Formulas from Calculus

Derivatives

$$
\begin{array}{lll}
\frac{d}{dx}[x^n] = nx^{n-1} & \frac{d}{dx}[e^x] = e^x & \frac{d}{dx}[\sin x] = \cos x \\
\frac{d}{dx}[c] = 0 & \frac{d}{dx}[b^x] = b^x \ln b & \frac{d}{dx}[\cos x] = -\sin x \\
\frac{d}{dx}[x] = 1 & \frac{d}{dx}[\ln x] = \frac{1}{x} & \frac{d}{dx}[\tan x] = \sec^2 x \\
\frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2} & \frac{d}{dx}[\log_b x] = \frac{1}{x \ln b} & \frac{d}{dx}[\sec x] = \tan x \sec x \\
\frac{d}{dx}\left[\frac{1}{x^2}\right] = -\frac{2}{x^3} & \frac{d}{dx}[\sinh x] = \cosh x & \frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \\
\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}} & \frac{d}{dx}[\cosh x] = \sinh x & \frac{d}{dx}[\arctan x] = \frac{1}{1+x^2} \\
\frac{d}{dx}\left[\frac{1}{\sqrt{x}}\right] = -\frac{1}{2x\sqrt{x}} & \frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x & \\
 & \frac{d}{dx}[\operatorname{arcsinh} x] = \frac{1}{\sqrt{1+x^2}} & \\
 & \frac{d}{dx}[\operatorname{arctanh} x] = \frac{1}{1-x^2} &
\end{array}
$$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

Chain Rule: $$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x) \quad \text{or} \quad \frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

Special Cases

$$\frac{d}{dx}[f(x)^n] = nf(x)^{n-1}f'(x)$$

$$\frac{d}{dx}\left[\frac{1}{g(x)}\right] = \frac{-g'(x)}{g(x)^2}$$

$$\frac{d}{dx}[\ln|f(x)|] = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}\left[e^{f(x)}\right] = f'(x)e^{f(x)}$$

Integrals

$$
\begin{aligned}
\int x^n \; dx &= \tfrac{1}{n+1}x^{n+1} + C & \int e^x \; dx &= e^x + C \\
\int \frac{1}{x} \; dx &= \ln|x| + C & \int b^x \; dx &= \tfrac{1}{\ln b} b^x \\
\int c \; dx &= cx + C & \int \sinh x \; dx &= \cosh x + C \\
\int x \; dx &= \tfrac{1}{2}x^2 + C & \int \cosh x \; dx &= \sinh x + C \\
\int x^2 \; dx &= \tfrac{1}{3}x^3 + C & \int \sin x \; dx &= -\cos x + C \\
\int \frac{1}{x^2} \; dx &= -\frac{1}{x} + C & \int \cos x \; dx &= \sin x + C \\
\int \sqrt{x} \; dx &= \tfrac{2}{3}x\sqrt{x} + C & \int \tan x \; dx &= \ln|\sec x| + C \\
\int \frac{1}{\sqrt{x}} \; dx &= 2\sqrt{x} + C & \int \sec x \; dx &= \ln|\tan x + \sec x| + C \\
\int \frac{1}{1+x^2} \; dx &= \arctan x + C & \int \sin^2 x \; dx &= \tfrac{1}{2}(x - \sin x \cos x) + C \\
\int \frac{1}{\sqrt{1-x^2}} \; dx &= \arcsin x + C & \int \cos^2 x \; dx &= \tfrac{1}{2}(x + \sin x \cos x) + C \\
\int \ln x \; dx &= x \ln x \; - x + C & \int \tan^2 x \; dx &= \tan x \; - x + C \\
\int x^n \ln x \; dx &= \tfrac{x^{n+1}}{n+1} \ln x \; - \tfrac{x^{n+1}}{(n+1)^2} + C & \int \sec^2 x \; dx &= \tan x + C
\end{aligned}
$$

Substitution $\displaystyle \int f(g(x))g'(x) \; dx = \int f(u) \; du = F(u) + C = F(g(x)) + C$

$$\int_a^b f(g(x))g'(x) \; dx = \int_{g(a)}^{g(b)} f(u) \; du$$

Special cases $\displaystyle \int \frac{f'(x)}{f(x)} \; dx = \ln|f(x)| + C$

$$\int e^{f(x)} f'(x) \; dx = e^{f(x)} + C$$

By parts $\displaystyle \int u \; dv = uv - \int v \; du$

$$\int_a^b u \; dv = uv]_a^b - \int_a^b v \; du$$

or $\displaystyle \int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x) \; dx$

$$\int_a^b f(x)g'(x)dx = f(x)g(x)]_a^b - \int_a^b g(x)f'(x) \; dx$$