1. Find the angle between the line $\mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and the plane $\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 3$.

2. Find the co-ordinates of the point, where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the yz-plane.

3. if

$$y = 5e^{7x} + 6e^{-7x}$$

, show that $\frac{d^2y}{dx^2} = 49y$.

4. if A is a square matrix of order 2 and $\left|A\right| = 4$, then find the value of $\left|2AA'\right|$, where A' is the transpose of matrix A.

5. Find the order of differential equation of the family of circles of radius 3 units.

6. Find the value of $(x-y)$ from the matrix equation $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

7. solve the equation differential equation

$$(y + 3x^2)\frac{dx}{dy} = x$$

.

8. Find

$$\int e^x(\cos x - \sin x)\csc^2 x dx$$

9. Using vectors, prove that the points $(2, -1, 3), (3, -5, 1)$ and $(-1, 11, 9)$ are collinear.

10. For any two vectors $\mathbf{a}$ and $\mathbf{b}$, prove that

$$(\mathbf{a} * \mathbf{b})^2 = \mathbf{a}^2\mathbf{b}^2 - (\mathbf{a}.\mathbf{b})^2$$

11. Find

$$\int \frac{x-1}{(x-2)(x-3)}\,dx$$

12. Integrate

$$\frac{e^x}{\sqrt{5-4e^x-e^{2x}}}$$

with respect to x.

13. If $P(A) = 0.6, P(B) = 0.5$ and $P(B|A) = 0.4$, find $P(A \cup B)$ and $P(A|B)$.

14. If an operation on the set of integers $\mathbb{Z}$ is defined by $a * b = 2a^2 + b$, then find (i) whether it is binary or not, and (ii) If a binary, then is it commutative or not.

15. Four cards are drawn one by one with replacement from a well-shuffeled deck of playing cards. Find the probability that atleast three cards are diamonds.

16. The Probability of two students A and B coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent, find the probability of only one of them coming to school on time.

17. if

$$x^p y^q = (x+y)^{p+q}$$

prove that $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$.

18. Find

$$\int (\sin x \sin 2x \sin 3x)dx$$

19. Difference

$$\tan^{-1} \frac{3x-x^3}{1-3x^2}$$

, $|x| < \frac{1}{\sqrt{3}}$ w.r.t $\tan^{-1} \frac{x}{\sqrt{1-x^2}}$

20. If

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

, $|x| < 1, |y| < 1$, show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

21. Find the particular solution of the differential equation :

$$x\frac{dy}{dx}\sin\frac{y}{x} + x - y\sin\frac{y}{x} = 0$$

, given that $y(1) = \frac{\pi}{2}$.

22. Find the particular solution of the differential equation :

$$(1+e^{2x})\,dy + (1+y^2)e^x\,dx = 0$$

given that $y(0) = 1$.

23. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation.

24. Show that the function f in $A = \mathbb{R} - \{\frac{2}{3}\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1}.

25. Find whether the function $f(x) = \cos(2x + \frac{\pi}{4})$; is increasing or decreasing in the interval $\frac{3\pi}{8} < x < \frac{5\pi}{8}$.

26. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

27. Prove that

$$\sin^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{12} + \cos^{-1}\frac{63}{65} = \frac{\pi}{2}$$

28.

$$\int_0^1 (|x-1| + |x-2| + |x-4|)\,dx$$

29. Using vectors find the value of x such that the four points $A(x, 5, -1)$, $B(3, 2, 1)$,$C(4, 5, 5)$ and $D(4, 2, -2)$ are coplanar.

30. If x, y, z are the different and $\triangle = \begin{vmatrix} x & x^2 & x^3 - 1 \\ y & y^2 & y^3 - 1 \\ z & z^2 & z^3 - 1 \end{vmatrix} = 0$, then using properties of determinants , show that $xyz = 1$.

31. Using integration, find the area of triangle ABC bounded by the lines $4x - y + 5 = 0$, $x + y - 5 = 0$ and $x - 4y + 5 = 0$.

32. Using integration, find the area of the following region:

$$(x, y) : x^2 + y^2 \leq 16a^2 and y^2 \leq 6ax$$

33. Find the vector equation of the line passing through $(2, 1, -1)$ and parallel to the line $\mathbf{r}$=$(\mathbf{i} + \mathbf{j}) + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$. Also, find the distance between these two lines.

34. Find the coordinates of the foot Q of the perpendicular drawn from the point $P(1, 3, 4)$ to the plane $2x - y + z + 3 = 0$. Find the distance PQ and the image of P treating the plane as a mirror.

35. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 4 hours for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours for assembling. The Profit for type A souvenirs is 100 rupees each and for type B souvenirs, profit is of 120 rupees each.How many souvenirs of each type should the company manufacture in order to maximize the profit ? Formulate the problem as LPP and then solve it graphically.

36. In answering a question on a multiple choice questons with four choices in each question, out of which only one is correct, a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability that he copies is also $\frac{1}{4}$. The probability that the answer is correct, given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to th question, given that he correctly answered it.

37. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

38. Using elementry row transformations, find the inverse of the matrix $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

39. Using matrices, solve the following system of linear equations: $2x + 3y + 10z = 4$, $4x + 6y + 5z = 1$, $6x + 9y - 20z = 2$