DESIGN AND ANALYSIS OF TIME-DELAYED CONTROL SYSTEM

End Semester Evaluation Report

BACHELOR OF TECHNOLOGY
IN
ELECTRICAL ENGINEERING

Submitted by

Aahan (192092)
Shivangi Pokhriyal (192090)
Vivek Faujdar (192094)
Vivek Sharma (192083)
Krishna Kumar (192123)

DEPARTMENT OF ELECTRICAL ENGINEERING
NATIONAL INSTITUTE OF TECHNOLOGY
HAMIRPUR

ACKNOWLEDGEMENT

Firstly, we would like to thank the Director of *National Institute of Technology Hamirpur, Himachal Pradesh* for giving us the opportunity to do a project within the organization. We also would like to thank all the people that worked along with us at the *National Institute of Technology Hamirpur Himachal Pradesh*. With their patience and openness, they created an enjoyable working environment. It is indeed with a great sense of pleasure and immense sense of gratitude that we acknowledge the help of these individuals. We are highly indebted to Director Prof. Hiralal Murlidhar Suryawanshi and Head of Department, Electrical Engineering Deptt. Dr. R .K. Jarial, for the facilities provided to accomplish this Project. We would like to thank Dr. Veena Sharma, Department of EE for their support and advice to get and complete the Project in the above-said organization. We are extremely grateful to our department staff members and friends who helped us in the successful completion of this Project.

Aahan
(Roll No: 192092)
Signature:

Shivangi Pokhriyal
(Roll No: 192090)
Signature:

Vivek Faujdar
(Roll No: 192094)
Signature:

Vivek Sharma
(Roll No: 192083)
Signature

Krishna Kumar
(Roll No: 192123)
Signature

NATIONAL INSTITUTE OF TECHNOLOGY HAMIRPUR (HP)

CANDIDATE'S DECLARATION

We hereby certify that the work which is being presented in the project titled "**Design and Analysis of Time Delayed Control System**" in partial fulfillment of the requirements for the award of the Degree of Bachelor of Technology and submitted in the Electrical Department, National Institute of Technology Hamirpur, is an authentic record of our own work carried out during a period from July 2022 to Dec 2022 under the supervision of **Dr. Veena Sharma**, Electrical Department, National Institute of Technology, Hamirpur.

The matter presented in this project report has not been submitted by us for the award of any other degree of this or any other Institute/University.

Aahan
(Roll no: 192092)

Shivangi Pokhriyal
(Roll no: 192090)

Vivek Faujdar
(Roll no: 192094)

Vivek Sharma
(Roll no: 192083)

Krishna Kumar
(Roll no: 192123)

This is to certify that the above statement made by the candidates is correct to the best of my knowledge.

(Dr. Veena Sharma)

NATIONAL INSTITUTE OF TECHNOLOGY HAMIRPUR
HAMIRPUR-177005, HIMACHAL PRADESH, INDIA
DEPARTMENT OF ELECTRICAL ENGINEERING

CERTIFICATE

This is to certify that the Major Project report entitled **"Design and Analysis of Time Delayed Control System"** submitted by Aahan (Roll no: 192092), Shivangi Pokhriyal (Roll no: 192090), Vivek Faujdar (Roll no: 192094), Vivek Sharma (Roll no: 192083) and Krishna Kumar (Roll no: 192123) is a record of bonafide work carried out by them in partial fulfillment of the requirement for the award of the degree of "***Bachelor of Technology in Electrical Engineering***".

Date: **Place: HAMIRPUR**

Signature

(Mentor)

Dr. Veena Sharma,

Associate Dean

Department of Electrical Engineering

NIT Hamirpur

Signature and date

Head of Department

(Department of Electrical Engineering

TABLE OF CONTENTS

ABSTRACT

Time delay is a delay between command response and the start of the output response. Time delay in systems arises from inherent time delays in the components of the systems, or from the deliberate introduction of time delays into the systems for control purposes.

Time-delay systems can be represented by delay differential equations (DDEs), which belong to the class of functional differential equations, and have been extensively studied over the past decades. Such time delays can limit and degrade the achievable performance of controlled systems, and even induce instability. Controlling such processes is challenging because delays cause linear phase shifts that limit the control bandwidth and affect closed-loop stability. Time-delay terms lead to an infinite number of roots of the characteristic equation, making systems difficult to analyze with classical methods, especially in checking the stability and designing stabilizing controllers. Thus, such problems are often solved indirectly by using approximation.

Most of the time, for non-linear systems, such as the heat exchanger process, plant models are affected by uncertainties and cannot be modeled exactly. These processes may exhibit gain and time-constant changes as well as varying time delays which need to be compensated robustly in order to maintain the closed loop specifications both under nominal conditions and under time delay uncertainties.

In order to deal with time delay we modeled the heating system using three different methods, i.e., PI controller, Smith Predictor, and Modified Smith Predictor.

INTRODUCTION

Time delay is a delay between the command response and the start of the output response. Time-delay systems arise from inherent time delays in the components of the systems, or from the deliberate introduction of time delays into the systems for control purposes.

Time-Delay Systems are also called systems with aftereffects or dead time, hereditary systems, equations with deviating arguments, or differential-difference equations.
Many processes involve dead times, also referred to as transport delays or time lags.

Control systems often operate in the presence of delays, primarily due to the time it takes to acquire the information needed for decision-making, create control decisions, and execute them. Actuators, sensors, and field networks involved in feedback loops usually introduce delays.

Thus, delays are strongly involved in challenging areas of communication and information technologies: stability of networked control systems (NCSs) or high-speed communication networks

Time-delay systems can be represented by delay differential equations (DDEs), which belong to the class of functional differential equations, and have been extensively studied over the past decades. Such time delays can limit and degrade the achievable performance of controlled systems, and even induce instability. Controlling such processes is challenging because delays cause linear phase shifts that limit the control bandwidth and affect closed-loop stability. Time-delay terms lead to an infinite number of roots of the characteristic equation, making systems difficult to analyze with classical methods, especially in checking the stability and designing stabilizing controllers. Thus, such problems are often solved indirectly by using approximation. A widely used approximation method is the ***Pade approximation***, which is a rational approximation and results in a shortened fraction as a substitute for the exponential time-delay term in the equation. However, such an approach constitutes a limitation in accuracy, can lead to instability of the actual system, and induce minimum phase and, thus, high-gain problems.
Prediction-based methods (e.g., ***Smith predictor***, ***finite spectrum assignment (FSA)***, and ***adaptive Posicast***) have been used to stabilize time-delay systems by transforming the problem into a non-delay system. Controllers have also been designed using the ***Lyapunov framework***.

2. APPROXIMATION METHODOLOGY

RATIONAL APPROXIMATION OF TIME DELAY

Time delay can be defined as a delay attributable to the time taken in transporting material or to the finite rate of propagation of a signal. The method of approximation of a time delay transfer function arises from the development of the exponential function.

