Econometric Modelling

SVAR – B Model

$$y_t = A_1 y_{t-1} + \cdots\cdots\cdots\cdots + A_p y_{t-p} + u_t$$

$$u_t = B\epsilon_t \qquad \epsilon_t \sim (0, \mathbf{I_K})$$

$$\Sigma_u = BB'$$

<u>Baseline VAR</u>

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \end{bmatrix}$$

$\}$ Two Monetary Policy Tools

$\}$ Measure of Output and Prices

Econometric Modelling (contd.)

Key Notations

- n – number of policy shocks that we are interested in
- K – number of variables in the SVAR
- Z_t – (n x 1) vector of instruments
- u_{1t} – (n x 1) vector of residual of monetary policy
- u_{2t} – (K – n x 1) vector of residual of non – monetary policy
- ϵ_{1t} – (n x 1) structural policy shocks
- ϵ_{2t} – (K – n x 1) non-structural policy shocks
- Σ_{AB} – represents the covariance matrix

Econometric Modelling (contd.)

Matrix - Visuals

$$u_t = \begin{bmatrix} u_t^1 \\ u_t^2 \\ u_t^3 \\ u_t^4 \\ u_t^5 \end{bmatrix} \begin{matrix} u_{1t} \; {}_{(n \times 1)} \\ \\ u_{2t} \; {}_{(k-n \times 1)} \end{matrix}$$

$$Z_t = \begin{bmatrix} z_t^1 \\ z_t^2 \end{bmatrix}$$

Z_t – (n x 1) vector of instruments

$$B = \begin{array}{cc} B_1 \; {}_{(k \times n)} & B_2 \; {}_{(k \times (k-n))} \end{array}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{55} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} \end{bmatrix} = \begin{array}{cc} B_1 & B_2 \\ \end{array} \begin{bmatrix} \boxed{\begin{matrix} B_{11} \\ B_{21} \end{matrix}} & \begin{matrix} B_{12} \\ B_{22} \end{matrix} \end{bmatrix}$$

Econometric Modelling (contd.)

Using External Instruments

Necessary Conditions:

The usage of external instruments require hunting for variables (instruments) that follow –

1. Correlation with monetary policy shocks
2. No correlation with the other shocks in the system

$$E[Z_t\epsilon_{1t}] \neq 0$$
$$E[Z_t\epsilon_{2t}] = 0$$

Econometric Modelling (contd.)

Using External Instruments

Derivation:

$$E[Z_t u_t{}'] \quad = \mathrm{E}\,[\mathrm{Z_t}]\,[\mathrm{u_{1t}}{}' \quad \mathrm{u_{2t}}{}']$$

$$= \mathrm{E}\,[Z_t u_{1t}{}' \quad Z_t u_{2t}{}']$$

$$= \mathrm{E}\,[\mathrm{Z_t}\epsilon_{1t}{}' \quad \mathrm{Z_t}\epsilon_{2t}{}']\,\mathrm{B}' \qquad \ldots\ldots(u_t = B\epsilon_t)$$

$$= \left[\Sigma_{\mathrm{Z}\epsilon_1}\mathrm{B}_{11}{}' + \Sigma_{\mathrm{Z}\epsilon_2}\,\mathrm{B}_{12}{}' \qquad \Sigma_{\mathrm{Z}\epsilon_1}\mathrm{B}_{21}{}' + \Sigma_{\mathrm{Z}\epsilon_2}\mathrm{B}_{22}{}'\right]$$

$$= \left[\Sigma_{Z\epsilon_1'}\,\mathrm{B}_{11}{}' \qquad \Sigma_{Z\epsilon_1'}\,\mathrm{B}_{21}{}'\right] \qquad \ldots\ldots(\Sigma_{Z\mathrm{u}_2{}'} \;=\; 0)$$

Econometric Modelling (contd.)

Using External Instruments

Comparing the values:

$$\Sigma_{Zu_1'} = \Sigma_{Z\epsilon_1'} \, B_{11}{}'$$

$$\Sigma_{Zu_2'} = \Sigma_{Z\epsilon_1'} \, B_{21}{}'$$

$$B_{21} = (\, \Sigma_{Zu_1'}^{-1} \; \Sigma_{Zu_2'})' \, B_{11}$$

From the above derived equation, we can compute $\Sigma_{Zu_1'}^{-1}\Sigma_{Zu_2'}$ using the following steps –

1. Estimating the reduced form VAR and obtaining the residuals $\hat{u}_t$
2. Regressing $\widehat{u_{2t}}$ on $\widehat{u_{1t}}$ using Z_t as instruments for $\widehat{u_{1t}}$

Econometric Modelling (contd.)

Using External Instruments

Computing $B_{21}B_{11}^{-1}$

$$U_2 = C\, U_1 + V$$

Using Z as instruments

$$U_2\, Z' = C\, U_1\, Z' + V\, Z'$$

Taking expectations

$$E\,[U_2\, Z'] = C\, E\,[U_1\, Z'] + E[V\, Z']$$

$$C = E\,[U_2\, Z']\, E\,[U_1\, Z']^{-1} \quad = \quad B_{21}B_{11}^{-1}$$

Econometric Modelling (contd.)

