This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2015.2508009, IEEE Transactions on Vehicular Technology

A Game Theoretic Approach to Replanning-aware Interactive Scene Prediction and Planning

Mohammad Bahram[1,2], Andreas Lawitzky[2], Jasper Friedrichs[1], Michael Aeberhard[1] and Dirk Wollherr[2]

***Abstract*—This work presents a novel cooperative-driving prediction and planning framework for dynamic environments based on the methods of game theory. The proposed algorithm can be used for highly automated driving on highways or as a sophisticated prediction module for advanced driver assistance systems without the need of inter-vehicle communication. The main contribution of this work is a model-based, interaction-aware motion prediction of all vehicles in a scene. In contrast to other state-of-the-art approaches, the system also models the replanning capabilities of all drivers. With that, the driving strategy is able to capture complex interaction between vehicles, planning maneuver sequences over longer time horizons. It also enables an accurate prediction of the traffic for the next immediate time step. The prediction model is supported by an interpretation of what other drivers intend to do, how they interact with traffic and the ongoing observation. As part of the prediction loop, the proposed planning strategy incorporates the expected reactions of all traffic participants, offering cooperative and robust driving decisions. By means of experimental results in simulated highway scenarios, the validity of the proposed concept and its real-time capability is demonstrated.**

***Index Terms*—cooperative systems, decision making, motion planning, advanced driver assistance systems, highly automated driving**

I. INTRODUCTION

As automated mobile platforms find their ways from research areas into real applications in human populated environments, more and more safety requirements must be fulfilled. Nevertheless, the benefits of automation of mobile platforms are conclusive by means of comfort, energy efficiency and safety [1].

Safe navigation in dynamic environments is one of the key challenges in the domains of mobile robotics, autonomous and highly-automated driving (HAD). The task is to *plan* safe trajectories in populated environments with only partial knowledge about future trajectories of the human traffic participants. The base for such knowledge can be acquired from real data by learning or from a model of human behavior. This knowledge is then used to *predict* the future human motion in the environment as accurately as possible, with growing uncertainty over time. This uncertainty is explicitly modelled in probabilistic approaches, which assign likelihoods to various motion options.

In the intuitive structure of independent prediction and planning phases, the planning of the future trajectory is performed on the predicted motion of other traffic. This does not take into account the mutual influence on each other's motion, decreasing the reliability of the safety assessment and, as a consequence, the safety and comfort of the system. Driving decisions influence the motion plans of the surrounding vehicles and vice versa. Of course, this interaction takes place between all traffic participants in the scene. It implies that prediction of *scenes* has to be performed instead of prediction of each individual vehicle.

Put differently, there is a feedback loop between own driving decisions and the evolution of the environment. It is shown in [2] that performance is limited if prediction and planning are treated independently. It means that accurate prediction and planning algorithms have to be aware of this interaction or suffer from low accuracy over expected time horizons, hence, risking safety.

The consideration of this work can be compared to chess, where each player plans his strategy for several moves into the future. He also assumes that his next move is noticed by the other player, who in turns adapts his strategy. Therefore, the problem formulation of the planing process (referred to as *interactive driving strategy* in the following) and its solution with *game theoretic* approaches is conclusive. Game theory can be defined as "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers" [3]. It is mainly used in economics, political science, as well as logic and computer science.

Games are described as interactive decision processes and can be used to model mutual decision making. Multiple vehicles with a predefined set of basic maneuvers (i.e. actions) are similar to players which mutually influence each other in their future decisions. The assumption that vehicles choose maneuvers according to an intention and risk assessment is consistent with a *rational player*. Like the previous comparison to chess, the idea of evaluating maneuver sequences over multiple time steps leads to *sequential games* where vehicles (players) alternately choose their actions. The concept of *extensive-form games* [4], [5] allows modeling of such sequences of simultaneous decision-making.

To provide a better understanding, the basic extensive-form game will be described using the following simple example: Two rational drivers (P_1, P_2) driving on two lanes next to each other. The current lane of the vehicle P_2 is ending. The only actions of each vehicle are here either keeping lane (k) or to change to left neighboring lane (l). This example is considered in the following as a simple two player game which is defined as an extensive-form game by the tupel $\Gamma = (K, P, A, U)$, where

- K is the game tree with a initial state x^1 as the root state,
- $P = \{P_1, P_2\}$ are the players,
- $A = \{k, l\}$ are the actions of the players,

The authors are with [1]BMW Group Research and Technology, D-80992 Munich, Germany, and [2]Chair of Automatic Control Engineering (LSR), Technische Universität München, D-80290 Munich, Germany. {mohammad.bahram, michael.aeberhard}@bmw.de, jasper.friedrichs@googlemail.com, {a.lawitzky, dw}@tum.de

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2015.2508009, IEEE Transactions on Vehicular Technology

- $U = \{1, -1\} \;\hat{=}\; \{\text{'safe','dangerous'}\}$, is the payoff function.