APPROXIMATION OF TIME DELAY TRANSFER FUNCTION USING TAYLOR SERIES

The Taylor series of a real or complex-valued function $f(x)$ that is infinitely differentiable at a real or complex number a is the power series

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots$$

where $n!$ denotes the factorial of n. In the more compact sigma notation, this can be written as

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$e^{-sT}=1-\frac{(sT)}{1!}+\frac{(sT)^2}{2!}+\cdots+(-1)^n\frac{(sT)^n}{n!}+\cdots$$

PADE APPROXIMATION METHOD

In mathematics, a **Padé approximant** is the "best" approximation of a function near a specific point by a rational function of a given order. Under this technique, the approximant's power series agrees with the power series of the function it is approximating.

An often-used method of rational approximation of the time delay transfer function is the application of the Padé approximant. The Padé approximant enables the approximation of more complex functions using rational functions. The coefficients of both polynomials are calculated so that the first terms vanish in the Taylor series of the approximated function.

Given a function f(x) and two integers $m \geq 0$ and, $n \geq 0$ the Pade approximant of the order [m/n] is the rational function R(x):

$$R(x)=\frac{\sum_{j=0}^{m}a_jx^j}{1+\sum_{k=1}^{n}b_kx^k}=\frac{a_0+a_1x+a_2x^2+\cdots+a_mx^m}{1+b_1x+b_2x^2+\cdots+b_nx^n}$$

PADE TABLE

A Padé table is an infinite two-dimensional array of Padé approximations to a given function. This method has an important feature in that there are no restrictions on the degrees m and n of both polynomials. They may be the same but also can differ from each other. The equation set can be solved in general and so the explicit formulas for the coefficients of both polynomials can be obtained. The formulas for the same degrees of polynomials (m=n) can be found in the form:

m — n	0	1	2	3
0	$\frac{1}{1}$	$\frac{1-x}{1}$	$\frac{1-x+\frac{x^2}{2}}{1}$	$\frac{1-x+\frac{x^2}{2}-\frac{x^3}{6}}{1}$
1	$\frac{1}{1+x}$	$\frac{1-\frac{x}{2}}{1+\frac{x}{2}}$	$\frac{1-\frac{2x}{3}+\frac{x^2}{6}}{1+\frac{x}{3}}$	$\frac{1-\frac{3x}{4}+\frac{x^2}{4}-\frac{x^3}{24}}{1+\frac{x}{4}}$
2	$\frac{1}{1+x+\frac{x^2}{2}}$	$\frac{1-\frac{x}{3}}{1+\frac{2x}{3}+\frac{x^2}{6}}$	$\frac{1-\frac{x}{2}+\frac{x^2}{12}}{1+\frac{x}{2}+\frac{x^2}{12}}$	$\frac{1-\frac{3x}{5}+\frac{3x^2}{20}-\frac{x^3}{60}}{1+\frac{2x}{5}+\frac{x^2}{20}}$
3	$\frac{1}{1+x+\frac{x^2}{2}+\frac{x^3}{6}}$	$\frac{1-\frac{x}{4}}{1+\frac{3x}{4}+\frac{x^2}{4}+\frac{x^3}{24}}$	$\frac{1-\frac{2x}{5}+\frac{x^2}{20}}{1+\frac{3x}{5}+\frac{3x^2}{20}+\frac{x^3}{60}}$	$\frac{1-\frac{x}{2}+\frac{x^2}{10}-\frac{x^3}{120}}{1+\frac{x}{2}+\frac{x^2}{10}+\frac{x^3}{120}}$

IMPORTANCE

Padé approximations are usually superior to Taylor series when functions contain poles because the use of rational functions allows them to be well-represented.

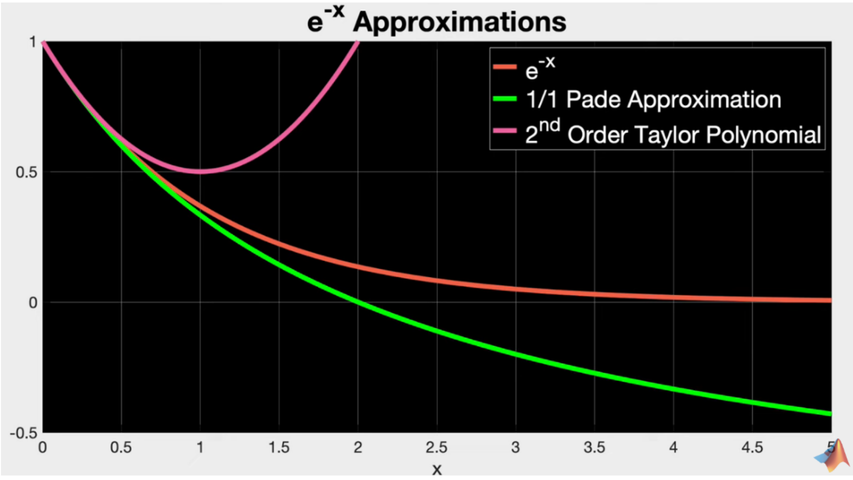
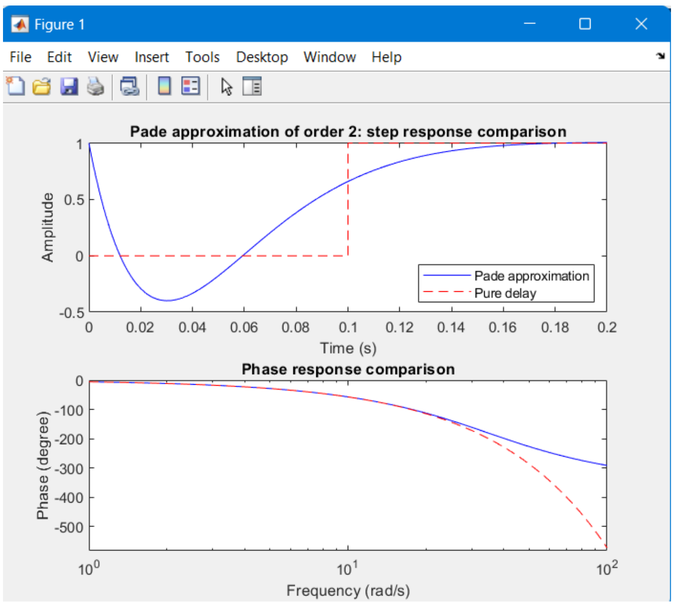


Figure 2.1

Matlab Simulation of Pade Approximation

```
tau=2;
Gs0=tf(12,conv([1,4],[1,6]));
Gs=Gs0*tf(1,1,'IODelay',tau);
Gs1=pade(Gs,1);
Gs2=pade(Gs,2);
Gs3=pade(Gs,3);
Gs1
Gs2
%step(Gs,'k',Gs1,'k',Gs2)
pade(0.1,2)
```



SMITH PREDICTOR FOR CONTROL PROCESS WITH LONG DEAD TIME

Smith predictor which was first presented by O. J. M. Smith in 1957 was introduced to improve control performances.

In order to eliminate the long delay time which significantly improves the performance of the system, the smith predictor with PID or Dead Time compensator is used which mainly works on the closed loop and can be used to analyze open loop characteristics of the system.

