Lecture 3: Experiment 2
EE380 (Control Systems)

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Part I

Procedure: Discussed in Lecture 3

Outline of the experiment

Want speed of motor to track a sinusoid. Steps:

- Least squares system identification (sys-id).
- Recognition and compensation of plant's dead zone.
- Design controller using loop-shaping.
- Simulation on PC.
- Deployment on experimental setup.

Tasks common to all 6 experiments

Simulation

- Perform PC-based simulation of CL system using GNU Octave.
- Perform PC-based simulation of digital control of a continuous-time system using GNU Octave.

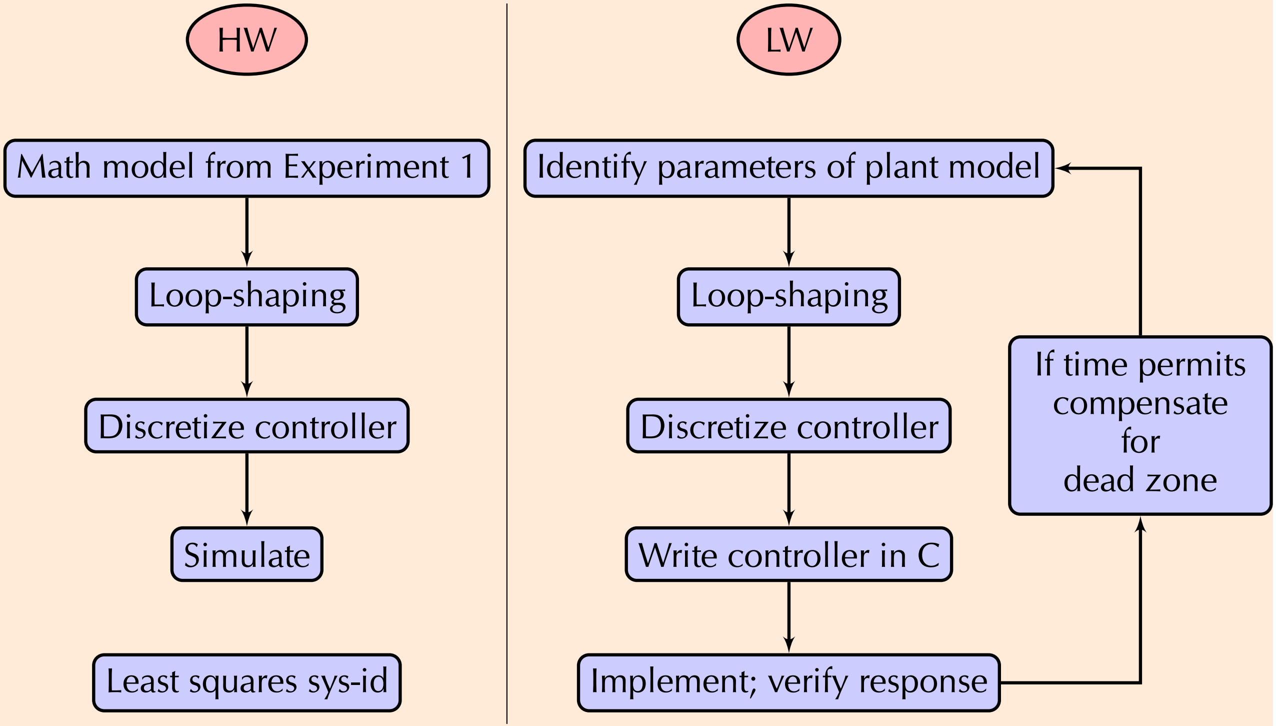
Realization on hardware

- Utilize the various components of an integrated development environment (IDE): editor, compiler, linker, debugger, and programmer to program a μC.
- Program controller using C language into μC.
- Monitoring: read data into PC from μC using UART modules.