Identifying Restrictions

$$B_{21} = (\,\Sigma_{Zu_1'}^{-1},\ \Sigma_{Zu_2'})'\,B_{11}$$

For n>1

$$u_t = B\epsilon_t$$

$$\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}$$

$$u_{1t} = B_{11}\,\epsilon_{1t} + B_{12}\,\epsilon_{2t}$$
$$u_{2t} = B_{21}\,\epsilon_{1t} + B_{22}\,\epsilon_{2t}$$

Rearranging the equations, we obtain –

$$u_{1t} = B_{12}B_{22}^{-1}\,u_{2t} + (B_{11} - B_{12}B_{22}^{-1}B_{21})\,\epsilon_{1t}$$

$$u_{2t} = B_{21}B_{11}^{-1}\,u_{1t} + (B_{22} - B_{21}B_{11}^{-1}B_{12})\,\epsilon_{2t}$$

Econometric Modelling (contd.)

Identifying Restrictions

Rewriting –

$$u_{1t} = \eta\, u_{2t} + S_1 \epsilon_{1t}$$

$$u_{2t} = \Lambda\, u_{1t} + S_2 \epsilon_{2t}$$

where

$$S_1 = (B_{11} - B_{12} B_{22}^{-1} B_{21}) \qquad S_2 = (B_{22} - B_{21} B_{11}^{-1} B_{12})$$

Imposing Restriction -

$$\begin{bmatrix} u_{1t}^1 \\ u_{1t}^2 \end{bmatrix} = \eta\, u_{2t} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \epsilon_{1t}^1 \\ \epsilon_{1t}^2 \end{bmatrix}$$

- Using the economic theory we set $S_{12} = 0$, which means that a structural shock in 1 year treasury rate will not have an immediate impact on the fed funds rate
- This requires estimation of the S_1 matrix

Econometric Modelling (contd.)

Identifying Restrictions

Rewriting –

$$u_{1t} = \eta\, u_{2t} + S_1\epsilon_{1t}$$

$$u_{2t} = \Lambda\, u_{1t} + S_2\epsilon_{2t}$$

where

$$S_1 = (B_{11} - B_{12}B_{22}^{-1}B_{21}) \qquad S_2 = (B_{22} - B_{21}B_{11}^{-1}B_{12})$$

$$B_{11}\, S_1^{-1} = (\, I - B_{12}B_{22}^{-1}\, B_{21}B_{11}^{-1})^{-1}$$

$$B_{21}\, S_1^{-1} = B_{21}B_{11}^{-1}\, (\, I - B_{12}B_{22}^{-1}\, B_{21}B_{11}^{-1})^{-1}$$

$$S_1S_1' = (\, I - B_{12}B_{22}^{-1}\, B_{21}B_{11}^{-1})\, B_{11}B_{11}'\, (\, I - B_{12}B_{22}^{-1}\, B_{21}B_{11}^{-1})\,'$$

Econometric Modelling (contd.)

Identifying Restrictions

Estimating S_1:

$$\Sigma_u = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \qquad \& \qquad \Sigma_u = BB' = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} B'_{11} & B'_{21} \\ B'_{12} & B'_{22} \end{bmatrix}$$

$$M = B_{21}B_{11}^{-1}\,\Sigma_{11}\,(B_{21}B_{11}^{-1})' - \left(\Sigma_{21}\,(B_{21}B_{11}^{-1})' + (B_{21}B_{11}^{-1})\,\Sigma'_{21}\right) + \Sigma_{22}$$

$$B_{12}B'_{12} = \left(\Sigma_{21} - B_{21}B_{11}^{-1}\,\Sigma'_{11}\right)' M^{-1} \left(\Sigma_{21} - B_{21}B_{11}^{-1}\,\Sigma'_{11}\right)$$

$$B_{22}B'_{22} = \Sigma_{22} + B_{21}B_{11}^{-1}\,(B_{12}B'_{12} - \Sigma_{11})\,(B_{21}B_{11}^{-1})'$$

$$\mathbf{B_{11}}\boldsymbol{B'_{11}} = \Sigma_{11} - B_{12}B'_{12}$$

$$\mathbf{B_{12}}\boldsymbol{B_{22}^{-1}} = \left[\left(B_{12}B'_{12}\,(B_{21}B_{11}^{-1})'\right) + \left(\Sigma_{21} - B_{21}B_{11}^{-1}\,\Sigma'_{11}\right)'\right](B_{22}B'_{22})^{-1}$$

Econometric Modelling (contd.)

Estimating Impact Matrix

$$S_1 S_1' = (I - B_{12}B_{22}^{-1} B_{21}B_{11}^{-1}) B_{11}B_{11}' (I - B_{12}B_{22}^{-1} B_{21}B_{11}^{-1})'$$

$$B_{21} S_1^{-1} = B_{21}B_{11}^{-1} (I - B_{12}B_{22}^{-1} B_{21}B_{11}^{-1})^{-1}$$

$$B_{11} S_1^{-1} = (I - B_{12}B_{22}^{-1} B_{21}B_{11}^{-1})^{-1}$$

- Using $B_{12}B_{22}^{-1}$, and $B_{11}B_{11}'$ and $B_{21}B_{11}^{-1}$ one can compute S_1S_1'
- Imposing triangular structure we obtain S_1
- Finally using S_1, we obtain B_{11} and B_{21}

$$\begin{array}{cc} & B_1 \\ B = & \begin{bmatrix} \boxed{\begin{matrix} B_{11} \\ B_{21} \end{matrix}} & \begin{matrix} B_{12} \\ B_{22} \end{matrix} \end{bmatrix} \end{array}$$