The proposed approach Smith Predictor combines PID Process will eliminate the Long delay time in the system. It is a model-based controller that is effective for processes with long dead time. Smith Predictor control is theoretically a good solution to the problem of controlling the time delay systems. This approach improves the Performance and robustness of the system in real-time applications.

The Smith predictor technique uses a process model to predict future values of the output variable. The control calculations are based on both the predicted values and the current value of the output.

Existing System

- The proportional integral derivative (PID) control algorithm is widely used in process industries because of its simplicity, robustness, and successful practical application.
- A PID control system can suffice for many industrial control loops.

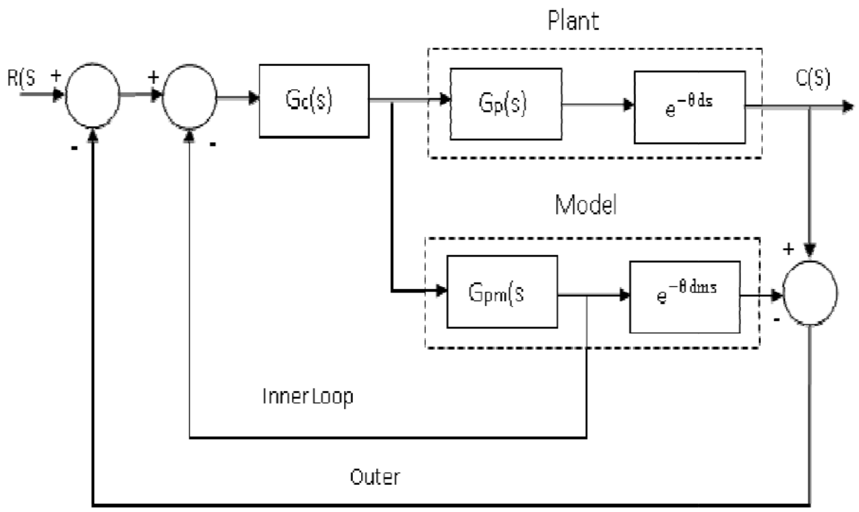
The PID controller is the most common controller used in industry. The following is the basic algorithm of the PID Controller which has several variants of the basic algorithm.

$u(t) = K[\, e(t) + 1/T_i \int e(\boldsymbol{\tau})d\boldsymbol{\tau} + T_d . de(t)/dt \,]$ ----------------------------- (1)

$e(t) = y_r(t) - y(t)$ ----------------------------- (2)

Where u is the control signal, e is the error signal, yr the reference signal, y is the process variable, K the gain, Ti the integration time, and Td the derivative time

The following diagram represents the structure of Smith Predictor



We can see that there is a time delay item e^-**θ**ds characteristic equation which could produce phase lag and make the system unstable

Then the closed loop transfer function with Smith predictor

$$\frac{C(s)}{R(s)} = \frac{G_c(s)G_p(s)e^{-\theta ds}}{1+G_c(s)G_p(s)+G_c(s)G_p(s)e^{-\theta ds}-G_c(s)G_{pm}(s)e^{-\theta dms}}$$

Smith predictor is in the dotted line textbox, and Gc(s) is a simple PID controller, Gp(s) is the transfer function plant, $e^{-\theta ds}$ is Dead time of the plant. Gpm(s) is the transfer function for the model of the plant. $e^{-\theta dms}$ is the dead time of the plant model. The Smith Predictor Plant and Model both have the same transfer function with dead time. If without the Smith predictor, the closed loop transfer function would be

$$\frac{C(s)}{R(s)} = \frac{G_c(s)G_p(s)e^{-\theta ds}}{1+G_c(s)G_p(s)e^{-\theta ds}}$$

And the close loop characteristic equation is:

$$1 + G_c(s)G_p(s)e^{-\theta ds} = 0$$

In the case of Gp = Gm and **θ**d = **θ**dm. The transfer function can be written as:

$$\frac{C(s)}{R(s)} = \frac{G_c(s)G_p(s)e^{-\theta ds}}{1+G_c(s)G_p(s)}$$

And the corresponding characteristic equation is:

$$1 + G_c(s)G_p(s) = 0$$

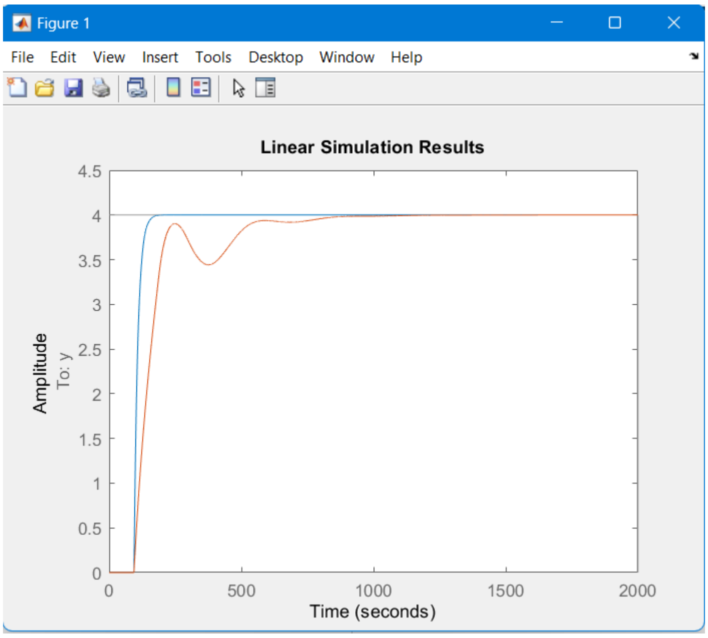
The time delay item is eliminated in the characteristic equation which will improve the control performance significantly.

Matlab Simulation of Smith Prediction

Editor - C:\Users\krish_ohljf3h\Downloads\smith.m *

smith.m *

```
% Smith predictor
s = tf('s');

% Building LTI blocks

P = exp(-93.9*s)*5.6/(40.2*s+1);
P.InputName = 'u';
P.OutputName = 'y';

Gp = 5.6/(40.2*s+1);
Gp.InputName = 'u';
Gp.OutputName = 'yp';

Dp = exp(-93.9*s);
Dp.InputName = 'yp';
Dp.OutputName = 'y1';

C = 0.5*(1+1/(40*s));
C.InputName = 'e';
C.OutputName = 'u';

F = 1/(20*s+1);
F.InputName = 'dy';
F.OutputName = 'dp';

% Building summation blocks

Sum1 = ss([1 -1 -1],'InputName',{'ysp','yp','dp'},'OutputName','e');
Sum2 = ss([1 -1],'InputName',{'y','y1'},'OutputName','dy');

% Interconnecting the model
Tsp = connect(P,Gp,Dp,C,F,Sum1,Sum2,'ysp','y'); % Tsp = y/ysp

% Closed-loop system with PI controller used for comparison
P = tf(5.6,[40.2 1],'OutputDelay',93.9);
C = 0.1*(1+tf(1,[100 0]));
Tpi = feedback(P*C,1);

% time and frequency analysis
t = 0:0.1:2000;
RefSig = (t>=0&t<2000)*4;%+(t>=1000&t<=2000)*8;  %reference signal
% plot(t,RefSig); % to see how the above command builds the signal

% Comparing the responses of Tpi and Tsp to the reference signal

% lsim(Tsp,RefSig,t);
% lsim(Tpi,RefSig,t);
lsim(Tsp,Tpi,RefSig,t);
```



Lyapunov Stability Theorem

Introduction:-

Lyapunov stability was developed by Aleksandr Mikhailovich Lyapunov in his doctoral thesis on "*The general problem of the motion*", in 1892. It is useful for determining the stability of nonlinear systems. It is also applicable to the stability testing of linear systems. In this approach, the stability of the system is determined without solving the differential equations.

