Case 1: VAR with $\lambda_1^* = 0.2\ \lambda_2^* = 0.4$

Table 1: Lag Order Frequency for VAR ($\lambda_1^* = 0.2\ \lambda_2^* = 0.4$)

	FPE								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.22	0.62	0.08	0.03	0.02	0.01	0.01	0.00	0.01
T=100	0.02	0.83	0.09	0.03	0.02	0.01	0.00	0.00	0.00
T==200	0.00	0.87	0.09	0.03	0.01	0.00	0.00	0.00	0.00

	AIC								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.22	0.61	0.08	0.03	0.02	0.01	0.01	0.01	0.01
T=100	0.02	0.83	0.09	0.03	0.02	0.01	0.00	0.00	0.00
T==200	0.00	0.86	0.09	0.03	0.01	0.00	0.00	0.00	0.00

	HQ								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.41	0.55	0.03	0.00	0.00	0.00	0.00	0.00	0.00
T=100	0.10	0.88	0.02	0.00	0.00	0.00	0.00	0.00	0.00
T==200	0.00	0.99	0.01	0.00	0.00	0.00	0.00	0.00	0.00

	SC								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.68	0.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T=100	0.32	0.68	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T==200	0.02	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2: MSPE Values for VAR(1) process ($\lambda_1^* = 0.2\ \lambda_2^* = 0.4$)

	FPE				AIC				HQ				SC		
	T=50	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**
h=1	2.20	2.12	1.99		2.21	2.12	1.99		2.16	2.11	1.99		2.19	2.15	1.99
h=2	2.11	2.05	2.04		2.12	2.05	2.04		2.05	2.02	2.03		2.04	2.02	2.03
h=4	2.11	2.07	2.04		2.12	2.07	2.04		2.08	2.06	2.03		2.07	2.06	2.03

Observations

- As length of time series increases from T=50 to T=200, lag order 1 is predicted with higher frequency for all the information criteria. This can be attributed to the fact that more data helps predict better
- The same fact could also be analyzed in observing MSPE values which decrease as length of Time series increase, suggesting more accurate forecasts as length of time series increases.
- The SC criteria, as expected, predicts lower lag orders as compared to other Information criteria

Case 2: VAR with $\lambda_1^* = 0.6\ \lambda_2^* = 0.8$

Table 3: Lag Order Frequency for VAR ($\lambda_1^* = 0.6\ \lambda_2^* = 0.8$)

	FPE								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.00	0.81	0.10	0.04	0.02	0.01	0.01	0.01	0.01
T=100	0.00	0.85	0.09	0.03	0.01	0.01	0.00	0.00	0.00
T==200	0.00	0.86	0.09	0.03	0.01	0.01	0.00	0.00	0.00

	AIC								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.00	0.79	0.10	0.04	0.02	0.01	0.01	0.01	0.01
T=100	0.00	0.85	0.09	0.03	0.01	0.01	0.00	0.00	0.00
T==200	0.00	0.86	0.09	0.03	0.01	0.01	0.00	0.00	0.00

	HQ								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.00	0.95	0.04	0.01	0.00	0.00	0.00	0.00	0.00
T=100	0.00	0.97	0.02	0.00	0.00	0.00	0.00	0.00	0.00
T==200	0.00	0.99	0.01	0.00	0.00	0.00	0.00	0.00	0.00

	SC								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.00	0.99	0.01	0.00	0.00	0.00	0.00	0.00	0.00
T=100	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T==200	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 4: MSPE Values for VAR(1) process ($\lambda_1^* = 0.6\ \lambda_2^* = 0.8$)

	FPE				**AIC**				**HQ**				**SC**		
	T=50	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**
h=1	2.20	2.12	2.00		2.21	2.12	2.00		2.13	2.10	1.99		2.11	2.09	1.98
h=2	2.22	2.11	2.06		2.24	2.11	2.06		2.14	2.09	2.05		2.12	2.09	2.05
h=4	2.26	2.14	2.08		2.29	2.14	2.08		2.19	2.13	2.08		2.18	2.12	2.08

Observations

- The HQ and SC criteria predict lag order 1 more frequently with second set of lambdas ($\lambda_1^* = 0.6\ \lambda_2^* = 0.8$) even for lower T=50 values. Hence we observe lower MSPE values (for h=1).
- For second set of lambda, the time series has higher range or more spread in values than one with lower lambdas. This provides more range of data for analysis and hence predictions may be better. We can observe this, as the second set of lambda value predict the VAR(1) model more than first set of lambda values
- For high h=4, the MSPE values for second lambda set is higher than MSPE values for first lambda set, across all time periods and Information criteria (comparing last row of MSPE of Table 1 and Table 2)

Case 3: MA with $\lambda_1^* = 0.2\ \lambda_2^* = 0.4$

Table 5: Lag Order Frequency for MA ($\lambda_1^* = 0.2\ \lambda_2^* = 0.4$)