Data Briefing

Baseline Specification

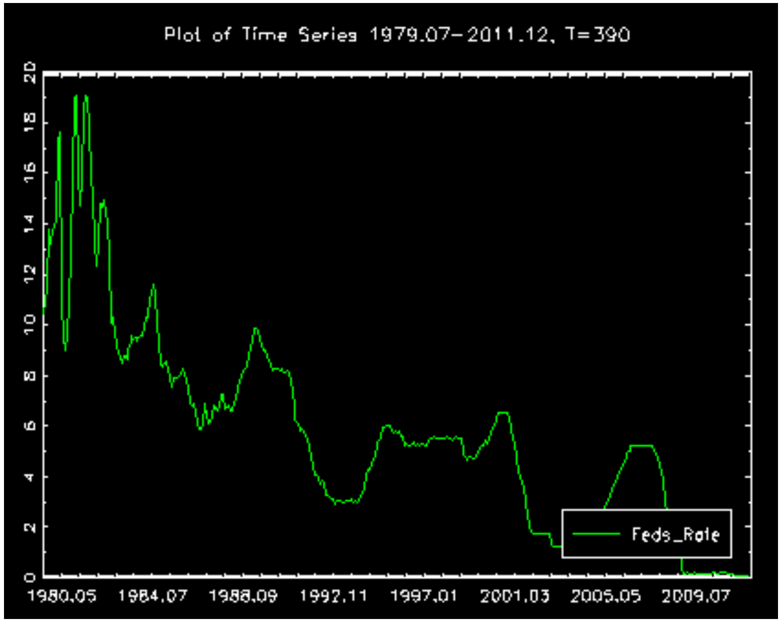
- Four variables – Fed Funds Rate, 1 year Treasury Rate, log CPI and log Industrial Production
- Time Period – July 1979 to December 2011 (T=390)
- Monetary policy Variables –
  - Conventional – Fed Funds Rate
  - Forward Guidance – 1 Year Treasury Rate

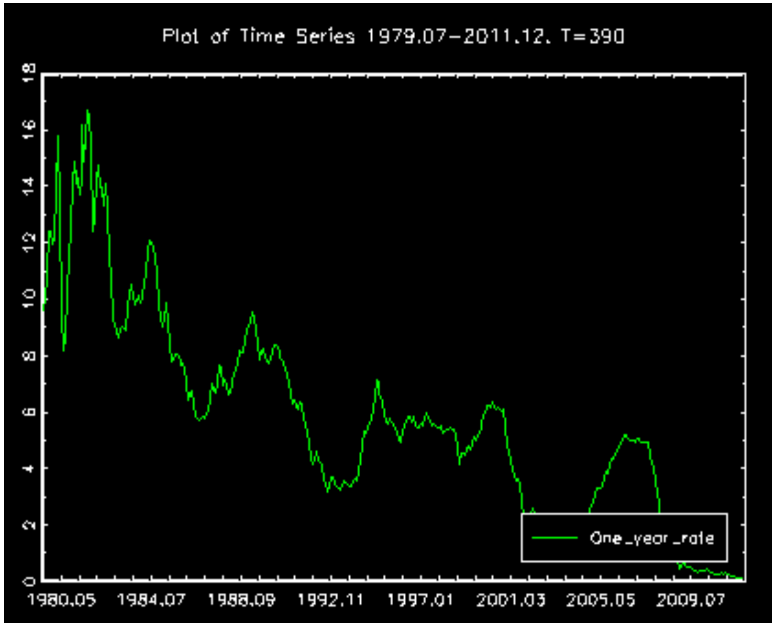
Instruments

- 2 Policy shocks – Futures price changes (5 Futures Contracts)
- Feds Funds Rate Shock – Current month FFR Contract
- Forward Guidance Shock – 3 month ahead FFR Contract

