1 Vector Algebra

1. Find the angle between the line $\overrightarrow{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and the plane $\overrightarrow{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.

2. Find the co-ordinates of the point where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the yz-plane.

3. Using vectors, prove that the points $(2, -1, 3)$, $(3, -5, 1)$ and $(-1, 11, 9)$ are collinear.

4. For any two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ prove that

$$(\overrightarrow{\mathbf{a}} \times \overrightarrow{\mathbf{b}})^2 = \overrightarrow{\mathbf{a}}^2 \overrightarrow{\mathbf{b}}^2 - (\overrightarrow{\mathbf{a}} . \overrightarrow{\mathbf{b}})^2$$

5. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

6. Find the vector equation of the line passing through $(2, 1, -1)$ and parallel to the line $\overrightarrow{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$. Also find the distance between these two lines.

7. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

8. Find the coordinates of the foot Q of the perpendicular drawn from the point $P(1, 3, 4)$ to the plane $2x - y + z + 3 = 0$. Find the distance PQ and the image of P treating the plane as a mirror.

9. Using vectors find the value of $\mathbf{x}$ such that the four points $A(x, 5, -1)$, $B(3, 2, 1)$, $C(4, 5, 5)$ and $D(4, 2, -2)$ are co-planar.

2 Matrices

1. If A is a square matrix of order 2 and $\left|A\right| = 4$, then find the value of $\left|2.AA'\right|$ where A' is the transpose of matrix A.

2. Find the value of $(x-y)$ from the matrix equation

$$2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} -3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$$

3. If x, y, z are the different and $\triangle = \begin{vmatrix} x & x^2 & x^3-1 \\ y & y^2 & y^3-1 \\ z & z^2 & z^3-1 \end{vmatrix} = 0$ then using properties of determinants show that $xyz = 1$.

4. Using elementary row transformations find the inverse of the matrix $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

5. Using matrices solve the following system of linear equations:

$$\begin{aligned} 2x + 3y + 10z &= 4 \\ 4x + 6y + 5z &= 1 \\ 6x + 9y - 20z &= 2 \end{aligned}$$

6. Find the value of $(x-y)$ from the matrix equation

$$2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} -3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$$

3 Functions and Relations

1. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ given by R=(a, b) : $|a - b|$ is even, is an equivalence relation.

2. Show that the function f in $A = R - \{\frac{2}{3}\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}.

3. Find whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ is increasing or decreasing in the interval $\frac{3\pi}{8} < x < \frac{5\pi}{8}$.

4. If an operation on the set of integers $\mathbb{Z}$ is defined by $a * b = 2a^2 + b$, then find (i) whether it is binary or not and (ii) If a binary then is it commutative or not.

4 Integrations

1. Find

$$\int e^x (\cos x - \sin x) \csc^2 x dx$$

2. Find

$$\int \frac{x-1}{(x-2)(x-3)}\, dx$$

3. Integrate

$$\frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}$$

with respect to x.

4. Find

$$\int (\sin x \sin 2x \sin 3x)\, dx$$

5. Evaluate

$$\int_0^1 (|x-1| + |x-2| + |x-4|)\, dx$$

6. Find :

$$\int e^x \left(\frac{2 + \sin 2x}{2\cos^2 x}\right) dx$$

5 Differentiation

1. If

$$y = 5e^{7x} + 6e^{-7x}$$

   show that $\dfrac{d^2y}{dx^2} = 49y$

2. if

$$x^p y^q = (x+y)^{p+q}$$

   and prove that $\dfrac{dy}{dx} = \dfrac{y}{x}$ and $\dfrac{d^2y}{dx^2} = 0$

3. Differentiate

$$\tan^{-1} \frac{3x - x^3}{1 - 3x^2}$$

   $|x| < \dfrac{1}{\sqrt{3}}$ w.r.t $\tan^{-1} \dfrac{x}{\sqrt{1-x^2}}$

4. If

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a\,(x-y) \quad |x| < 1, \left|y\right| < 1$$

   show that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$

5. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of the triangle is maximum when $\theta = \dfrac{\pi}{6}$.

6 Differential Equations

1. Find the order of differential equation of the family of circles of radius 3 units.

2. Solve the equation differential equation

$$\left(y + 3x^2\right)\frac{dx}{dy} = x\,dx$$

3. Find the particular solution of the differential equation :

$$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0$$

given that $y(1) = \frac{\pi}{2}$

4. Find the particular solution of the differential equation :

$$\left(1 + e^{2x}\right)dy + \left(1 + y^2\right)e^x dx = 0$$

given that $y(0) = 1$

5. Find the differential equation representing the family of curves

$$y = -A\cos 3x + B\sin 3x$$

6. Find the differential equation of the function $\cos^{-1}(\sin 2x)$ w.r.t. x.

7 Probability

1. If $P(A) = 0.6, P(B) = 0.5$ and $P\left(\frac{B}{A}\right) = 0.4$ find $P(A \cup B)$ and $P\left(\frac{A}{B}\right)$.

2. Four cards are drawn one by one with replacement from a well-shuffled deck of playing cards. Find the probability that at least three cards are diamonds.

3. The Probability of two students A and B coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$ respectively, Assuming that the events 'A coming on time' and 'B coming on time' are independent find the probability of only one of them coming to school on time.

4. In answering a question on a multiple choice questions with four choices in each question out of which only one is correct a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability that he copies is also $\frac{1}{4}$. The probability that the answer is correct given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to the question given that he correctly answered it.

5. If A and B are independent events with $P(A) = \frac{3}{7}$ and $P(B) = \frac{2}{5}$, then find $P(A' \cap B')$

8 Trigonometric Identities

1. Prove that

$$\sin^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{12} + \cos^{-1}\frac{63}{65} = \frac{\pi}{2}$$

9 Linear Programming

1. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 4 hours for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours for assembling. The Profit for type A souvenirs is 100 rupees each and for type B souvenirs, profit is of 120 rupees each. How many souvenirs of each type should the company manufacture in order to maximize the profit? Formulate the problem as LPP and then solve it graphically.