Figure 1 shows the game tree in its extensive-form. The states show the index of the player whose turn it is, while the transitions between states are labeled with a chosen action. The payoff function is given next to the terminal state. The first entry corresponds to the payoff for player P_1 while the second entry corresponds to the payoff for P_2. The obvious strategy for each driver is to perform a lane change in order to maximize the entire payoff, since the other strategies cause critical situation for the one or both of them.

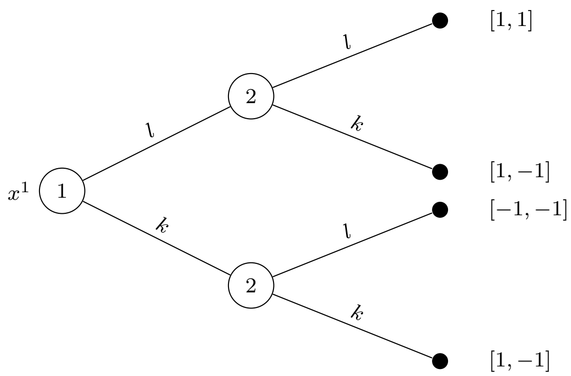


Fig. 1. Game tree of the simple extensive-form game defined by Γ.

In contrast to the numerous applications of game theory in traffic micro-behavior simulation (i.e. traffic situations, in which only few road users are in the "game" in a very limited space) [6], the concept of sequential games is applied here to the approach of interactive driving strategy in order to model the *replanning* capabilities of each agent.

The algorithm presented in this work thus explicitly respects this mutual influence. First, an overview of related publications is given, the problem of prediction and planning under mutual influence is defined and a solution is proposed using methods from game theory. In contrast to former works, this approach fully exploits the implications of the interaction loop, respecting an ongoing mutual influence of the driving decisions. The approach is evaluated in simulated scenarios, pointing out the benefits. Finally, a complexity analysis of the approach is given and the real-time capability is shown.

II. RELATED WORK

As pointed out before, motion prediction and prediction-based planning are key elements of today's robotics and autonomous driving research. It is therefore not surprising that various approaches have been suggested over the recent years. The authors of [7] classify the existing methods for motion prediction in three levels: *Physics-based*, *Maneuver-based* and *Interaction-aware* models.

The physics-based approaches are the simplest motion models. They consider that the future motion of vehicles only depends on the laws of physics. However, the intent of drivers is completely neglected. The maneuver-based motion models eliminate this drawback. Here, each driver is represented as a maneuvering entity which executes its intended maneuvers independently from other traffic. Finally, the interaction-aware approaches regard the dependencies between vehicles' maneuvers. The higher abstraction level and computational complexity are two main challenges of interaction-aware approaches.

Motion prediction in highway scenarios is most commonly based on predefined motion primitives weighted by the vehicles past movement [8], learning based approaches [9]–[11], psychologically motivated human models [12] or optimization-based models [13]. But, in many architectures, e.g. in [14]–[20], navigation consists of separate modules for prediction and for planning. In [19], the planning process itself is realized by a nonlinear model predictive approach which is solved using combinatorial optimization formulation. The mutual influence on each other's motion and thus modeling of replanning capability was not regarded in this work.

Different approaches have been developed which take this interactive coupling explicitly into account. In [21], the interactive human walking navigation is analyzed from a game theoretic perspective. The agents are assumed to make their decisions once and simultaneously. Hence, this approach does not model the replanning capability of the humans.

In [22], the authors investigated the collision free control strategy between two AGVs (Automated Guided Vehicles) for a simplified road junction without traffic lights. The approach is based on the idea of zero-sum games. However, the replanning capability was also not considered in this work.

In [17], the authors look into a highly cooperative scenario, merging of vehicles onto a highway. It combines a cost-based driving strategy with a prediction model. Unfortunately, this approach assumes a simplified road geometry with a fixed position of where a merging vehicle can enter the highway and relies on more parameters than the one presented in this work. Furthermore, the mutual influence between traffic participants is not taken into account.

The approach in [23] considers the interaction for vehicle prediction and risk assessment at road intersections. The authors clearly separate a driver's high-level intention and interaction-aware prediction. With the help of Dynamic Bayesian Networks, high complexity is handled. Inspired by this work, [24] suggests a maneuver-based approach for highway scenarios. The idea in this work is to consider the interaction between the road users by finding an optimal predicted scene in terms of minimizing the risk for all the traffic participants. It lacks, however, a prediction over multiple time steps as given in this work.