Preliminary Data Checks

Eyeball Econometrics





Preliminary Data Checks

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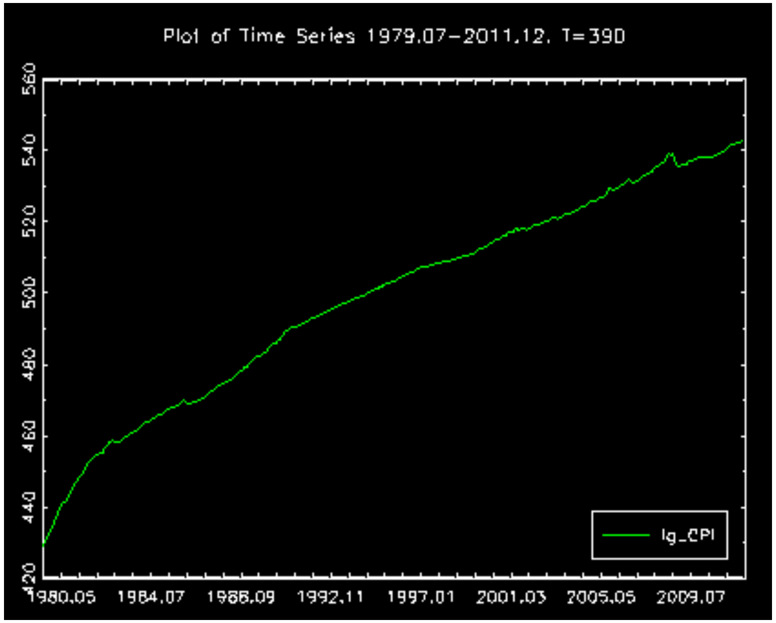
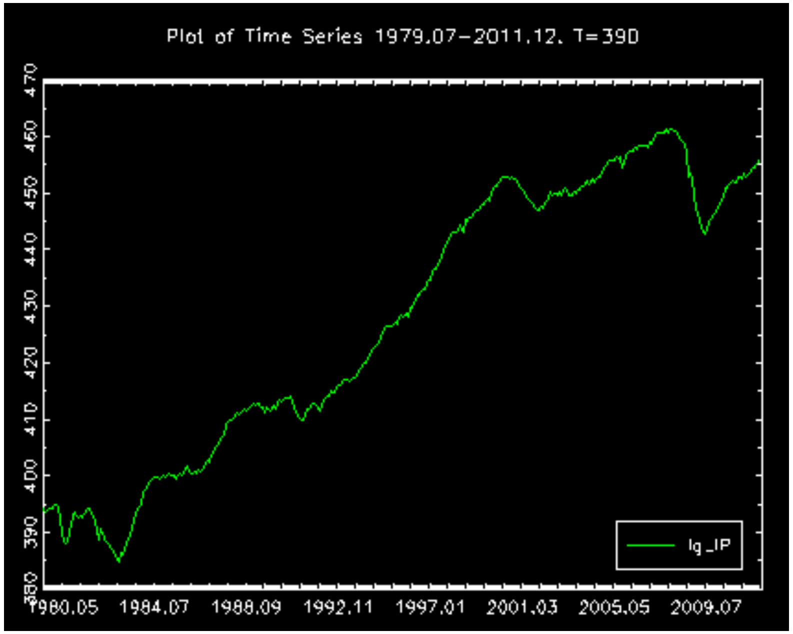




Initial Analysis

Order of Integration

Feds Rate

Critical value: -3.41

Statistic: -2.58

Optimal VAR(p) = 12

Decision: ***Fail to reject*** the null

<u>First Diff</u>

Critical value: -2.86

Statistic: -6.92

Decision: ***Reject*** the null

1 yr Treasury Rate

Critical value: -3.41

Statistic: -4.07

Optimal VAR(p) = 12

Decision: ***Reject*** the null

log CPI

Critical value: -3.41

Statistic: -5.66

Optimal VAR(p) = 3

Decision: ***Reject*** the null

log IP

Critical value: -3.41

Statistic: -2.10

Optimal VAR(p) = 5

Decision: ***Fail to reject*** the null

<u>First Diff</u>

Critical value: -2.86

Statistic: -6.00

Decision: ***Reject*** the null

Model Estimation

$$Y_t = [\ FFR_t\ ,\ TR1_t\ ,\ \ln CPI_t\ ,\ \ln Ip_t\ ]'$$

Lag order estimation using Information criteria (p_{max} =12)	
AIC = 12	SC = 2

Comparative Analysis

- Std residual analysis
- Autocorrelogram
- Diagnostics test

Johansen Trace Test

r	LR	p-value	90%	95%	99%
0	119.36	0	60	63.66	70.91
1	50.66	0.0059	39.73	42.77	48.87
2	24.43	0.0732	23.32	25.73	30.67
3	6.35	0.4282	10.68	12.45	16.22

