Collaborative Review Task M4 Solutions

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Solution to Question 2 - We will start with question 2.

An equivalent martingale measure P^* is characterized by numbers $P^*(a) =: \alpha$, $P^*(b) =: \beta$ and

$P^*(c) =: \gamma$, $P^*(d) = \delta$ such that

$\alpha, \beta, \gamma, \delta > 0$ $\quad \alpha + \beta + \gamma + \delta = 1$

and the martingale property on X^1 and X^2

$$18\alpha + 18\beta + 9\gamma + 9\delta = 12$$
$$36\alpha + 9\beta + 12\gamma + 8\delta = 12$$

Solving this system of equations gives us,

$$\gamma + \delta = \frac{2}{3}, \quad \alpha + \beta = \frac{1}{3}$$

Using these three equations, it is impossible to find unique values of $\alpha, \beta, \gamma, \delta$ so that they all depend on just one variable.

So if $\gamma = P_1$, $\delta = \frac{2}{3} - P_1$

and, $\alpha = P_2$, $\beta = \frac{1}{3} - P_2$

Also if $P_P^* = \left(P_2, \frac{1}{3} - P_2, P_1, \frac{2}{3} - P_1\right)$

where $0 < P_1 < \frac{2}{3}$, $0 < P_2 < \frac{1}{3}$ is an EMM

then,

$$E^{*P}(H^1) = 6P_2 + \frac{1}{3} - P_2 + 8P_1 + 4\left(\frac{2}{3} - P_1\right)$$
$$= 5P_2 + \frac{1}{3} + 4P_1 + \frac{8}{3}$$
$$= 5P_2 + 4P_1 + 3$$

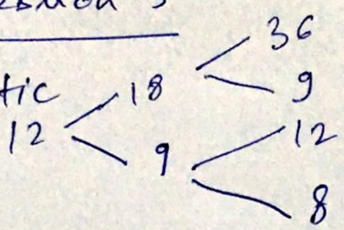
which depends on P_1 and P_2, so H^1 has no unique no-arbit. Pric

Solution to Question 1 -

Since none of the values p_1 and p_2 determined in question 2 satisfy these contingent claims $(6, 1, 8, 4)$. And this claim is not attainable. Also p_1 and p_2 are such that equations mentioned in question 2 have infinitely many solutions so the market is not complete.

Solution to question 3

tree of the Stochastic process:



$$\phi_0 + 18\phi_1 + 36\phi_2 = 6$$
$$\phi_0 + 18\phi_1 + 9\phi_2 = 1$$
$$\phi_0 + 9\phi_1 + 12\phi_2 = 8$$
$$\phi_0 + 9\phi_1 + 8\phi_2 = 4$$

which after solving lead to $\phi_2 = \frac{5}{27}$ and $\phi_2 = 1$ which is not possible together as ϕ_2 can take only 1 value. In this case, it is not possible to find a replicating strategy H.

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