Analysis

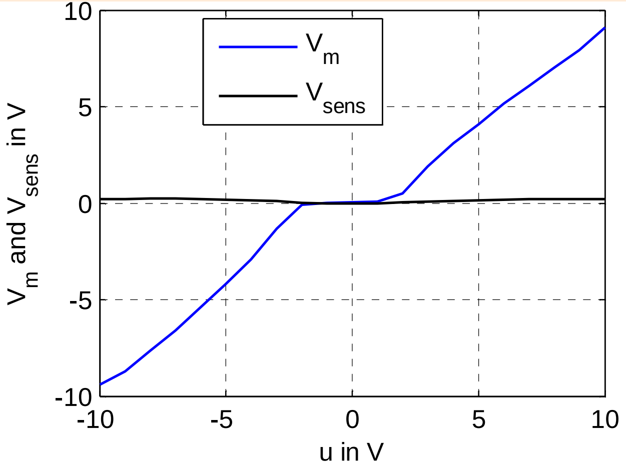
- Compare actual performance with predicted performance.

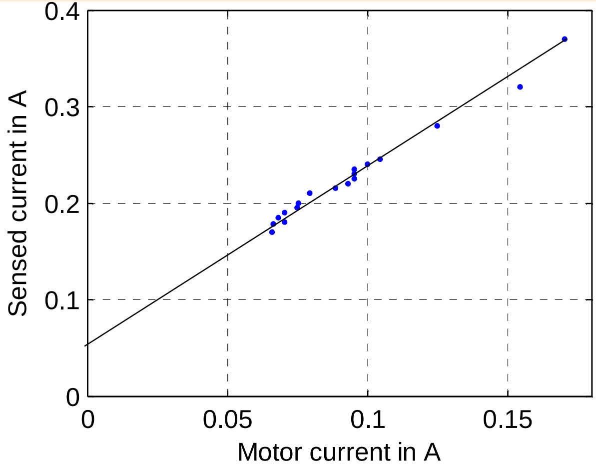
Homework (HW) vs. Lab work (LW)



Dead zone and how to overcome it

If $u < 2$ V when $V_s = 12$ V, then V_m will be zero.





Overcoming dead zone

```
if(u<0&&u>-2)
        u =  u - 2;
else if(u>0&&u<2)
        u =  u + 2;
```

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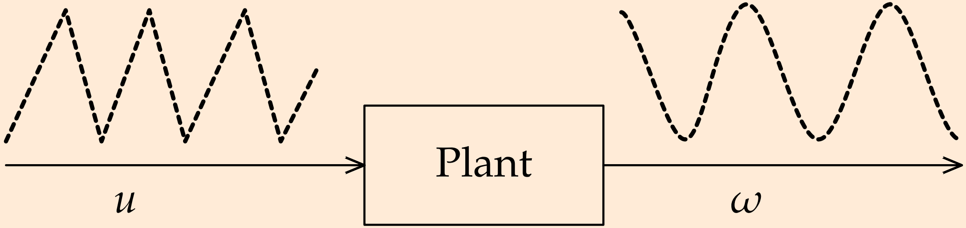
Least squares sys-id



- Place C code of triangular waveform generator in `main-prog.c`.
- Prepare setup to apply u in OL and collect ω from motor.
- Input $u_1, u_2, u_3, \ldots$ forming the triangular waveform.
- Collect $u_1, u_2, u_3, \ldots$ and $\omega_1, \omega_2, \omega_3, \ldots$ into `terminal.log`.
- Form `readSID.m` by replacing plots-related section of `readplot.m` with sys-id code from `sysid.m`.
- Execute `readSID.m` to obtain K, a, b of $\frac{K}{s^2+as+b}$.