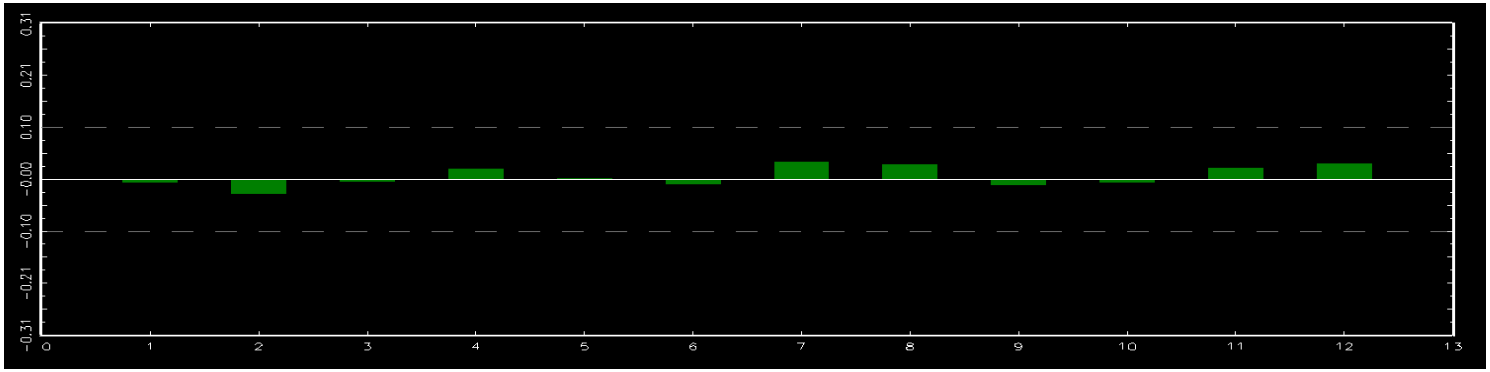
Note

- Do not transform series into first differences
- Run regression on levels

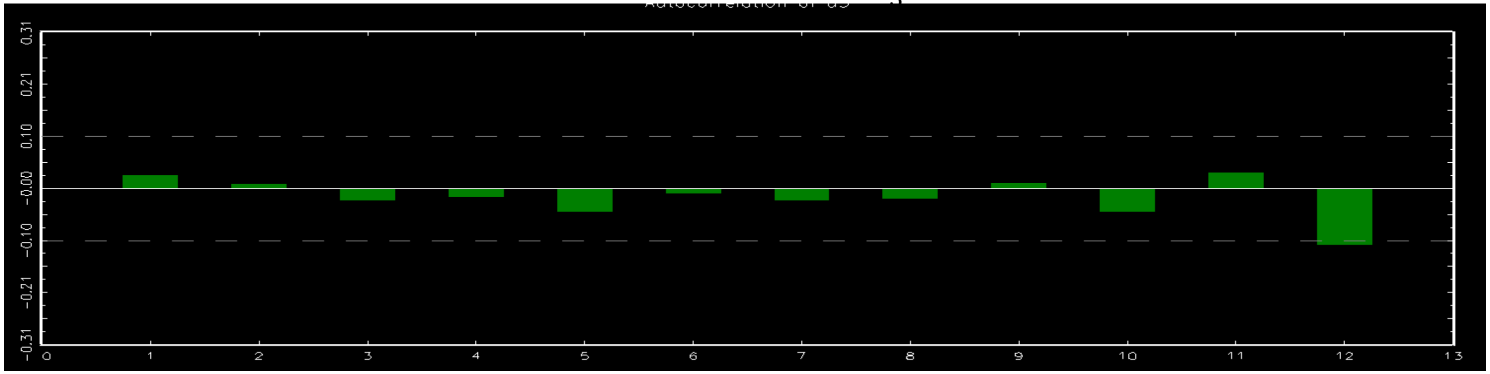
Model Checking

Using AIC lag order – p=12

Autocorrelation for u_4



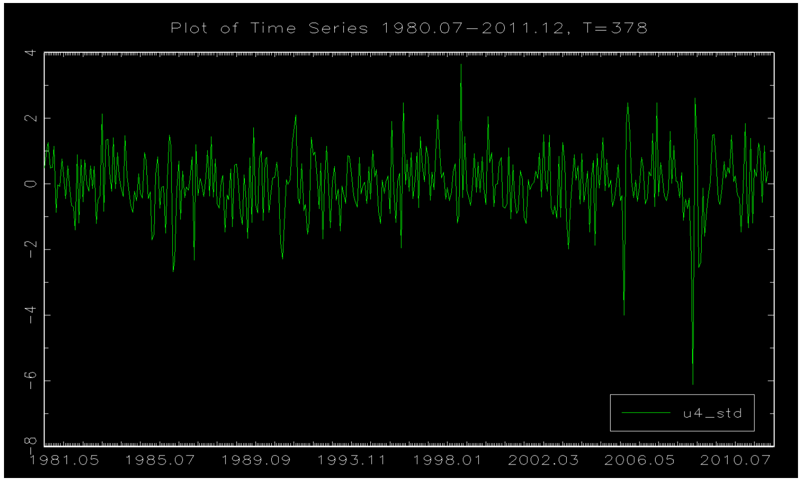
Autocorrelation for u_3



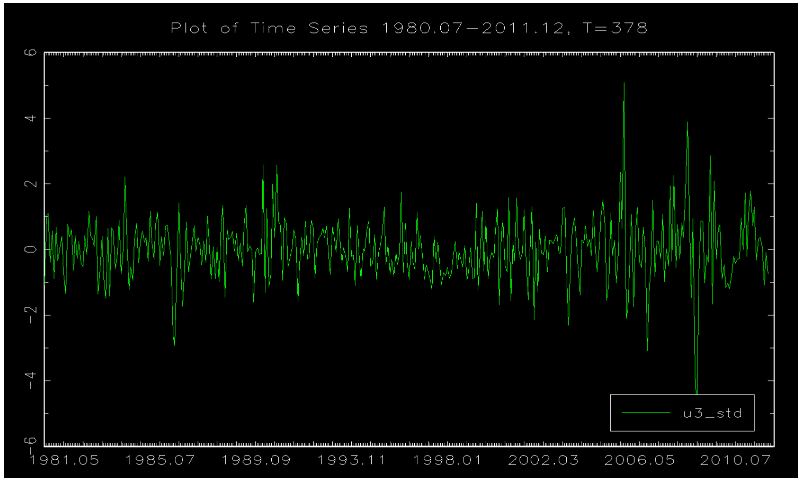
Model Checking

Using AIC lag order – p=12

Std residuals u_4



Std residuals u_3



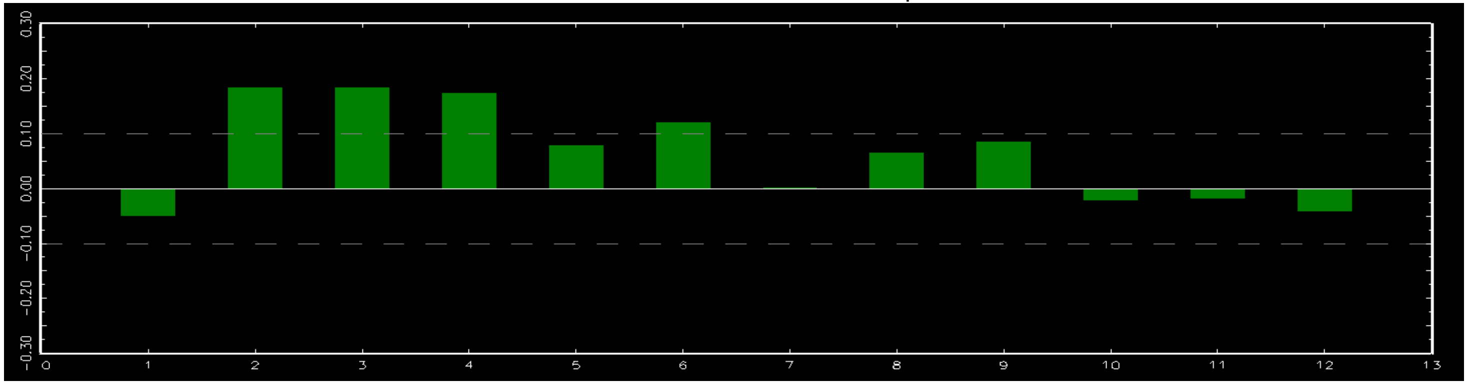
Diagnostics Check

Portmanteau Test	Non-normality test
Statistic: 615.6 p value: 0.40 Decision: ***Fail to reject*** the null	Statistic: 748.82 p value: 0.000 Decision: ***Reject*** the null