While inter-vehicle communication unarguably does bring benefits to cooperative driving, since crucial information will be available as a priori knowledge [25]–[27], there is also "need for significant penetration before [inter-vehicle communication systems] can become effective" [28]. However, the disadvantages due to the high cost of the required infrastructure, the scalability problem of the networks and the related security issues make its industrialization very difficult [29]. Furthermore, a cooperative interaction with older generation vehicles, which are not able of inter-vehicle communication, must be made possible. Therefore, a prediction and planning framework is required which enables a reliable cooperative driving in certain situations (e.g. highway entrance) without a specific need of inter-vehicle communication.

Nevertheless, [24] builds up the foundations on which the presented approach is based on. Both approaches share the use of an interaction-unaware and interaction-aware maneuver prediction. The approach in this paper, however, allows the integration of traffic laws when combining the two (e.g. right of way when merging onto highways). Furthermore, this work suggests a framework for combining different prediction models. The most important difference is that this approach regards maneuver sequences over multiple time steps instead of a single next maneuver. Thus, the *replanning* capability of other traffic participants is considered in the planning of the cognitive vehicle.

In the next section, the problem of interactive scene prediction and planning is discussed and the novel approach is presented for merging onto a highway as an illustrative example, where a strong mutual interaction between vehicles occurs. The algorithm, however, can be applied to other scenarios as well.

III. APPROACH

Given a scene with multiple traffic participants, the task of prediction is to make statements about their future motion based on all observable states. The probabilistic nature of the prediction arises from non-observable states, e.g. the driver intention. The task of safe motion planning is to find a trajectory to act according to some cost function while avoiding collisions. Note, that prediction and planning must incorporate all affected traffic participants and cannot be done independently. This mutual influence of each other's decisions at each point in time has to be modelled in order to predict the scene as accurately as possible.

The approach in this work offers a prediction and planning algorithm which explicitly models the continuous mutual dependence of all traffic participants over multiple time steps. The following assumption describes the idea of this approach:

Assumption 1. *The motion planning of each traffic participant corresponds to its own most likely predicted maneuver in the prediction of the scene, taking into account the mutual dependencies. In other words, interactive planning and prediction are equivalent problems.*

As the cognitive (host) vehicle is a part of the scene, the above assumption can be applied to it as well. Therefore, the most likely maneuver of the cognitive vehicle will at the same time be its planned maneuver. The subsequent distinction between planning and prediction serves a better overview of the approach.

The task of the interactive driving strategy is to find a *maneuver sequence* π_c^{*T} of the cognitive vehicle up to the *planning horizon* T, which represents the best trade-off between intention of the cognitive vehicle and risk assessment with respect to the replanning ability of the other traffic. At the same time, this maneuver sequence is the most likely from the point of view of the surrounding traffic.

Fig. 2 visualizes the idea of the approach. The goal is to plan a maneuver sequence over multiple discrete time steps for automated driving on highways that will be passed to the *operational unit* for execution. The planned maneuver sequence is interaction-aware, i.e. it depends on the prediction of other traffic participants I. The prediction itself is interactive, as it depends on the prediction of the cognitive vehicle by others III. This interactive prediction implies interaction awareness, meaning the mutual dependence between traffic participants is considered.

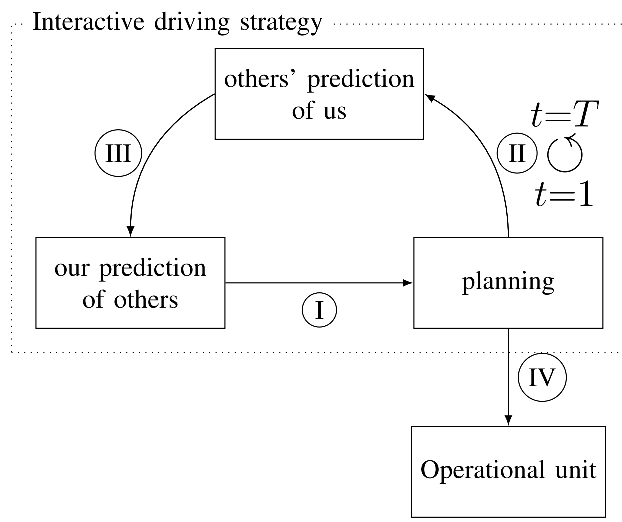


Fig. 2. Closed feedback loop of the interactive driving strategy.