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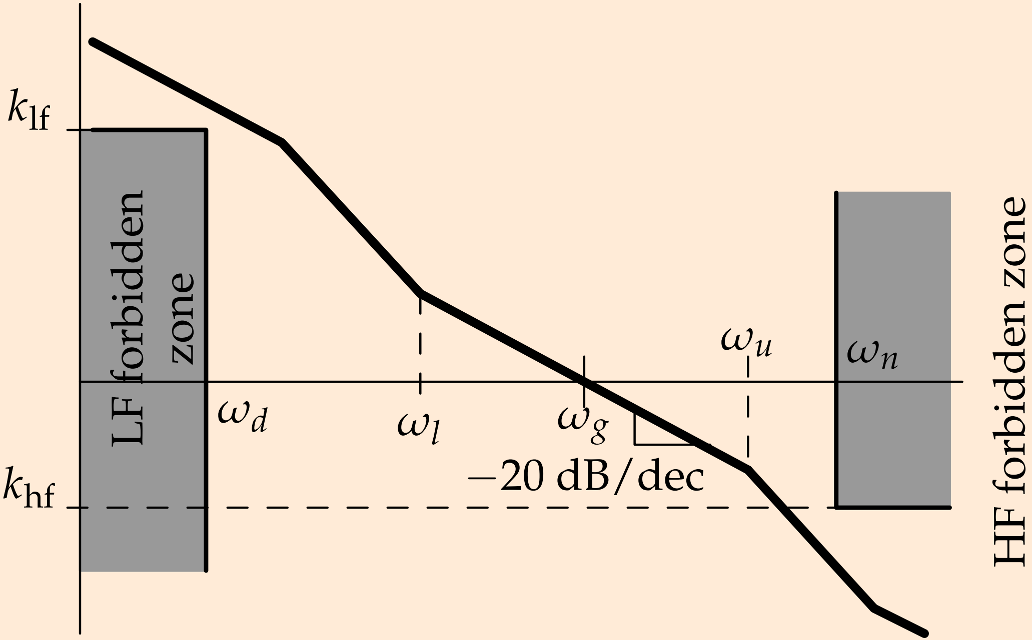
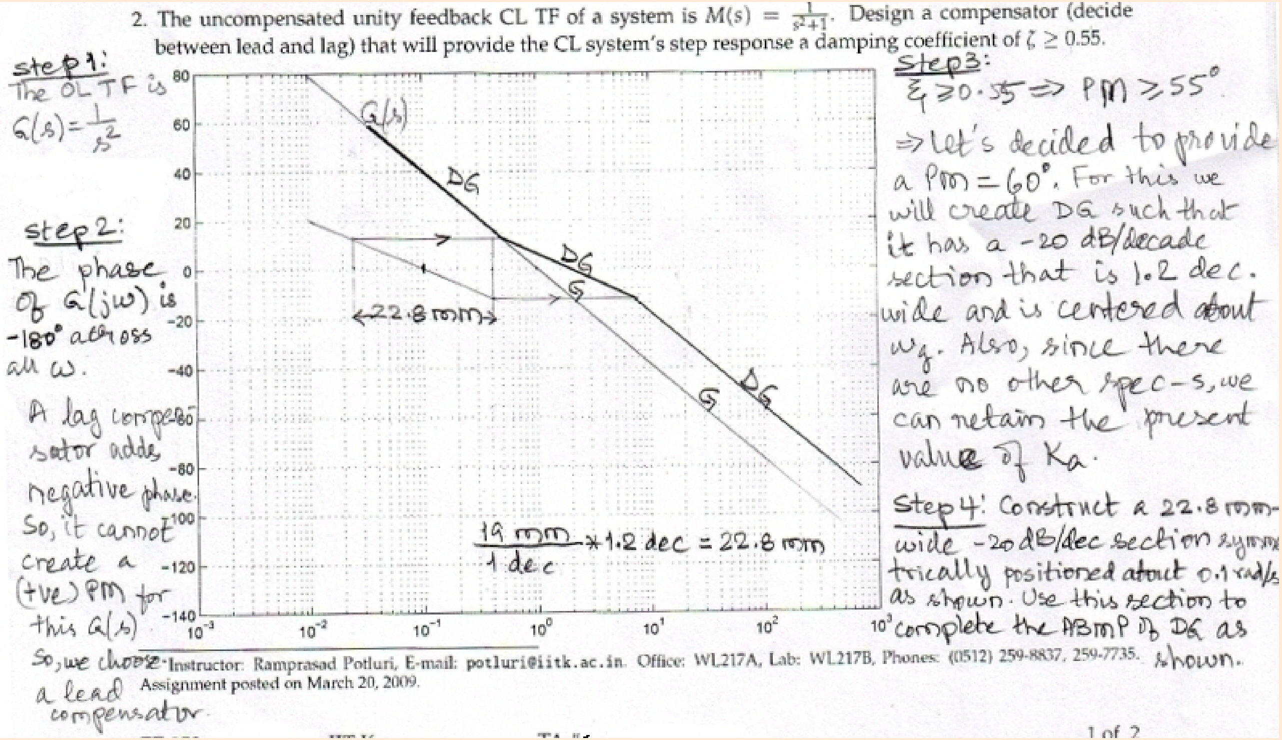
Loop-shaping (1/5): Typical G_{des}



