Group Work Project

Submission 1

The History of Measure-Theoretic Probability and Martingales

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Abstract

In this documents we are trying to express short review on the history of the probability on the measure theoretic prospective. Here we have mainly focused on the three famous mathematicians namely Kolmogorov, A. Markov and J L Doob and their contribution to the development of the modern approach to the probability.

Introduction

The history of probability theory begins with James Bernoulli (1654-1705) and his work, Ars Conjectandi (The Art of Guessing) published in 1713, in which he proved (quite rigorously) the first Iimit theorem of probability theory, the law of large numbers; and of De Moivre (1667-1754), Miscellanea Analytica Supplementum (1730), in which the central Iimit theorem was stated and proved for the first time. One of the famous Russian Mathematician, Kolmogorov (1903–1987) who saw the connection between the ideas of Borel and Lebesgue and probability theory and he gave probability theory its rigorous measure theory basis on the twenty century. After Kolmogorov, Paul Levy (1886–1971), the French mathematician, established the outline for modern Probability with his innovative work on Stochastic Processes and also studied the characteristic of functions and limit theorems. Kolmogorov put probability into the wider context of measure theory by developing the axioms on the probability theory. When we are measuring something, we are assigning a number to some sort of mathematical objects. In a similar way, probability is also a way of assigning a number to a mathematical objects. Kolmogorov's formulation express that the mathematical theory of measures could encompass the theory of probability as a special case. On the devolvement of the probability theory, more than a decade ago, a Russian Mathematician Andrey Markov (1856 – 1922) founded a new branch of probability theory by applying Mathematics to the Poetry. Markov process named after Mathematician Markov, is a random process whose outcome is independent of the past. It is a stochastic process that satisfies characteristics of memoryless, such process are the process whose future outcome is independent of the Past. Furthermore, J.L. Doob (1910-2004), an American Mathematician, further contributed for the devolvement of the Probability theory. In his work is also known as Doob Martingales. Doob Martingales possess the Martingale properties and yields a Stochastic Process, which approximates a random variables.

Kolmogorov and Axioms of Probability

For the development of the probability theory, Kolmogorov made very significant contribution by developing three axioms. An axioms are statements which are used to interpret and deduce the

logic and prove theorems and hypothesis. The first axioms defines the non-zero property of the probability. This states that, the probability of an event is non- zero real number. It means the probability of and event P (E) is greater than zero but less than infinity. i.e. $0 < P(E) < Infinity$. And second, is the definition of Probability of entire sample space. Which says the probability that at least one of the event in the entire sample space will occur is one. I.e. $P\ (\Omega) = 1$. It implies that it is 100 % sure that, at least one event will occur. The third Axioms defines the property of Mutual exclusive Events. Two events which don't have anything in common are called mutually exclusive events. States that, the union of the probability of all mutual exclusive events are equal to the sum of probability of occurring each events. Suppose we have $E1\ and\ E2$ are two mutually exclusive events then,

$$P\ (E1\ U\ E2) = P(E1) + P(E2)$$

These axioms are universal and still applicable till dates. But they have some shortcoming. They do not tell how to use these axioms for mathematical calculations and lacked the insights on the random process.

Markov and Markov Process

Markov process is the random process which follow the Markov's properties of Memorylessness. A Markov Process is a stochastic process whose outcome is finite, independent of the past values and probabilities are constants over time. Consider a vector X0, which represents the initial state of the system of matrix M such that Mx0 is the vector after first iteration. After nth iteration we get the states vectors as, x0, Mx1, M2x0 … Mnx0. where Mnx0 is the vector after nth iteration. This chain is called Markov Chain and this Matrix M is called Transition Matrix. The development of has been milestone for the development of Economics and Finance. It does not required the past data for future predictions, which significantly reduce the number of the parameters required to make predictions. It has been used widely but let focus on Economic and Financial use here. In finance they are often used in microeconomic situations like Market Crash, Predicting Assets and Option

Pricing and Calculating the Credit Risks and the most important use of Markov chain are to measure the Credit Risks.

J. L. Doob and the development of martingales

J L Doob devolved the probability theory as a major discipline of Mathematical Education. Apart from his ground-breaking research in probability theory, he also wrote three books published in 1953, 1984 and in 1994. The first book, Stochastic Processes, published in the Wiley series in 1953 was a very influential book in the development of probability theory in the US and the world. A famous French probabilist, P A Meyer, on his work, Stochastic Processes, "The bible of new probability". In his publication, Doob insights that measure theory, appropriately developed, provides the basics and allows one to solve a number of problems in probability theory. In the same book, Doob develops the theory of martingales – both discrete and continuous time – and a detailed study of Ito's stochastic differential equations with respect to Brownian motion. This Doob Martingale helps to constructs a stochastic process. This process approximates a random variables and also satisfies the property of martingale with respect to the filtration. It may be thought of as the evolving sequence of best approximations to the random variable based on information accumulated up to a certain time.

Conclusion

Many other mathematicians were also involved in the development of the probability measure theory and still to come. This theory is still widely used by contemporary mathematician and finance professionals and is still relevant. Which is the foundation of the Kolmogorov Axioms and these Axioms subsequently helped for the devolvement of the stochastic process and Martingales.