	FPE								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.29	0.46	0.16	0.04	0.02	0.01	0.01	0.01	0.00
T=100	0.04	0.60	0.27	0.06	0.02	0.01	0.01	0.00	0.00
T==200	0.00	0.47	0.40	0.09	0.02	0.01	0.00	0.00	0.00

	AIC								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.28	0.45	0.16	0.05	0.02	0.01	0.01	0.01	0.01
T=100	0.04	0.60	0.27	0.06	0.02	0.01	0.01	0.00	0.00
T==200	0.00	0.47	0.40	0.09	0.02	0.01	0.00	0.00	0.00

	HQ								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.54	0.39	0.06	0.01	0.00	0.00	0.00	0.00	0.00
T=100	0.19	0.71	0.10	0.01	0.00	0.00	0.00	0.00	0.00
T==200	0.01	0.81	0.18	0.00	0.00	0.00	0.00	0.00	0.00

	SC								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.82	0.17	0.01	0.00	0.00	0.00	0.00	0.00	0.00
T=100	0.51	0.48	0.01	0.00	0.00	0.00	0.00	0.00	0.00
T==200	0.08	0.89	0.03	0.00	0.00	0.00	0.00	0.00	0.00

Table 6: MSPE Values for MA(1) process ($\lambda_1^* = 0.2\ \lambda_2^* = 0.4$)

	FPE				AIC				HQ				SC		
	T=50	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**
h=1	2.01	1.95	1.84		2.02	1.95	1.84		1.96	1.95	1.83		1.99	1.99	1.85
h=2	2.15	2.16	2.03		2.16	2.16	2.03		2.08	2.12	2.03		2.06	2.10	2.03
h=4	2.13	2.04	2.02		2.14	2.04	2.02		2.09	2.03	2.01		2.09	2.02	2.01

Observations

- From Table 3: comparing row h=1 with row h=2 and 3, we observe that MSPE values are smaller for h=1.
- Also, value of MSPE tend to decrease as we increase the length of time series from T=50 to T=200
- AIC and FPE predict more dispersed as well as higher lag orders compared to HQ and SC.

Case 4: MA with $\lambda_1^* = 0.6\ \lambda_2^* = 0.8$

Table 7: Lag Order Frequency for MA ($\lambda_1^* = 0.2\ \lambda_2^* = 0.4$)

	FPE								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.00	0.11	0.42	0.17	0.15	0.06	0.04	0.02	0.02
T=100	0.00	0.01	0.24	0.28	0.26	0.10	0.07	0.03	0.02
T==200	0.00	0.00	0.02	0.16	0.35	0.22	0.15	0.06	0.04

	AIC								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.00	0.10	0.40	0.17	0.15	0.06	0.05	0.03	0.04
T=100	0.00	0.01	0.24	0.27	0.25	0.10	0.07	0.03	0.03
T==200	0.00	0.00	0.02	0.16	0.35	0.22	0.15	0.06	0.04

	HQ								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.01	0.30	0.51	0.10	0.05	0.01	0.01	0.00	0.00
T=100	0.00	0.07	0.58	0.24	0.09	0.01	0.00	0.00	0.00
T==200	0.00	0.00	0.26	0.41	0.27	0.05	0.01	0.00	0.00

	SC								
	p=0	**p=1**	**p=2**	**p=3**	**p=4**	**p=5**	**p=6**	**p=7**	**p=8**
T=50	0.08	0.58	0.32	0.02	0.00	0.00	0.00	0.00	0.00
T=100	0.00	0.36	0.59	0.05	0.00	0.00	0.00	0.00	0.00
T==200	0.00	0.03	0.69	0.24	0.04	0.00	0.00	0.00	0.00

Table 8: MSPE Values for MA(1) process ($\lambda_1^* = 0.6\ \lambda_2^* = 0.8$)

	FPE				AIC				HQ				SC		
	T=50	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**		**T=50**	**T=100**	**T=200**
h=1	1.69	1.51	1.46		1.71	1.51	1.46		1.63	1.50	1.48		1.66	1.53	1.49
h=2	2.43	2.21	2.15		2.45	2.21	2.15		2.33	2.14	2.13		2.25	2.13	2.13
h=4	2.23	2.13	2.08		2.25	2.13	2.08		2.13	2.07	2.06		2.08	2.04	2.04

Observations

- AIC and FPE predict more dispersed as well as higher lag orders compared to HQ and SC.
- Comparing for MSPE for lambda set 1 and second lambda set for h=1 (first row from Table 3 and first row from Table 4), the MSPE values for second lambda set are lower. However for higher h (2 and 4) the MSPE values of first lambda set are better
    - This may be because in second lambda set MSE matrix higher has higher values (because of bigger lambda values) and so the inv is lower, thereby making the overall MSPE lower for second sets as compared to first set of lambdas
- Also, the MA(1) may have a VAR infinity representation and as we have higher T values the predicted lag orders are also in the higher range such as p=5,6,7 etc. And at the same time MSPE values are lower for higher T values, thereby suggesting MA(1) process may be better represented with higher lag order of a VAR(p) process