1), we get





To determine  let us take  so that  radians



(3) Expand  in a series of powers of  up to the term containing . Hence find an approximate value of .

The Taylors’s series expansion of  in power series about  is



It is required to expand  in powers of . This expansion is

 ..... (1)

Given , 







Substituting these in (1) we get





To determine  Let us take  so that  radians



(4) Expand  in a series of powers of( upto the term contesting . .

Solution :

The Taylors’s series expansion of  in power series about  is



It is required to expand  in powers of . This expansion is

 ..... (1)

Given,  





,



Substituting these in (1) we get





To determine . Let us take  so that  radians



(5) Expand  in a series of powers of  up to the term containing.

Hence find an approximate value of  (VTU 2017)

Solution: The Taylor’s series expansion of  in power series about  is



It is required to expand  in powers of . This expands is

 ..... (1)

Given, , 

, 

, 

substituting these in (1), we get





To determine  let us take  so that 



(6) Expand  in a series of powers of  up to the term containing fourth degree.

Solution: The Taylor’s series expansion of  in power series about  is



It is required to expand  in powers of . This expansion is

 ..... (1)

Given, , 

. 

, 

Substituting these in (1), we get





(7) Obtain Taylor’s series expansion of  about the point  up to fourth degree term. (VTU 2017)

solution: The Taylor’s series expansion of  in power series about  is



It is required to expand  about the point This expansion is

 .... (1)

Given, , 

. 

, 



,





Substituting these in (1), we get





(8) Obtain Taylor’s series expansion of  about the point  up to fourth degree term.

solution: The Taylor’s series expansion of  in power series about  is



It is required to expand  about the point This expansion is

 .... (1)

Given, , 

. 

, 



,









(9) Use Taylor’s series to prove that

 where 

solution: The Taylor’s series expansion of  in power series is

Given : 



Differentiating w r t 



Let, 











And so on.

Substituting these values in (1), we get



(10) Use Taylor’s series, to prove that



Solution: The Taylor’s series expansion of  in power series is



Let, ,



And so on.

Substituting these values in (1), e get



(11) Obtain Maclaurin’s series expansion of  up to the term containing.

Solution: The Maclaurin’s series expansion of  in power series about  is



Let,  

, 

, 

, 

, 

, 

And so on

Substituting these values in (1), we get





(12) Obtain Maclaurin’s series expansion of up to the term containing.

Solution: The Maclaurin’s series expansion of  in power series about  is

 .... (1)

Let , , 

, 

, 

, 

, 

, 

, 

And so on

Substituting these values in (1), we get





(13) Obtain Maclaurin’s series expansion of  up to term containing .

Solution: The Maclaurin’s series expansion of  in power series about  is



Let, , 

,

,























And so on.

Substituting these values in (1), we get





(14) Obtain Maclaurin’s series expansion of  up to the term containing 

The Maclaurin’s series expansion of  in power series about  is



Let, ,  





















And so on

Substituting these values in (1), we get





(15) Obtain Maclaurin’s series expansion of  up to term containing 

Solution: The Maclaurin’s series expansion of in power series about 

 ..... (1)

Let, , 

, 

And so on

Substituting these values in (1), we get





(16) Obtain Maclaurin’s series expansion ofup to the term containing 

solution: The Maclaurin’s series expansion of in power series about is



Let,  

And so on

Substituting these values in (1), we get





(18) Obtain maclaurin’s series expansion of up to the term containing 

solution: The Maclaurin’s series expansion of in power series about is

 .... (1)

Let,







And so on

substituting these values in (1), we get





19. Obtain Maclaurin’s series expansion of up to the term containing .

Solution: The Maclaurin’s series expansion of in power series about is



Let,  

, 



,



, 

, 

And so on

Substituting these values in (1), we get





20. Obtain Maclaurin’s series expansion of in power series about is

 .... (1)

Let, 

And so on

Substituting these values in (1), we get





21. Obtain Maclaurin’s series expansion of up to the term containing .

solution: The Maclaurin’s series expansion of in power series about is

 ... (1)

Let,  



















And so on

substituting these values in (1), we get





22. Obtain Maclaurin’s series expansion of up to the term containing .

solution: The Maclaurin’s series expansion of in power series about is



Let,  



















And so on

substituting these values in (1), we get





23. Obtain Maclaurin’s series expansion of up to the term containing 

Solution: The Maclaurin’s series expansion of in power series about is



Let,  

























And so on

substituting these values in (1), we get





24. Obtain Maclaurin’s series expansion of up to the term containing 

Solution : The Maclaurin’s series expansion of in power series about is



If then by Leibnitz’s theorem, we have

 [Refer Example 1.20]

Putting  we get



Putting we get



And so on

Substituting these values in (1), we get





EXERCISE

1. Expandin powers of up to four terms

2. Obtain Taylor’s series expansion of  in ascending powers of up to four terms

3. Prove that 

4. Prove that 

5. Prove that 

6. Obtain the Maclaurin’s series expansion of the following functions:

(i)  (ii)  (iii)  (iv)  (v)  (vi)

(vii)  (viii)  (ix)  (x) (xi)  (xii) 

7. prove that 

8. Prove that 

9. Prove that 

10. Prove that 

ANSWERS

1.  2. 