Bibliography

[1]. Jan Oboj, Peter Spoida, Nizar Touz, Martingale Inequalities for the Maximum via Path wise Arguments, 2015, Online, Available at:

http://www.cmap.polytechnique.fr/~touzi/OblojSpoidaTouzi_23_03_2015.pdf

[Accessed on: 2019-03-12]

[2]. K B Athreya, Joseph L Doob and Development of Probability Theory, RESONANCE, April 2015, Online, Available at:

https://www.ias.ac.in/article/fulltext/reso/020/04/0286-0288

[Accessed on: 2019-03-12]

[3]. C.K.Taylor, The three probability axioms, ThoughtCo, 2019, Online, Available at:

https://www.thoughtco.com/what-are-probability-axioms-3126567 ,

[Accessed on: 2019-03-12]

[4]. CHRISTOPHER P. PORTER, KOLMOGOROV ON THE ROLE OF RANDOMNESS IN PROBABILITY THEORY, PDF, Online Available at:

http://www.cpporter.com/wp-content/uploads/2013/08/Porter-Kolmogorov-Role-of-Randomness.pdf

[Accessed on: 2019-03-12]

[5]. Probaility Axioms, Worform Mathwords, wolfram research, 2019, Online, available at:

http://mathworld.wolfram.com/ProbabilityAxioms.html

[Accessed on: 2019-03-12]

[6]. Maths in a minute: The axioms of probability theory, Plus Magaine, 2019, online, Available at:

https://plus.maths.org/content/maths-minute-axioms-probability

[Accessed on: 2019-03-12]

[7]. A. L. Kuzemsky, BIOGRAPHY OF Andrej N. KOLMOGOROV, 2017, Online, Available at:

http://theor.jinr.ru/~kuzemsky/ankolmogbio.html

[Accessed on: 2019-03-12]

[8]. Stephanie,Statistics How To, Axiomatic Probability: Definition, Kolmogorov's Three Axioms, 2018, Online, available at :

https://www.statisticshowto.datasciencecentral.com/axiomatic-probability/

[Accessed on: 2019-03-12]

[9]. MICHAEL J. NEELY, On Probability Axioms and Sigma Algebras, EE 465, UNIVERSITY OF SOUTHERN CALIFORNIA, SPRING 2012, Online, Available at:

http://ee.usc.edu/stochastic-nets/docs/probability-axioms-sigma-algebras.pdf

[Accessed on: 2019-03-12]

[10]. Morey, Edward. 3 Basic Definitions of Probability Theory, Online, Available at :

https://www.colorado.edu/economics/morey/6818/3defprob.pdf

[Accessed on: 2019-03-12]

[11]. Myers, Daniel. CS 547 Lecture 6: Axioms of Probability, Online, available at:

http://pages.cs.wisc.edu/~dsmyers/cs547/lecture_6_probability.pdf

[Accessed on: 2019-03-12]

[12]. Universitat Zurich, Axiomatic Probability, Online, Available at:

https://www.math.uzh.ch/index.php?file&key1=45741

[Accessed on: 2019-03-12]

[13]. Axioms of Probability, MBA Skool, 2019, Online, available at:

https://www.mbaskool.com/business-concepts/statistics/7511-axioms-of-probability.html

[Accessed on: 2019- 03- 13]

[14].Wisdom Jobs, AXIOMATIC OR MODERN APPROACH TO PROBABILITY QUANTITATIVE TECHNIQUES FOR MANAGEMENT, Online, Available at:

https://www.wisdomjobs.com/e-university/quantitative-techniques-for-management-tutorial-297/axiomatic-or-modern-approach-to-probability-9944.html

[Accessed on: 2019- 03- 13]

[15]. R. G. Gallager, “Finite-state Markov chains” in Stochastic processes: theory for applications. United Kingdom: Cambridge university press, 2013, 16. Online, Available at:

http://www.rle.mit.edu/rgallager/documents/6.262-4vaw.pdf

[Accessed on: 2019- 03- 13]

[16]. Brian Hayes, First Links in the Markov Chain, Sigma Xi, The Scientific Research Honor Society, 2018, Online, Available at:

https://www.americanscientist.org/article/first-links-in-the-markov-chain

[Accessed on: 2019- 03- 13]

[17]. J. Hoek & R. J. Elliott, “Asset Pricing Using Finite State Markov Chain Stochastic Discount Functions”, Stochastic Analysis and Applications, 30:5, 2012, Online, Available at:

http://dx.doi.org/10.1080/07362994.2012.704852

[Accessed on: 2019- 03- 13]

[18]. Christian Maes, An introduction to the theory of Markov processes, Instituut voor Theoretische Fysica, KU Leuven, Belgium, 2013, Online, Available at :

https://fys.kuleuven.be/itf/staff/christ/files/pdf/markovlect123.pdf

[Accessed on: 2019- 03- 13]

[19]. T K Siu , W K Ching , S. Eric Fung & Michael K. Ng, " On a multivariate Markov chain model for credit risk measurement," Quantitative Finance, 5:6, s. 543-556, Feb 2007, Online, Available:

http://dx.doi.org/10.1080/14697680500383714

[Accessed on: 2019- 03- 13]

[20]. Yariv Ephraim, and Neri Merhav, Hidden Markov Processes, IEEE, 2002, Online, Available at:

https://pdfs.semanticscholar.org/5c23/9544407f4a9569b09d8dfa2927e9fef6219c.pdf

[Accessed on: 2019- 03- 13]

[21]. Deju Zhang, Xiaomin Zhang, “Study on forecasting the stock market trend based on stochastic analysis method”, International Journal of Business and Management, vol 4, nr 6, s. 163-164, 2009 Online Available at:

http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.654.166&rep=rep1&type=pdf

[Accessed on: 2019- 03- 13]