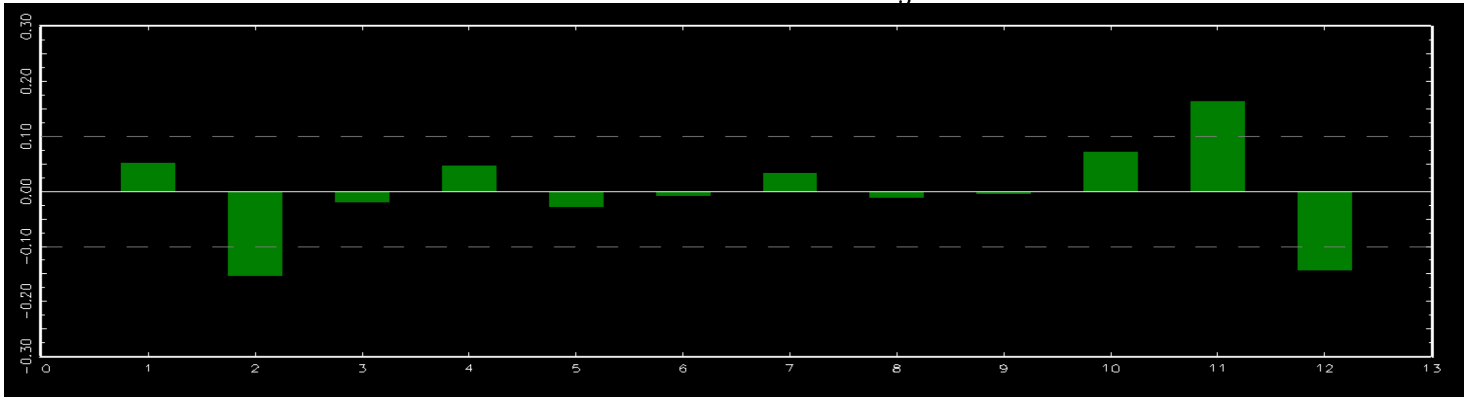
Model Checking

Using AIC lag order – p=2

Autocorrelation for u_4



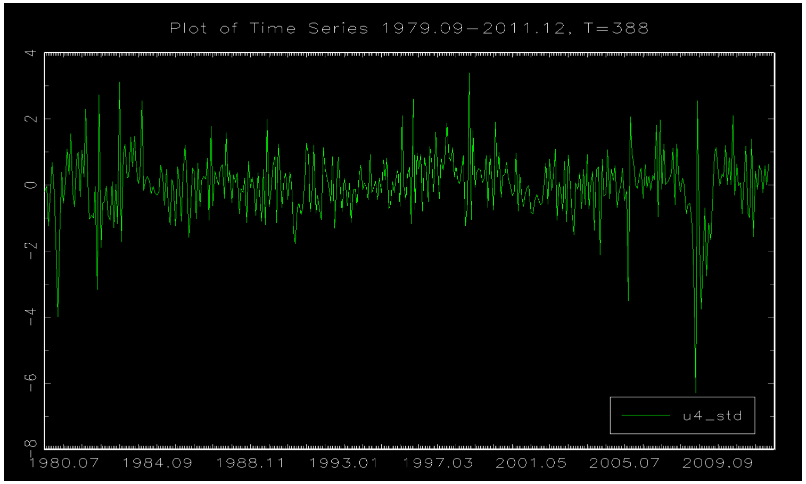
Autocorrelation for u_3



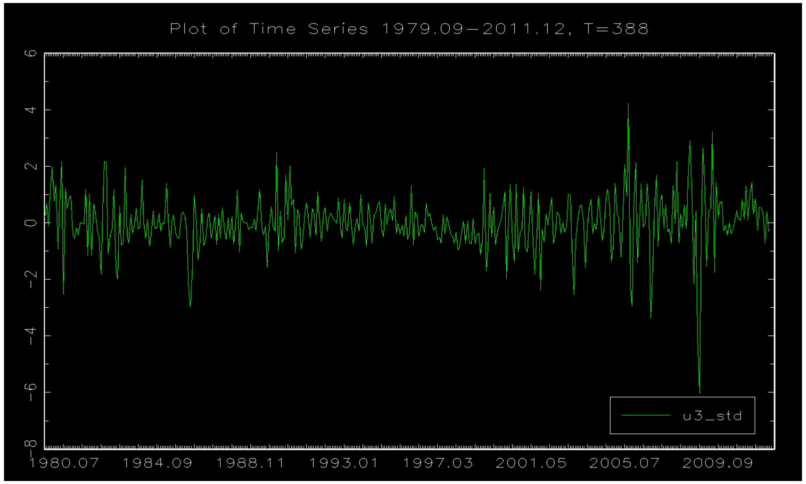
Model Checking

Using AIC lag order – p=2

