1. Find the angle between the line $\overrightarrow{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and the plane $\overrightarrow{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.

2. Find the co-ordinates of the point, where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the yz-plane.

3. if

$$y = 5e^{7x} + 6e^{-7x}$$

, show that $\frac{d^2y}{dx^2} = 49y$.

4. if A is a square matrix of order 2 and $\left|A\right| = 4$, then find the value of $\left|2.AA^1\right|$, where A' is the transpose of matrix A.

5. Find the order of differential equation of the family of circles of radius 3 units.

6. Find the value of $(x - y)$ from the matrix equation

$$2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} -3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$$

7. solve the equation differential equation

$$\left(y + 3x^2\right)\frac{dx}{dy} = x$$

.

8. Find

$$\int e^x (\cos x - \sin x) \csc^2 x dx$$

9. Using vectors, prove that the points $(2, -1, 3)$, $(3, -5, 1)$ and $(-1, 11, 9)$ are collinear.

10. For any two vectors $\vec{a}$ and $\vec{b}$, prove that

$$\left(\vec{a} \times \vec{b}\right)^2 = \vec{a}^2 \vec{b}^2 - \left(\vec{a} . \vec{b}\right)^2$$

11. Find

$$\int \frac{x - 1}{(x - 2)(x - 3)} \, dx$$

12. Integrate

$$\frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} \tag{1}$$

with respect to x.

13. If $P(A) = 0.6, P(B) = 0.5$ and $P(B|A) = 0.4$, find $P(A \cup B)$ and $P(A|B)$.

14. If an operation on the set of integers $\mathbb{Z}$ is defined by $a * b = 2a^2 + b$, then find (i) whether it is binary or not, and (ii) If a binary, then is it commutative or not.

15. Four cards are drawn one by one with replacement from a well-shuffeled deck of playing cards. Find the probability that atleast three cards are diamonds.

16. The Probability of two students A and B coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent, find the probability of only one of them coming to school on time.

17. if

$$x^p y^q = (x + y)^{p+q}$$

and prove that $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$.

18. Find

$$\int (\sin x \sin 2x \sin 3x)\, dx$$

19. Differentiate

$$\tan^{-1} \frac{3x - x^3}{1 - 3x^2}$$

$, |x| < \dfrac{1}{\sqrt{3}}$ w.r.t $\tan^{-1} \dfrac{x}{\sqrt{1 - x^2}}$

20. If

$$\sqrt{1 - x^2} + \sqrt{1 - y^2} = a\,(x - y)\, |x| < 1, |y| < 1$$

, show that $\dfrac{dy}{dx} = \sqrt{\dfrac{1 - y^2}{1 - x^2}}$

21. Find the particular solution of the differential equation :

$$x\frac{dy}{dx} \sin\left(\frac{y}{x}\right) + x - y \sin\left(\frac{y}{x}\right) = 0$$

, given that $y(1) = \dfrac{\pi}{2}$.

22. Find the particular solution of the differential equation :

$$\left(1 + e^{2x}\right) dy + \left(1 + y^2\right) e^x dx = 0$$

given that $y(0) = 1$.

23. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation.

24. Show that the function f in $A = R - \{\dfrac{2}{3}\}$ defined as $f(x) = \dfrac{4x + 3}{6x - 4}$ is one-one and onto. Hence, find f^{-1}.

25. Find whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$; is increasing or decreasing in the interval $\dfrac{3\pi}{8} < x < \dfrac{5\pi}{8}$.

26. Find the equation of the plane passing through the point $((-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

27. Prove that

$$\sin^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{12} + \cos^{-1}\frac{63}{65} = \frac{\pi}{2}$$

28. Evaluate

$$\int_0^1 (|x-1| + |x-2| + |x-4|)\,dx$$

29. Using vectors find the value of x such that the four points $A\,(x, 5, -1)$, $B\,(3, 2, 1)$,$C\,(4, 5, 5)$ and $D\,(4, 2, -2)$ are coplanar.

30. If x, y, z are the different and $\triangle = \begin{vmatrix} x & x^2 & x^3-1 \\ y & y^2 & y^3-1 \\ z & z^2 & z^3-1 \end{vmatrix} = 0$, then using properties of determinants , show that $xyz = 1$.

31. Using integration, find the area of $\triangle ABC$ bounded by the lines $4x-y+5 = 0$, $x + y - 5 = 0$ and $x - 4y + 5 = 0$.

32. Using integration, find the area of the following region: $(x, y) : x^2 + y^2 \leq 16a^2$ and $y^2 \leq 6ax$

33. Find the vector equation of the line passing through $(2, 1, -1)$ and parallel to the line $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$. Also, find the distance between these two lines.

34. Find the coordinates of the foot Q of the perpendicular drawn from the point $P\,(1, 3, 4)$ to the plane $2x-y+z+3 = 0$. Find the distance PQ and the image of P treating the plane as a mirror.

35. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 4 hours for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours for assembling. The Profit for type A souvenirs is 100 rupees each and for type B souvenirs, profit is of 120 rupees each.How

many souvenirs of each type should the company manufacture in order to maximize the profit ? Formulate the problem as LPP and then solve it graphically.

36. In answering a question on a multiple choice questons with four choices in each question, out of which only one is correct, a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability that he copies is also $\frac{1}{4}$. The probability that the answer is correct, given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to th question, given that he correctly answered it.

37. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

38. Using elementry row transformations, find the inverse of the matrix $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

39. Using matrices, solve the following system of linear equations:

$$\begin{aligned} 2x + 3y + 10z &= 4 \\ 4x + 6y + 5z &= 1 \\ 6x + 9y - 20z &= 2 \end{aligned}$$