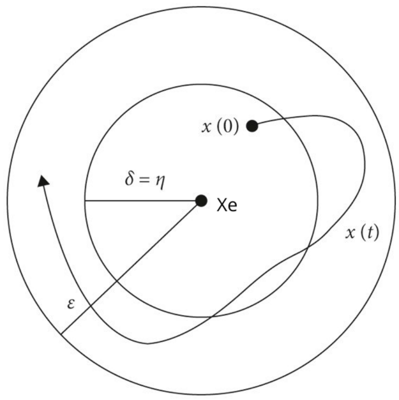
Concept:-

Lyapunov Stability Theorem is based on the concept of energy. If the total energy of the system is dissipated, then the system is always stable.

ENERGY↓ ↔ STABLE SYSTEM

Definitions:-

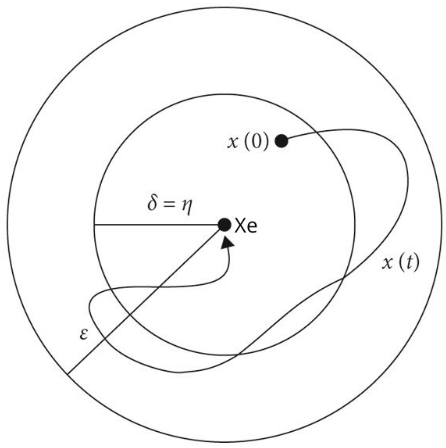
Lyapunov Stability:-



An equilibrium state X_e of an autonomous dynamic system is stable (or stable in the sense of Lyapunov) if for every ε>0, there exists a **δ**>0 where **δ** depends only on ε such that $||X-X_e|| < \boldsymbol{\delta}$ results in $||X(t_0, x_0) - X_e|| \leq \varepsilon$ for all $t \geq t_0$

"If there exist a ε such that no **δ** can be found to satisfy Lyapunov Stability condition, then the Equilibrium Point is Unstable."

Asymptotic Stability:-



An equilibrium state X_e of an autonomous dynamic system is stable if

1. It is Lyapunov Stable
2. There is a number **δ** such that every motion starting in the neighborhood **δ** of X_e and converges to X_e as t tends to ∞

There are two Lyapunov theorems which are often used:

(1) Lyapunov-Krasovskii Stability Theorem

(2) Lyapunov-Razumikhin Stability Theorem

<u>State Space Equations of Nonlinear Time Delay System</u>:-

$$\begin{aligned} \dot{x}(t) &= f(t, x(t), x_t), \quad \forall t \geq 0 \\ x_0(\theta) &= \varphi(\theta), \quad \forall \theta \in [-h, 0], \qquad \ldots\ldots(1) \end{aligned}$$

Lyapunov-Krasovskii Stability Theorem:-

Suppose that f maps R×(bounded sets in C_n) into bounded sets of R^n, and u, v, w : $R^+ \rightarrow R^+$ are continuous, nondecreasing functions with u(0) = v(0) = 0 and u(α) > 0, v(α) > 0, for α > 0. If there exists a continuous functional V: $R \times C_n \rightarrow R$ such that

1) $u(|\varphi(0)|) \leq V(t, \varphi) \leq v(||\varphi||_c)$; (means positive definite)
2) $\tilde{V}(t, \varphi) \leq -w(|\varphi(0)|)$, (negative definite or negative semidefinite)

where $\tilde{V}(t, \varphi) = \lim_{\Delta t \rightarrow 0+} \frac{1}{\Delta t}(V(t + \Delta t, x_{t+\Delta t}(t, \varphi)) - V(t, \varphi))$,

then the trivial solution of Eq. (1) is uniformly stable.

If w(α) > 0 for α > 0, then the trivial solution of Eq. (1) is uniformly asymptotically stable.

Additionally, if $\lim_{\alpha \rightarrow \infty} u(\alpha) = \infty$, then the trivial solution of Eq. (1) is globally uniformly asymptotically stable.

Lyapunov-Razumikhin Stability Theorem:-

Suppose that f maps R×(bounded sets in C_n) into bounded sets of R^n, and u, v, w : $R^+ \rightarrow R^+$ are continuous, nondecreasing functions with u(0) = v(0) = 0 and u(α) > 0, v(α) > 0, for α > 0, and v is strictly increasing. If there exists a continuous functional V : $R \times C_n \rightarrow R$ such that

1) $u(|x|) \leq V(t, x) \leq v(|x|)$;
2) $\tilde{V}(t, x(t)) \leq -w(|x(t)|)$, if $V(t + \theta, x(t + \theta)) \leq V(t, x(t))$, for $\theta \in [-h, 0]$,

where $\tilde{V}(t, x(t)) = \frac{d}{dt}V(t, x(t)) = \frac{\delta V(t, x(t))}{\delta t} + \frac{\delta V(t, x(t))}{\delta x} f(x, x_t)$,

then the trivial solution of Eq. (1) is uniformly stable.

(a) If w(α) > 0 for α > 0, and there exists a continuous nondecreasing function p(α) > 0 for α > 0, and the above condition 2 is strengthened to $\tilde{V}(t, x(t)) \leq -w(|x(t)|)$, if $V(t+\theta,x(t+\theta)) \leq p(V(t,x(t)))$ for $\theta \in [-h, 0]$, then the trivial solution of Eq. (1) is uniformly asymptotically stable.

(b) Additionally, if $\lim_{\alpha \rightarrow \infty} u(\alpha) = \infty$, then the trivial solution of Eq. (1) is globally uniformly asymptotically stable.

HEATING SYSTEM

A **heating system** is a mechanism for maintaining temperatures at an acceptable level; by using thermal energy. Transport and heat exchange phenomena occurring in a heat exchanger can be modeled as first-order partial differential equations (PDEs). We have proposed a time-delay system modeling of the flow temperatures of a heat exchanger.

Let us derive the formula for the transfer function of the thermal system and the mathematical model of the thermal System:

List of symbols used in the thermal system.

q = Heat flow rate, Kcal/sec.
θ_1 = Absolute temperature of an emitter, °K.
θ_2 = Absolute temperature of receiver, °K.
$\Delta\theta$ = Temperature difference, °C.
A = Area normal to heat flow, m^2
K = Conduction or Convection coefficient; Kcal/sec-°C.
Kr = Radiation coefficient, Kcal/sec-°C.
H = K/A=Convection coefficient, Kcal/m2-sec-°C.
H = Thermal conductivity, K cal/m-sec-°C.
ΔX = Thickness of conductor,m
R = Thermal resistance, °C-sec/Kcal
C = Thermal capacitance, Kcal/°C

Heat Flow Rate: Thermal systems are those that involve the transfer of heat from one substance to another. There are three different ways of heat flow from one substance to another. They are conduction, convection, and radiation.