Std residuals u_4



Std residuals u_3

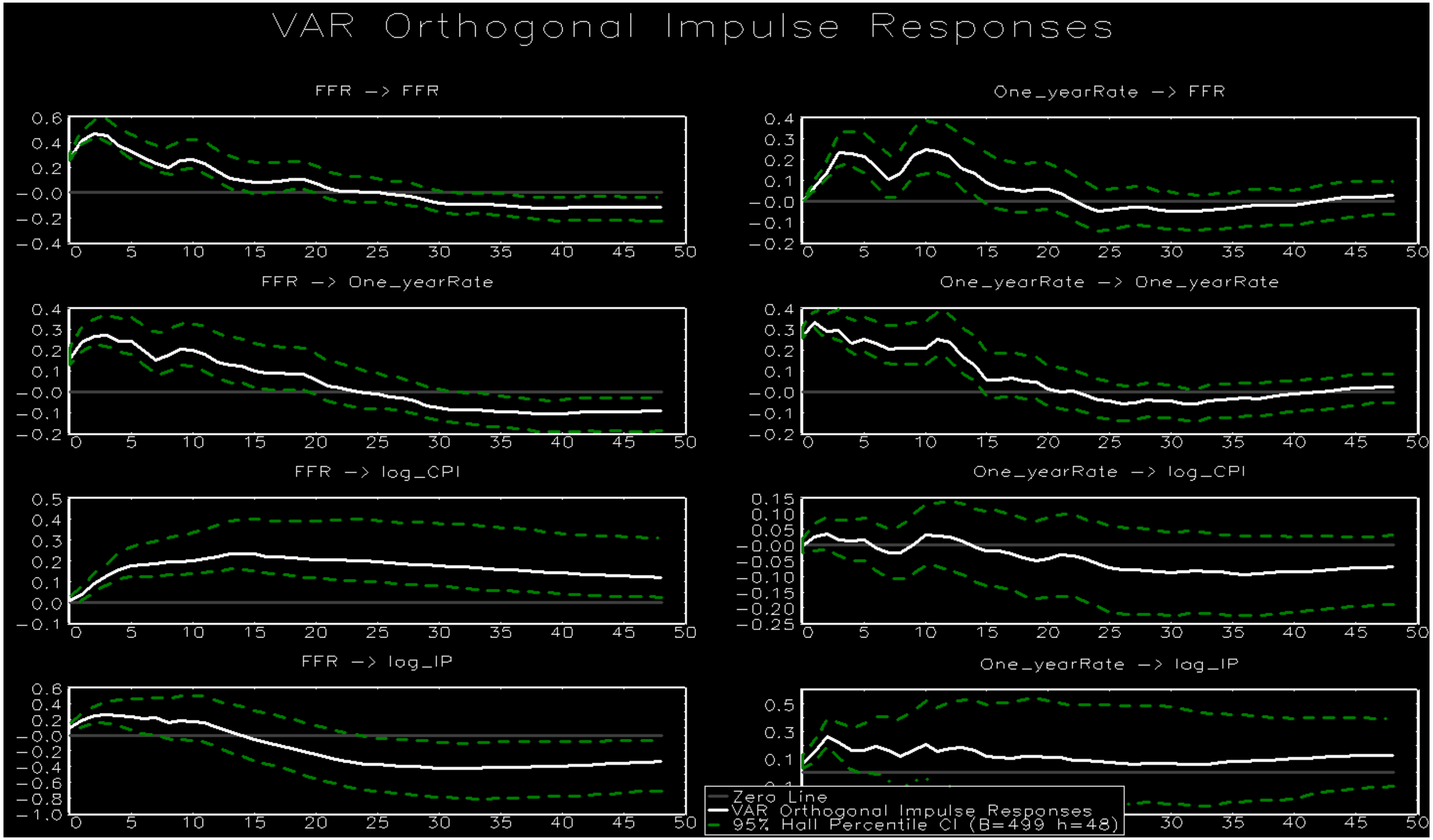


Diagnostics Check

Portmanteau Test	Non-normality test
Statistic: 928.87 p value: 0.000 Decision: ***Reject*** the null	Lutkepohl - Statistic: 14143 p value: 0.000 Decision: ***Reject*** the null