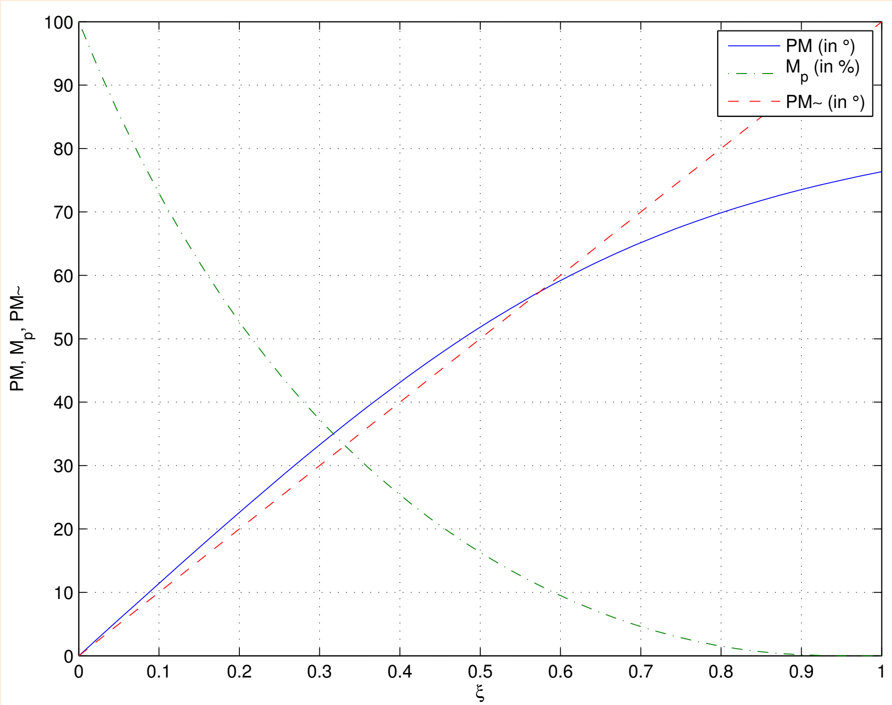
Loop-shaping (2/5): Example



$$DG = G_{\text{des}}$$

Loop-shaping (3/5): $\zeta \longleftrightarrow M_p \longleftrightarrow \mathrm{PM}$