For conduction, Heat flow rate = $q = K\,\Delta\theta = KA/\Delta X$

For convection, Heat flow rate, $q = K\,\Delta\theta = HA\,\Delta\theta$

For radiation, Heat flow rate, $q = Kr(\theta_1^4 - \theta_2^4)$

If $\theta_1 \gg \theta_2$ then, $q = Kr\,\theta^4$, where $\theta = (\theta_1^4 - \theta_2^4)^{1/4}$

Note: θ is called effective temperature difference of the emitter and receiver.

Mathematical model of the thermal system

The model of thermal systems is obtained by using thermal resistance and capacitance which are the basic elements of the thermal system. The thermal resistance and capacitance are distributed in nature. But for simplicity in analysis lumped parameter models are used.

In the lumped parameter model it is assumed that the substances that are characterized by resistance to heat flow have negligible heat capacitance and the substances that are characterized by heat capacitance have negligible resistance to heat flow. The thermal resistance, R for heat transfer between two substances is defined as the ratio of change in temperature and change in heat flow rate.

The thermal resistance, R for heat transfer between two substances is defined as the ratio of change in temperature and change in heat flow rate.

$$\text{Thermal resistance, } R = \frac{\text{Change in Temperature, °C}}{\text{Change in heat flow rate, Kcal / sec}}$$

For conduction or convection,

Heat flow rate, $q = K\Delta\theta$

On differentiating we get,

$$dq = Kd(\Delta\theta)$$

$$d(\Delta\theta)/dq = 1/K$$

But thermal resistance, $R = d(\Delta\theta)/dq$

Thermal resistance, $R = 1/K$ for conduction

$= 1/K = 1/HA =$ for convection

For radiation,

Heat flow rate, $q = Kr\theta^4$

On differentiating we get

$$dq = Kr4\theta^3 d\theta$$

$$d\theta/dq = 1/Kr4\theta^3$$

But thermal resistance, $R = d\theta/dq$

Thermal resistance, $R = 1/4Kr\,\theta^3$ (for radiation)

Thermal Capacitance, C is defined as the ratio of change in heat stored and change in temperature.

$$\text{Thermal capacitance, } C = \frac{\text{Change in heat stored, Kcal}}{\text{Change in temperature, °C}}$$

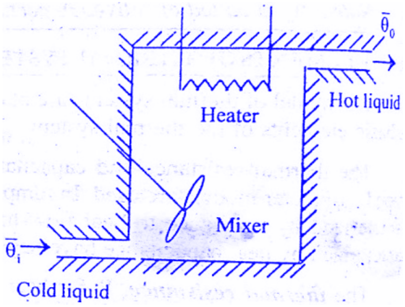
Let M = Mass of substance considered, Kg

c = Specific heat of substance, Kcal/Kg -°C

Now, Thermal capacitance, C = Mcp

Transfer function of Thermal system

Consider a simple thermal system shown in the below figure. Let us assume that the tank is insulated to eliminate heat loss to the surrounding air, there is no heat storage in the insulation, and the liquid in the tank is kept at a uniform temperature by perfect mixing with the help of a stirrer. Thus, a single temperature is used to describe the temperature of the liquid in the tank and of the outflowing liquid. The transfer function of the thermal system can be derived as shown below.



Let θ_1 = Steady state temperature of inflowing liquid, °C

θ_0 = Sandy state temperature of outflowing liquid, °C

G = Steady state liquid flow rate, Kg/sec

M = Mass of liquid in the tank, Kg

c = Specific heat of the liquid, Kcal/Kg °C

R = Thermal resistance, °C – sec/Kcal

C = Thermal capacitance, Kcal/°C

Q = Steady state heat input rate, Kcal/Sec

Let us assume that the temperature of inflowing liquid is kept constant. Let the heat input rate to the thermal system supplied by the heater is suddenly changed from Q to Q + q1. Due to this, the heat output flow rate will gradually change from Q to Q + q0. The temperature of the outflowing liquid will also be changed from θ0 to θ0 + θ.

For this system the equation for q0, C, and R are obtained as follows,

$$\left.\begin{matrix}\text{Change in output}\\ \text{heat flow rate,}\end{matrix}\right\} q_0 = \begin{matrix}\text{Liquid flow}\\ \text{rate, G}\end{matrix} \times \begin{matrix}\text{Specific heat of}\\ \text{liquid, c}\end{matrix} \times \begin{matrix}\text{Change in}\\ \text{temperature, } \theta\end{matrix}$$

$$= Gc\theta$$

Thermal capacitance, C = Mass, M x Specific heat of the liquid, c = Mc

$$\text{Thermal resistance, } R = \frac{\text{Change in temperature, } \theta}{\text{Change in heat flow rate, } q_0} = \frac{\theta}{q_0}$$

On substituting for q_0 from equation (1) in equation (3) we get

$$R = \frac{\theta}{Gc\theta} = \frac{1}{Gc}$$

In this thermal system, rate of change of temperature is directly proportional to change in heat input rate.

$$\therefore \frac{d\theta}{dt} \; \alpha \; q_i - q_0$$

$$\therefore C\frac{d\theta}{dt} = q_i - q_0$$

The constant of proportionality is capacitance C of the system.
Equation (5) is the differential equation governing the system. Since equation (5) is of the first-order equation, the system is the first-order system.
From equation (3), R = θ/q0
q0 = θ/R

On substituting for q0 from equation (6) in equation (5) we get,

$$C\frac{d\theta}{dt} = q_i - \frac{\theta}{R} \quad \Rightarrow \quad C\frac{d\theta}{dt} = \frac{Rq_i - \theta}{R} \quad \Rightarrow \quad RC\frac{d\theta}{dt} = Rq_i - \theta$$

$$\therefore RC\frac{d\theta}{dt} + \theta = Rq_i$$

Let, L {θ} = θ(s); L{dθ/dt} = sθ(s); L{q1} = Q1(s)

On taking Laplace transform of equation (7)

RC s θ(s) + θ(s) = R Q1(s)

θ(s) [sRC + 1] = R Q1(s)

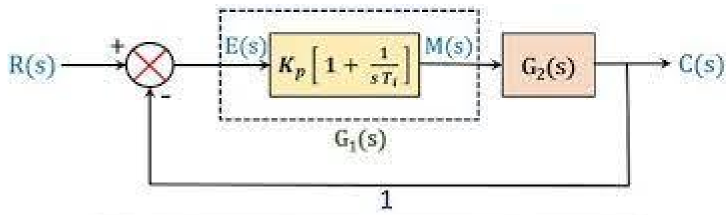
θ(s)/Q1(s) is the required transfer function of the thermal system.

$$\therefore \frac{\theta(s)}{Q_i(s)} = \frac{R}{sRC + 1} = \frac{R}{RC\left(s + \frac{1}{RC}\right)} = \frac{\frac{1}{C}}{s + \frac{1}{RC}}$$