Impulse Response – B Model

Orthogonalized Forecast Error Impulse response to 2 Structural Monetary Shocks



Impulse Response – B Model

Orthogonalized Forecast Error Impulse response to 2 Structural Monetary Shocks

Effects of feds rate shock –

- significant positive impact on (short run) – Feds rate and 1 year rate
    - Rates go to zero after one year
- Significant positive impact on price level (prolonged/permanent effect)
- Significant positive impact on production levels (short run)!!!!. But negative effect on production in longer run

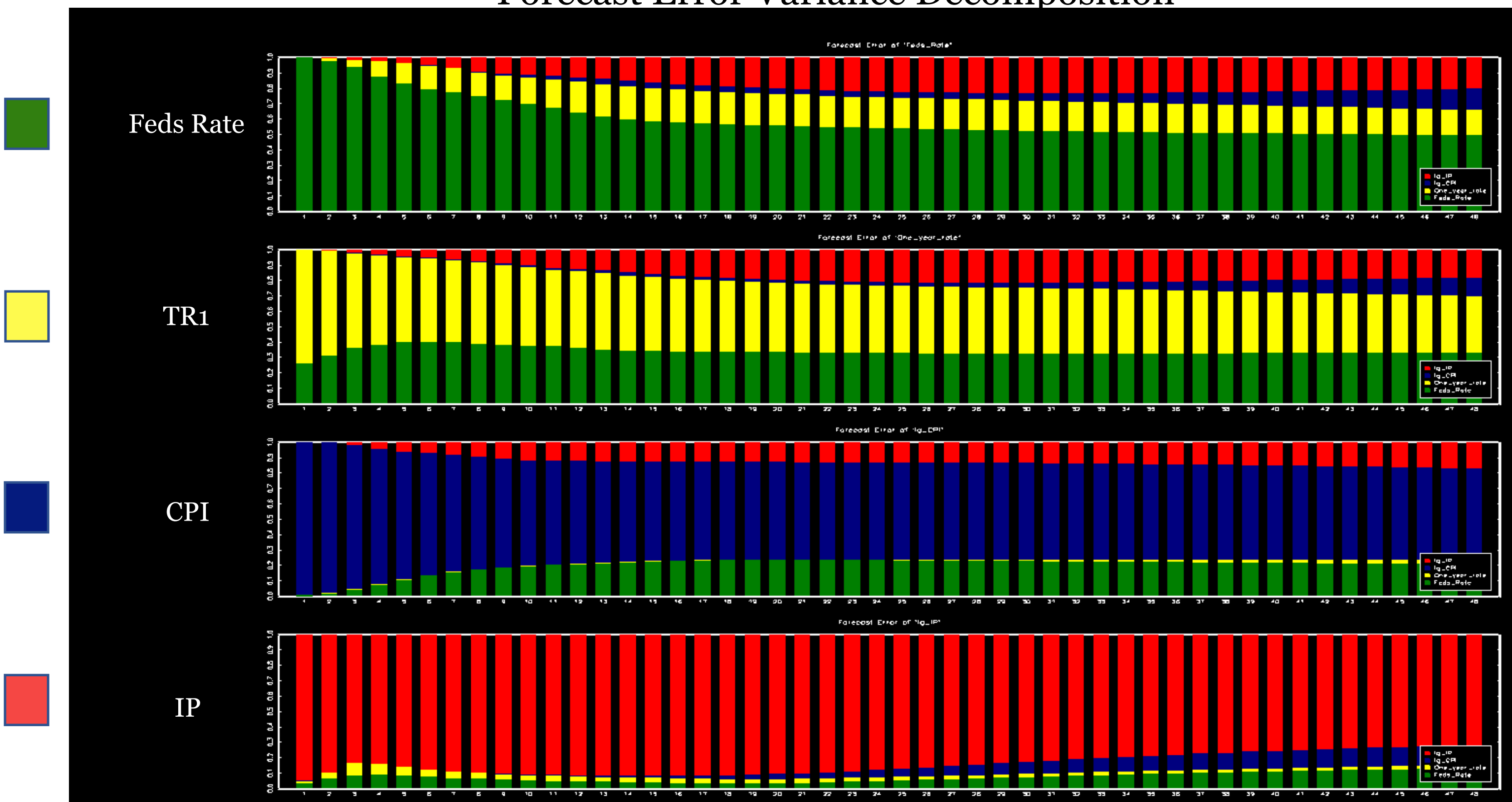
Forward Guidance Shock -

- Significant impact on both the rates.
    - Impact zeros down after 15 months
- No significant impact on price levels
- However, it does have significant positive impact on production in very short run!!!

Impulse Response – B Model

FEVD - 2 Structural Monetary Shocks

Forecast Error Variance Decomposition



Impulse Response – B Model

FEVD - 1 Structural Monetary Shocks – FFR Shock

Forecast Error Variance Decomposition