6. (i)  (ii) 

(iii)  (iv) 

(v)  (vi) 

(vii)  (viii) 

(ix)  (x) 

(xi)  (xii) 

INDETERMINATE FORMS

The quantities of the form  which do not represent any real value are called indeterminate forms.

Limit which lead to indeterminate forms are generally evaluated by employing a standard rule known as L ‘Hospital’s (French Mathematician) rule which is stated as below:

L ‘Hospital’ s Rule

If and  are two functions of ‘x’ such that

(i)  and 

(ii)  and exist and 

Then, 

Extension of L ‘Hospital’s Rule

If  and  then



And so on

Evaluation of limits of the form 

The L ‘Hospital’s rule can be employed to evaluate  whenever we encounter the limits of the form 

In many situations it may require to employ the L ‘Hospital’s rule repeatedly to determine a definite value for the limit.

Note: For the evaluation of  we can use the following standard limits.

(1)  (2)  (3)  (4) 

(5)  (6)  (7) 

(8) 

Worked Examples

1. Evaluate 

solution:  (indeterminate form)

By applying L ‘Hospital’s rule, we have



Again applying L ‘ Hospital’s rule, we have





2. Evaluate 

Solution:  (indeterminate form)

By applying L ‘Hospital’s rule, we have





3. Evaluate 

Solution:  (indeterminate form)

Before applying L’Hopital’s rule, we can use a standard limit



(multiply and divide by



By applying L ‘Hospital’s rule, we have





4. Evaluate 

Solution:  (indeterminate form)

Before applying L’Hospital’s rule, we can use a standard limit

 (multiply and divide by 







5. Evaluate 

Solution:  (indeterminate form)

Before applying L’Hospital’s rule, we can use a standard limit

 (multiply and divide by 



Before applying L’Hospital’s rule, we have





6. Evaluate 

Solution:  (indeterminate form)

By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





7. Evaluate 

Solution:  (indeterminate form)

By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





8. Evaluate 

 (indeterminate form)

By applying L ‘Hospital’s rule, we have





Again applying L’Hospital’s rule, we have





9. Evaluate 

Solution:  (indeterminate form)

Before applying L’Hospital’s rule, we can use a standard limit

 (multiply and divide by X)



By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





10. Evaluate 

Solution:  (indeterminate form)

Before applying L’Hospital’s rule, we can use a standard limit

 (multiply and divide by 



By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





11. Evaluate 

Solution:  (indeterminate form)

Before applying L’Hospital’s rule, we can use a standard limit

 (multiply and divide by x)  

By applying L ‘Hospital’s rule, we have





12. Evaluate 

 (indeterminate form)

Before applying L’Hospital’s rule, we can use a standard limit

 (Multiply and divide by



By applying L ‘Hospital’s rule, we have





Again applying L’Hospital’s rule, we have





13. Evaluate 

Solution:  (indeterminate form)

By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





14. Evaluate 

Solution:  (Indeterminate form)

Before applying L’Hospital’s rule, we can use a standard limit

 (Multiply and divide by x)



By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





15. Evaluate 

Solution:  (indeterminate form)

By applying L ‘Hospital’s rule, we have



 (Multiply and divide by 





Again applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





16. Evaluate 

Solution:  (Indeterminate form)

Before applying L’Hospital’s rule, we can use a standard limit

 (Multiply and divide by

By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





Note: (1)  (2)  (3)  (4)  (5) 

17. Evaluate 

 (Indeterminate form)

By applying L ‘Hospital’s rule, we have





18. Evaluate 

Solution:  (Indeterminate form)

By applying L ‘Hospital’s rule, we have









19. Evaluate 

Solution:  (indeterminate form)

By applying L ‘Hospital’s rule, we have





20. Evaluate 

Solution: 

 (Indeterminate form)

By applying L ‘Hospital’s rule, we have





21. Evaluate 

Solution: 

 (Indeterminate form)

By applying L ‘Hospital’s rule, we have







22. Evaluate 

Solution: 

 (Indeterminate form)

By applying L ‘Hospital’s rule, we have







23. Evaluate 

Solution:  (Indeterminate form)

By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





Evaluation of limits of the form 

The limits which are of the form  can be evaluated by taking L.C.M first and reducing it to the form  or . Then apply L’Hospital’s rule.