Heating system modeling using PI controller

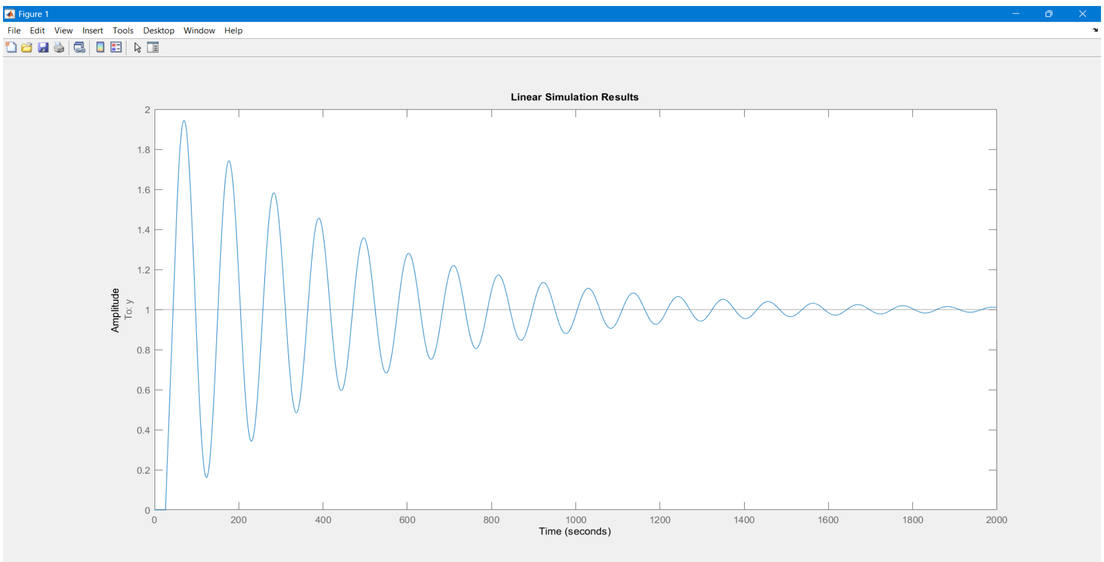
Heating_System.m *

```
% Smith predictor
s = tf('s');

% Building LTI blocks
delay=26;
P = exp(-delay*s)*1/(21.3*s+1);
P.InputName = 'u';
P.OutputName = 'y';

Gp = 1/(21.3*s+1);
Gp.InputName = 'u';
Gp.OutputName = 'yp';

Dp = exp(-delay*s);
Dp.InputName = 'yp';
Dp.OutputName = 'y1';



C = 1.18333333*(1+1/(21.3*s));
C.InputName = 'e';
C.OutputName = 'u';

F =tf(1) ;
F.InputName = 'dy';
F.OutputName = 'dp';

% Closed-loop system with PI controller used for comparison
Tpi = feedback(P*C,1);

% time and frequency analysis
t = 0:0.1:2000;
RefSig = (t>=0&t<2000);%+(t>=1000&t<=2000)*8;  %reference signal
% plot(t,RefSig); % to see how the above command builds the signal

% Comparing the responses of Tpi and Tsp to the reference signal

% lsim(Tsp,RefSig,t);
% lsim(Tpi,RefSig,t);
lsim(Tpi,RefSig,t);
```



Block Diagram of Control System with PI controller

Electronics Coach

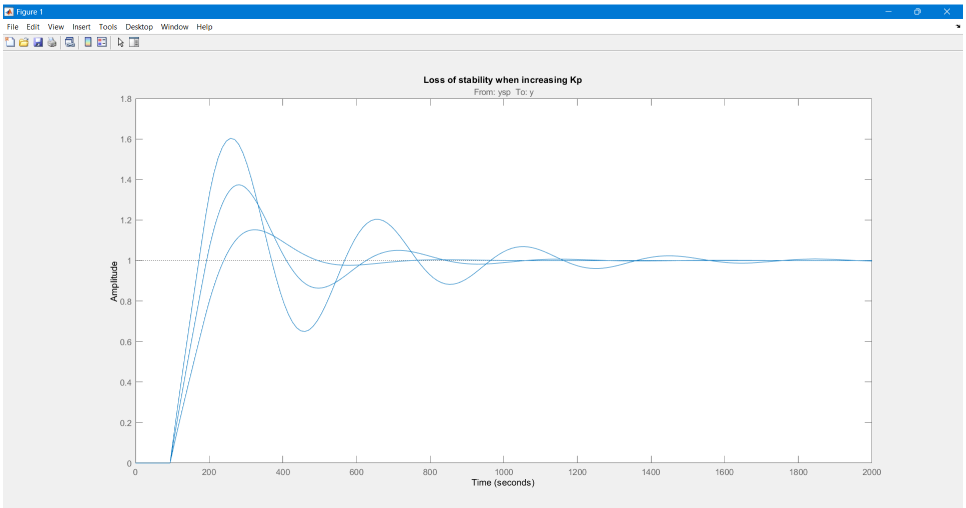


From the output, we can observe that the PI controller is not working properly as we increase the dead time. The settling time and peak overshoot of the response have increased.

Effect of Kp on PI Controller

Stability_IncreasingKp.m

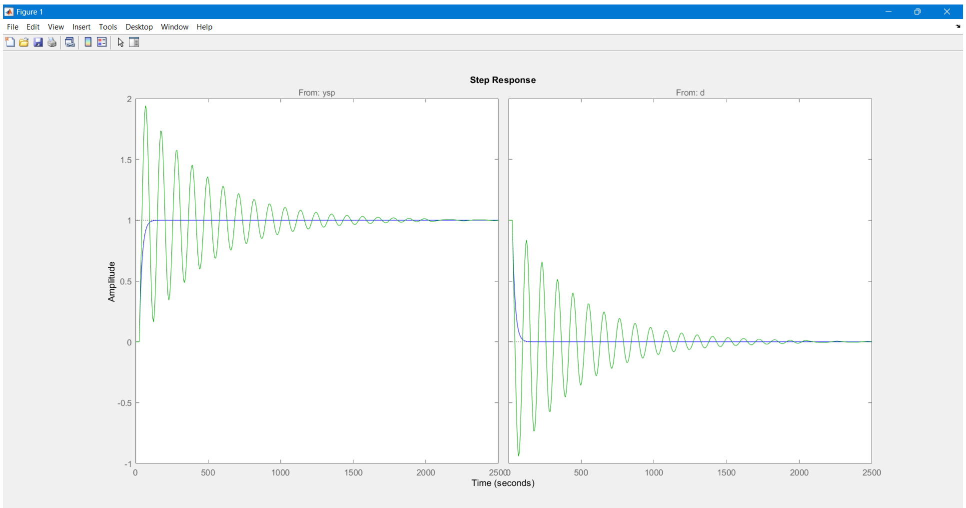
```
s = tf('s');
P = exp(-93.9*s) * 5.6/(40.2*s+1);
P.InputName = 'u';
P.OutputName = 'y';
P
Cpi = pidtune(P,pidstd(1,1),0.006);
Cpi
      % try three increasing values of Kp

Tpi = feedback([P*Cpi,1],1,1,1);  % closed-loop model [ysp;d]->y
Tpi.InputName = {'ysp' 'd'};

step(Tpi), grid on
Kp3 = [0.06;0.08;0.1];      % try three increasing values of Kp
Ti3 = repmat(Cpi.Ti,3,1);   % Ti remains the same
C3 = pidstd(Kp3,Ti3);       % corresponding three PI controllers
T3 = feedback(P*C3,1);
T3.InputName = 'ysp';

step(T3)
title('Loss of stability when increasing Kp')
```



With the increase in Kp, the delay causes a decrease in phase margin which implies a lower damping ratio and a more oscillatory response for the closed-loop system. Further, it decreases the gain margin thus moving the system to instability.