From Lecture 26 of EE250-2011



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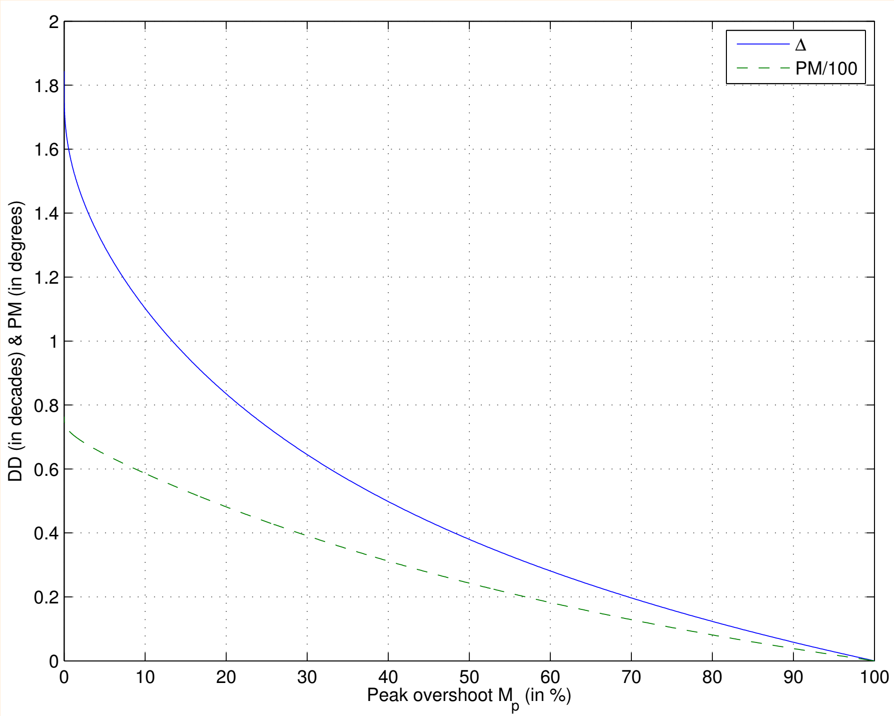
Loop-shaping (4/5): $M_p \longleftrightarrow$ DD $\longleftrightarrow$ PM

From Lecture 26 of EE250-2011



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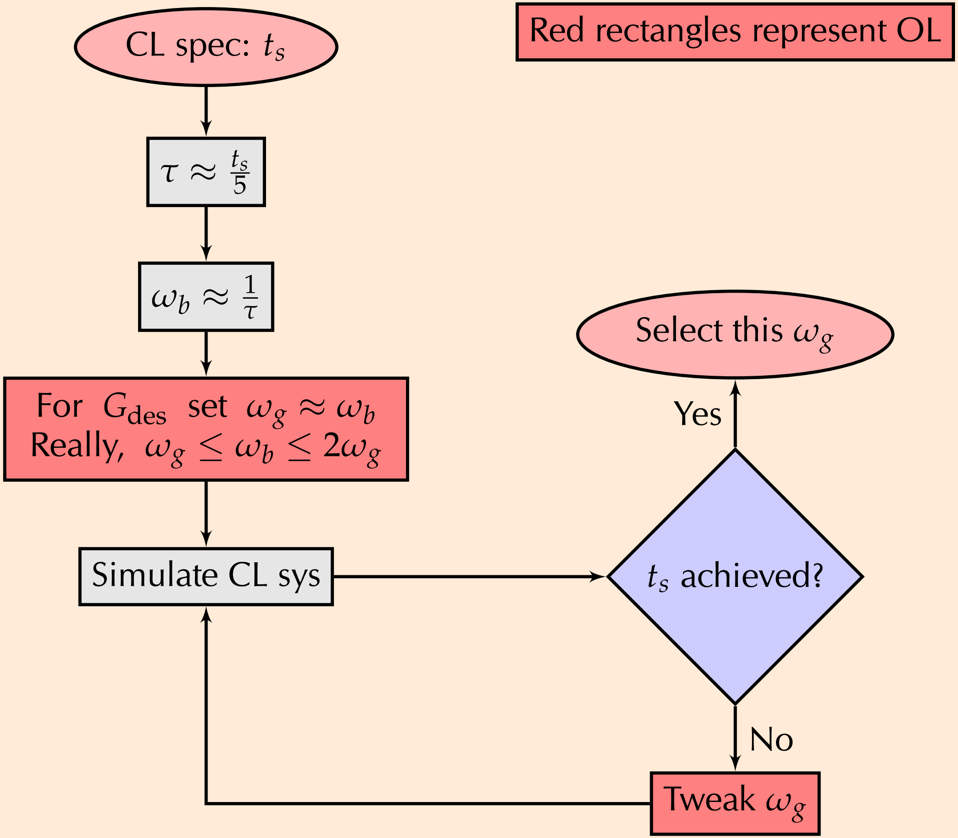
Loop-shaping (5/5): Determination of ω_g



Discretization

$$\frac{a_1 s + a_0}{s^2 + b_1 s + b_0}$$

Simulation diagram

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -b_0 x_1 - b_1 x_2 + u \\ y &= a_0 x_1 + a_1 x_2 \end{aligned}$$