Example

1. Evaluate 

Solution:  (Indeterminate form)

First take L.C.M



By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





2.Evaluate 

Solution:  (Indeterminate form)

First take L.C.M



By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





3. Evaluate 

Solution:  (Indeterminate form)

First take L.C.M



Before applying L’Hospital’s rule, we can use a standard limit

 (Multiply and divide by 



By applying L ‘Hospital’s rule, we have



Again applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





4. Evaluate 

Solution:  (Indeterminate form)

First take L.C.M



Before applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





5. Evaluate 

Solution:  (Indeterminate form)

First take L.C.M



Before applying L’Hospital’s rule, we can use a standard limit

(Multiply and divide by 

(Multiply and divide by 



By applying L ‘Hospital’s rule, we have







6. Evaluate 

Solution:  (Indeterminate form)

First take L.C.M



Before applying L’Hospital’s rule, we can use a standard limit

 (Multiply and divide by 



By applying L ‘Hospital’s rule, we have





Again applying L’Hospital’s rule, we have





7. Evaluate 

Solution: (Indeterminate form)

First take L.C.M



Before applying L’Hospital’s rule, we can use a standard limit

 (Multiply and divide by 



By applying L ‘Hospital’s rule, we have





8. Evaluate 

Solution: (Indeterminate form)

First take L.C.M



By applying L ‘Hospital’s rule, we have





9. Evaluation of limits of the form 

The limits which are of the form  can be evaluated by taking any one term (other than log) to denominator first and reducing it to the form  or . Then apply L’Hospital’s rule.

1. Evaluate 

Solution:  (Indeterminate form)

Take the term other than log term to denominator



By applying L ‘Hospital’s rule, we have







2. Evaluate 

Solution:  (Indeterminate form)

Take the term other than log term to denominator



By applying L ‘Hospital’s rule, we have







3. Evaluate 

Solution:  (Indeterminate form)

Take the term other than log term to denominator



By applying L ‘Hospital’s rule, we have





4. Evaluate 

Solution:  (Indeterminate form)

Take the term other than log term to denominator



Before applying L’Hospital’s rule, we can use a standard limit

 (Multiply and divide by 





By applying L ‘Hospital’s rule, we have





5. Evaluate 

Solution:  (indeterminate form)

Take any one term to denominator



By applying L ‘Hospital’s rule, we have





6. Evaluate 

Solution:  (Indeterminate form)

Take any one term to denominator



By applying L ‘Hospital’s rule, we have





Evaluation of limits of the form 

If the limit  takes any one of the form  then by taking log on both side, we get



It takes of the form then it can be reduced to the form  or by taking function to denominator. Then apply L’Hospital’s rule.

1. Evaluate 

Solution:  (Indeterminate form) 

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we get







2. Evaluate  (VTU 2017)

Solution:  (Indeterminate form) 

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we have







3. Evaluate 

Solution:  (Indeterminate form)

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have









4. Evaluate 

Solution:  (Indeterminate form) 

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we have







5. Evaluate  (VTU 2017)

Solution:  (Indeterminate form) 

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we have







6. Evaluate 

Solution:  (Indeterminate form)

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have







7. Evaluate 

Solution:  (Indeterminate form)

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have







8. Evaluate 

Solution:  (Indeterminate form)

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have







9. Evaluate 

Solution:  (Indeterminate form)

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have









10. Evaluate 

Solution:  (Indeterminate form)

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have









11. Evaluate  (VTU 2017)

Solution:  (Indeterminate form)

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have









12. Evaluate 

Solution:  (Indeterminate form)

Let 

Taking log on both sides, we get



By applying L’Hospital’s rule, we have







13. Evaluate 

Solution:  (Indeterminate form)

Put  as  implies 

Let 

Taking log on both sides, we get



Take the term other than log term to denominator



By applying L’Hospital’s rule, we have







14. Find the constant a and b such that  may be equal to unity.

Solution: Given that, 

 (Indeterminate form)

By applying L’Hospital’s rule, we have



Let  (1)

Again applying L’Hospital’s rule, we have



Again applying L’Hospital’s rule, we have





Let  (2)

Solving (1) & (2), we get



15. Find the constant a, and b such that  may be equal to 2.

Solution: Given that, 

Let  (1)

By applying L’Hospital’s rule, we have



Let  (2)

Again applying L’Hospital’s rule, we have



Let  (3)

Solving (1) & (2), we get



EXERCISE

Evaluate the following limits

1. 2.  3.  4. 

5.  6.  7.  8. 

9.  10.  11.  12. 

13.  14.  15.  16. 

17.  18.  19. 

20. 

21. Show that the constants *a* and *b* satisfy the identity  such that 

22. Find the constants *a* and *b* such that  may be equal to unity.

ANSWERS

1.  2.  3.  4.  5.  6. 7.  8.  9.  10.  11.  12. 

13.  14.  15.  16. 17.  18.  19.  20.  22. 