Heating system modeling using Smith Predictor

```matlab
% Smith predictor
s = tf('s');
delay=26;
% Building LTI blocks

P = exp(-delay*s)/(21.3*s+1);
P.InputName = 'u';
P.OutputName = 'y0';

Gp = 1/(21.3*s+1);
Gp.InputName = 'u';
Gp.OutputName = 'yp';

Dp = exp(-delay*s);
Dp.InputName = 'yp';
Dp.OutputName = 'y1';

C = 1.18333333*(1+1/(21.3*s));
%C=pidtune(P,'PI');
C.InputName = 'e';
C.OutputName = 'u';

F =tf(1) ;
F.InputName = 'dy';
F.OutputName = 'dp';

Ds = exp(-35*s)*1/(25*s+1);
Ds.InputName = 'yd';
Ds.OutputName = 'd';

% Building summation blocks

Sum1 = sumblk('e = ysp - yp - dp');
Sum2 = sumblk('y = y0 + d');
Sum3 = sumblk('dy = y - y1');

% Interconnecting the model
Tsp = connect(P,Gp,Dp,C,F,Sum1,Sum2,Sum3,{'ysp','d'},'y'); % Tsp = y/ysp

% Closed-loop system with PI controller used for comparison
Tpi = feedback([P*C,1],1,1,1);  % closed-loop model [ysp;d]->y
Tpi.InputName = {'ysp' 'd'};
step(Tsp,'b',Tpi,'g');
```



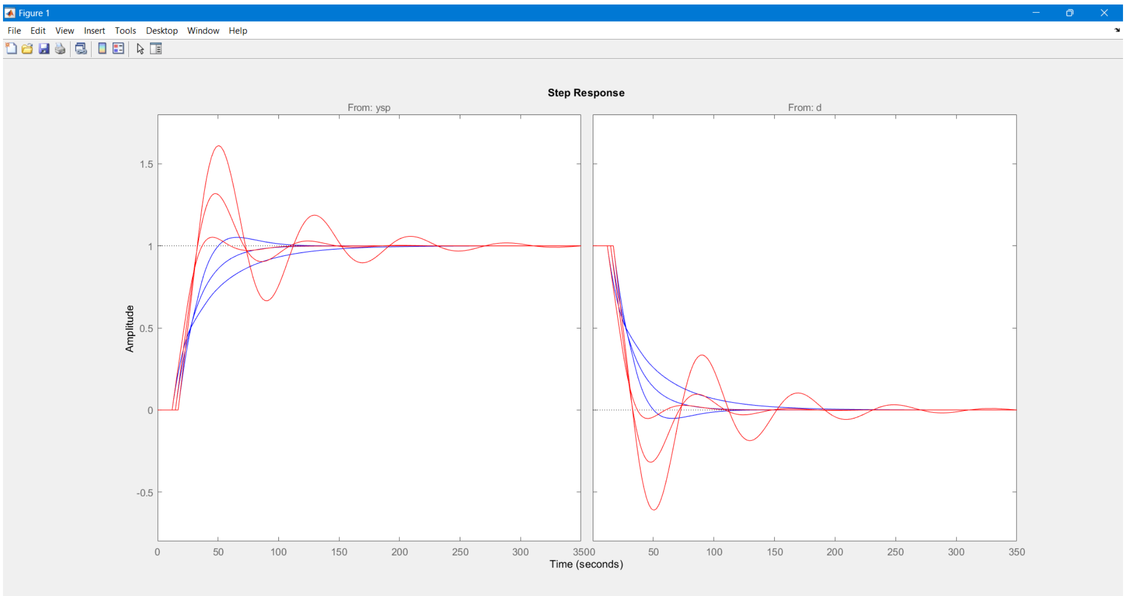
Here the green line denotes PI controller and the blue line denotes Smith Predictor. The figure above shows the comparison of the PI controller and Smith Predictor. From the output, we can see that for a long dead time, Smith Predictor works better in comparison to the PI controller because the settling time and peak overshoot of the Smith Predictor is less than the PI controller.

Model Mismatch

Editor - C:\Users\krish_ohljf3h\Downloads\modifiedhs_mywork.m *

modifiedhs_mywork.m *

```
% Smith predictor
s = tf('s');

% Building LTI blocks
P = exp(-14.7*s)*1/(21.3*s+1);
P.InputName = 'u';
P.OutputName = 'y0';

P1 = exp(-17*s) * 1.2/(23*s+1);
P2 = exp(-12*s) * 0.8/(18*s+1);

Plants = stack(1,P,P1,P2);

Gp = 1/(21.3*s+1);
Gp.InputName = 'u';
Gp.OutputName = 'yp';

Dp = exp(-14.7*s);
Dp.InputName = 'yp';
Dp.OutputName = 'y1';

C = 1.18333333*(1+1/(21.3*s));
C.InputName = 'e';
C.OutputName = 'u';

F =tf(1) ;
F.InputName = 'dy';
F.OutputName = 'dp';

Ds = exp(-35*s)*1/(25*s+1);
Ds.InputName = 'yd';
Ds.OutputName = 'd';

% Building summation blocks
Sum1 = sumblk('e = ysp - yp - dp');
Sum2 = sumblk('y = y0 + d');
Sum3 = sumblk('dy = y - y1');

% Interconnecting the model
Tsp = connect(Plants,Gp,Dp,C,F,Sum1,Sum2,Sum3,{'ysp','d'},'y'); % Tsp = y/ysp

% Closed-loop system with PI controller used for comparison
Tpi = feedback([Plants*C,1],1,1,1);  % closed-loop model [ysp;d]->y
step(Tsp,'b',Tpi,'r');
%Tpi = feedback(P*C,1);
```

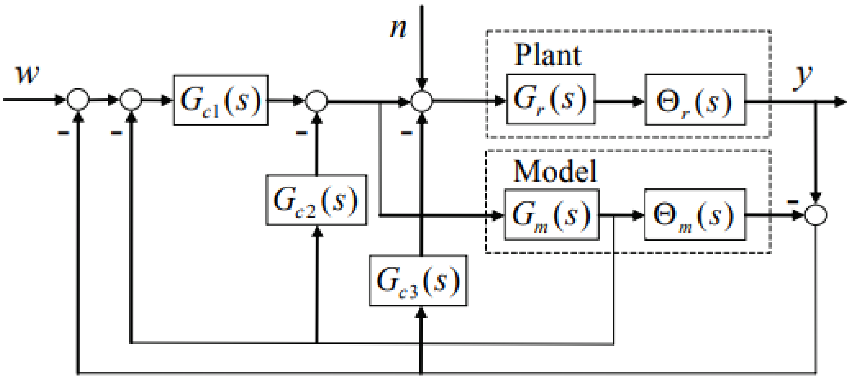


Here the red line denotes PI controller and the blue line denotes Smith Predictor.