Euler's approximation

$$\begin{aligned} x_{1(k+1)} &= x_{1(k)} + \Delta t \; x_{2(k)} \\ x_{2(k+1)} &= -b_0 \Delta t \; x_{1(k)} + (1 - b_1 \Delta t) x_{2(k)} + \Delta t \; u_k \\ y_k &= a_0 x_{1(k)} + a_1 x_{2(k)} \end{aligned}$$

Simulate; LW: C code, Implement, Analyze

- Simulation: `easysim.m`
- Discretized controller $\longrightarrow$ C code:

- Implement: As in demo slides
- Analyze: Compare results

$$\boxed{\begin{aligned} x_1(k+1) &= a_{11}x_1(k) + a_{12}x_2(k) + b_1u(k) \\ x_2(k+1) &= a_{21}x_1(k) + a_{22}x_2(k) + b_2u(k) \\ y(k) &= c_1x_1(k) + c_2x_2(k) + du(k) \end{aligned}}$$

In `main-prog.c` before `main()` insert `float x1[2],x2[2];`
In `main()` insert `x1[0] = x2[0] = 0;`

$$\boxed{\begin{aligned} \mathtt{x1}[1] &= \mathtt{a11} * \mathtt{x1}[0] + \mathtt{a12} * \mathtt{x2}[0] + \mathtt{b1} * \mathtt{u}; \\ \mathtt{x2}[1] &= \mathtt{a21} * \mathtt{x1}[0] + \mathtt{a22} * \mathtt{x2}[0] + \mathtt{b2} * \mathtt{u}; \\ \mathtt{y} &= \mathtt{c1} * \mathtt{x1}[0] + \mathtt{c2} * \mathtt{x2}[0] + \mathtt{d} * \mathtt{u}; \\ \mathtt{x1}[0] &= \mathtt{x1}[1]; \\ \mathtt{x2}[0] &= \mathtt{x2}[1]; \end{aligned}}$$

Work involved

- Unless order of controller changes in redesign, don't have to use log paper in lab.
- If prepared how to combine `readplot.m` & `sysid.m`, then:

$$\begin{array}{rcl} Q9 + Q10 & \leq 30 & \text{minutes} \\ Q11 & \leq 30 & \text{minutes} \\ Q12 & \leq 20 & \text{minutes} \\ Q13 + Q15 & \leq 10 & \text{minutes} \\ \hline \multicolumn{3}{c}{\text{Total: 1 hour 30 minutes}} \\ \hline \end{array}$$

- Remove compensation of dead zone and repeat.

Part II

Least squares sys-id theory: Part of Lecture 4

Bilinear transform and Z-transform

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
g(t) \arrow[rrr, "\text{Sampling at } T_s \text{ seconds}"] \arrow[d, bend left, "\text{Laplace Transform } \mathcal{L}"] & & & g(k) \arrow[d, bend left, "Z\text{-transform } \mathcal{Z}"] \\
G(s) \arrow[u, bend left, "\text{Inverse Laplace Transform } \mathcal{L}^{-1}"] \arrow[rrr, leftrightarrow, "\text{Bilinear Transform} = \text{Tustin's method}"] & & & G(z) \arrow[u, bend left, "\text{Inverse } Z\text{-transform } \mathcal{Z}^{-1}"]
\end{tikzcd}
$$

$$s = \frac{2}{T_s}\frac{z-1}{z+1} \quad \text{or} \quad z = \frac{1+\frac{sT_s}{2}}{1-\frac{sT_s}{2}}$$

- Both s-domain & z-domain are fictitious domains.
- s-domain simplifies working with differential equations; z-domain simplifies working with difference equations.
- Bilinear transform is not the only way to go $G(s) \leftrightarrow G(z)$.
- T_s is constrained by Nyquist sampling frequency.