If there isn't an exact match between the plant and the internal model used by the Smith Predictor, then it is important to study the robustness of the Smith Predictor to uncertainties in the process dynamics and dead time. From the above figure, we can see that Smith Predictor is not working properly for uncertainties.

Heating system modeling using modified Smith Predictor

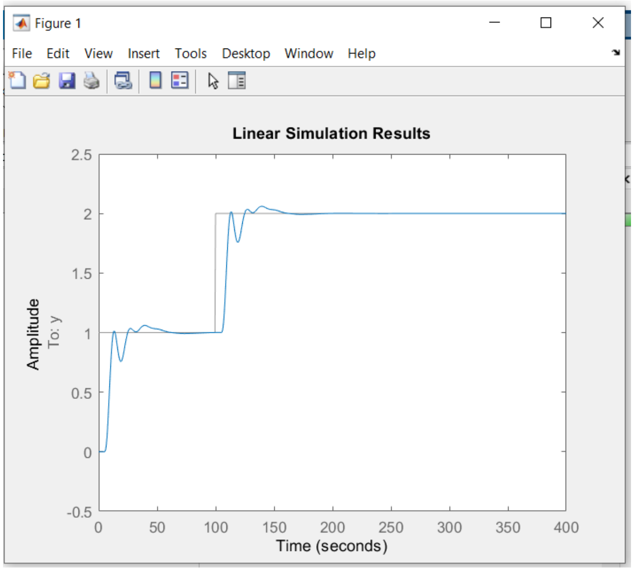
The modified Smith Predictor has improved the classical Smith predictor loop using a more sophisticated and complicated structure with additional controllers. Naturally, all the methods also use mathematical models of really controlled plants including time-delay term in the inner loop. The really controlled plant is formally divided into two blocks representing the time-delay-free transfer function $G_r(s)$ and time-delay term $\Theta_r(s)$. Analogically, its mathematical model in the inner loop consists of $G_m(s)$ and $\Theta_m(s)$. Signals w, n, and y denote reference value, disturbance in the input of the controlled plant, and output signal, respectively. It utilizes the structure with a trio of controllers, where $G_{c1}(s)$ is a PI controller used to stabilize the unstable pole, $G_{c2}(s)$ is a PD (or only P where it is appropriate) controller and $G_{c3}(s)$ is the disturbance controller. $G_{c2}(s)$ and $G_{c3}(s)$ ensure reference tracking and disturbance rejection, respectively, by considering the inner loop as an open-loop stable system. Moreover, the signal outgoing from controller $G_{c2}(s)$ can be interpreted as an estimation of the disturbance n.



The figure below shows the matlab program for implementing Modified smith predictor.

```
Editor - C:\Users\HP LAPTOP\Documents\MATLAB\time delay\smithCorrect.m*
smith.m    smithCorrect.m*

% Smith predictor
clear;
s = tf('s');
delay=5;
% Building LTI blocks

P = exp(-delay*s)*0.1/(s*s+0.4*s-0.05);
P.InputName = 'x';
P.OutputName = 'y';

Gc1=(.1*s+1)/(s);
Gc1.InputName = 'z';
Gc1.OutputName = 'b';

Gc2=(2.873*s + 4.7733);
Gc2.InputName = 'h';
Gc2.OutputName = 'd';

Gc3= 1.4142*s + .7071;
Gc3.InputName = 'k';
Gc3.OutputName = 'f';

M = 0.1/(s*s+0.4*s-0.05);
M.InputName = 'l';
M.OutputName = 'h';

D = exp(-5.5*s);
D.InputName = 'h';
D.OutputName = 'i';

% Building summation blocks

Sum1 = sumblk('j = w-k');
Sum2 = sumblk('z = j - h');
Sum3 = sumblk('l= b-d');
Sum4 = sumblk('x= l-f');
Sum5 = sumblk('k= y-i');

% Interconnecting the model
Tsp = connect(P,M,D,Gc1,Gc2,Gc3,Sum1,Sum2,Sum3,Sum4,Sum5,'w','y'); %

% time and frequency analysis
t = 0:0.5:400;
RefSig = (t>=0&t<100)+(t>=100&t<=400)*2 ;  %reference signal
plot(t,RefSig); % to see how the above command builds the signal

% Comparing the responses of Tpi and Tsp to the reference signal
lsim(Tsp,RefSig,t);
```

The figure shows the response of the modified smith predictor.



CONCLUSION

A heating system is a mechanism for maintaining temperatures at an acceptable level; by using thermal energy. We have compared three methods for dealing with time delays in heating systems. When the delay is small the PI controller gives the desired performance response. But on increasing the dead time of the system the PI controller shows a huge overshoot and the settling time is also increased. The response shows oscillatory behavior making the system unstable.

To eliminate the long dead time we modeled the plant with a smith predictor. The smith predictor is a model-based controller that is effective for processes with long dead time. It has an inner loop with the main controller that can be simply designed without the dead time. The effects of load disturbance and modeling error are corrected through an outer loop.
The smith predictor compares the original plant with the predicted model with an estimated time delay. When the predicted output equals the plant output, the signal from the plant model without delay can be used for control so that the controller can be designed without considering time delay and control performance can be significantly improved.

For modeling the higher-order systems we have to use a modified smith predictor. The modified Smith Predictor improves the classical Smith predictor loop using a more sophisticated and complicated structure with additional controllers.

REFERENCES

[1] M.GnanaMurugan, "Smith Predictor for Control of the Process with Long Delay Time," 2014.

[2] C. Song, W. Ji, X. Gong, and Z. Hu, "Research on Stability for Linear Control System," in *International Conference on Mechatronics and Automation*, Beijing, 2011.

[3] Alevisakis, G., &Seborg, D. (1973). “An extension of the Smith predictor to multivariable linear systems containing time delays", International Journal of Control, 3, 541–551.

[4] Qing-Chang Zhong (2006), *Robust Control of Time-delay Systems*, Springer

[5] Hang, C.C. (2004), *Smith Predictor and Its Modifications*. in Control Systems, Robotics, And Automation, Vol. II, edited by Heinz Unbehauen, Eolss Publishers

[6] Kaya, I. and D.P. Atherton. 1999. “A new PI-PD Smith predictor for control of processes with long dead time”. In *Proceedings of the 14th IFAC World Congress* (Beijing, China).

[7] Hamamci, S.E.; I. Kaya and D.P. Atherton. 2001. “Smith predictor design by CDM”. In *Proceedings of the European Control Conference* (Porto, Portugal).