$G(s) \longleftrightarrow G(z)$

- Consider definitions of $\mathcal{L}$ and $\mathcal{Z}$

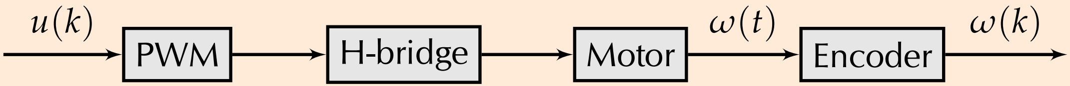
$$Y(s) = \mathcal{L}\{y(t)\} \triangleq \int_{t=0}^{\infty} y(t)e^{-st}dt$$

$$Y(z) = \mathcal{Z}\{y(k)\} \triangleq \sum_{k=0}^{\infty} y(k)z^{-k}$$

- Comparison suggests $z = e^{sT_s}$.
- To convert $G(s)$ to $G(z)$, can substitute $s = \frac{\ln z}{T_s}$.
- Easier to work with an approximation

$$z = e^{sT_s} = e^{\frac{sT_s}{2}} e^{\frac{sT_s}{2}} = \frac{e^{\frac{sT_s}{2}}}{e^{-\frac{sT_s}{2}}} = \frac{1 + \frac{\left(\frac{sT_s}{2}\right)}{1!} + \frac{\left(\frac{sT_s}{2}\right)^2}{2!} + \cdots}{1 + \frac{\left(-\frac{sT_s}{2}\right)}{1!} + \frac{\left(-\frac{sT_s}{2}\right)^2}{2!} + \cdots} \approx \frac{1 + \frac{sT_s}{2}}{1 - \frac{sT_s}{2}}$$

How Z-transform used in our sys-id



- $u(k)$ denotes sample of $u(t)$ at sampling instant $t = kT_s$.
- Let $u(k) \to \omega(k)$ TF be $G(z)$.
- Use $u(k), \omega(k)$ pairs to build $G(z)$.
- Use bilinear transform to go from $G(z)$ to $G(s)$.

Important property of Z-transform used:

$$z^{-l}X(z) \leftrightarrow x(k-l) \;\text{ given }\; X(z) \leftrightarrow x(k).$$

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What is least squares sys-id? (1/2)

- Let $G(z) = \dfrac{b_1 z^2 + b_2 z + b_3}{z^3 + a_1 z^2 + a_2 z + a_3} = \dfrac{Y(z)}{U(z)}$.
- Cross multiply:

$$b_1 z^2 U(z) + b_2 z U(z) + b_3 U(z) = z^3 Y(z) + a_1 z^2 Y(z) + a_2 z Y(z) + a_3 Y(z).$$

- Multiply throughout by z^{-3}:

$$b_1 z^{-1} U(z) + b_2 z^{-2} U(z) + b_3 z^{-3} U(z) = \\ Y(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) + a_3 z^{-3} Y(z).$$

- Take $\mathcal{Z}^{-1}$ to obtain difference equation

$$b_1 u(k-1) + b_2 u(k-2) + b_3 u(k-3) = \\ y(k) + a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3).$$

What is least squares sys-id? (2/2)

Consider $$b_1 u(k-1) + b_2 u(k-2) + b_3 u(k-3) = y(k) + a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3). \quad (1)$$

- Let $\sigma = [b_1 \quad b_2 \quad b_3 \quad -a_1 \quad -a_2 \quad -a_3]^\mathsf{T}$.
- Suppose we have data of $u(k)$ and $y(k)$ for $k = 0, 1, \ldots, N$.
- Problem: Find σ such that (1) holds for this data.

  I.E., find parameters of a TF that fits to input-output data.
- Let error in the fit be
$$\varepsilon(k,\sigma) = b_1 u(k-1) + b_2 u(k-2) + b_3 u(k-3) - y(k) - a_1 y(k-1) - a_2 y(k-2) - a_3 y(k-3).$$
- Modified problem: Find σ to minimize $\mathcal{J}(\sigma) \triangleq \sum_{k=0}^{N} \varepsilon^2(k,\sigma)$.
- If $\mathcal{J}(\sigma = \sigma_0) = 0$, then find best estimate $\hat{\sigma}$ of σ_0.