The essential contribution of the approach is that the cycle of prediction and planning is continued with respect to maneuver options previously determined by the planning module of the cognitive vehicle II. The main idea is that other traffic participants will observe the previous maneuver of the cognitive vehicle and have the option of replanning themselves. The prediction and planning loop can thus be continued over multiple time steps up to the planning horizon T before passing the maneuver sequence with the highest interaction-aware probability (i.e. most likely maneuver sequence) as the *driving goals* to the operational unit IV.

This maneuver sequence is the best trade-off between intention and collision avoidance in compliance with mutual interaction to other traffic participants. From a control perspective, prediction of multiple, partially cooperating vehicles is part of a closed feedback loop of acting, sensing and prediction of all traffic participants. As a consequence, prediction of other vehicles over reasonable time horizons has to incorporate own driving decisions as well. Thus, the driving strategy based on this approach shows inherently a cooperative behavior towards the surrounding traffic.

The operational unit comprises the two modules of trajectory planning and motion control. The trajectory planning generates a set of collision-free trajectories with minimum jerk based on the provided driving goals from the interactive driving strategy. The approach is based on a combined optimization of lateral and longitudinal movements using discretized terminal manifolds [30]. The motion control finally performs the first temporal section of the resulting trajectory. Before performing the remainder, the planning and prediction loop is repeated, resulting in a (possibly updated) maneuver sequence (i.e. driving goals) based on the currently available data. Thus, a receding horizon is realized which satisfies the two important requirements of reactivity and prediction.

In the following, the definitions and notations used in this work is first presented. Subsequently, the approach of the interactive driving strategy is discussed in detail.

A. *Definitions and Notations*

Since highways can be seen as fairly structured environments, drivers are modeled in the following to a good approximation using a *finite set of basic maneuvers* $\mathcal{M}$. Thus, each j-th maneuver of the v-th vehicle at time step t belongs to a different set as, $m_{j,v}^t \in \mathcal{M}_v^t$.[1]

Basic maneuver sets are sufficient for planning over only a single time step, however, planning over multiple time steps requires the definition of maneuver sequences. The *set of maneuver sequences* of the v-th vehicle over several time steps is defined as the Cartesian product of its basic maneuver sets for all time steps up to t as

$$\Pi_v^t := \prod_{i=1}^{t} \mathcal{M}_v^i. \tag{1}$$

A maneuver sequence is defined as a t-tuple of maneuvers

$$\pi_v^t \in \Pi_v^t := \left(m_{j,v}^1, m_{k,v}^2, \ldots, m_{n,v}^t\right), \tag{2}$$

where each maneuver is of a different time step.

The focus on interaction requires a relation of maneuver sequences of multiple vehicles. The result of all vehicles performing a maneuver sequence is defined as a *scene*. The output scenes of the prediction module (see I in Fig. 2) represent the prediction of surrounding traffic by the cognitive vehicle c. The *set of prediction scenes* $\mathcal{P}_c^t$ is thus defined as the Cartesian product over entire sets of maneuver sequences, except the set of maneuver sequences of the cognitive vehicle, as

$$\mathcal{P}_c^t := \prod_{\substack{v \\ v\neq c}} \Pi_v^t. \tag{3}$$

A prediction scene $p^t \in \mathcal{P}_c^t$ represents therefore the expected reaction of traffic to the preceding planned maneuver sequence of the cognitive vehicle. In Fig. 3 the set of prediction scenes from perspective of the cognitive vehicle is exemplarily illustrated.

The prediction scenes from perspective of other traffic participants (see III in Fig. 2) differ from those of the cognitive vehicle. It is based on a previously planned maneuver sequence of the cognitive vehicle. This means that at time step t other traffic participants are assumed to have observed the planned maneuver sequence of the cognitive vehicle up to this point. It leads to the definition of the *set of host-dependent prediction scenes* $\mathcal{P}_{x,\pi_c}^t$ of the traffic participant x. The set of

[1]To simplify the notation, it is ignored that the set of basic maneuvers of each vehicle is dependent on its current state. However, this fact is taken into account in the implementation.

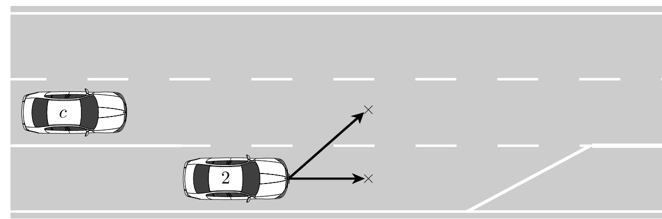


Fig. 3. The set of prediction scenes $\mathcal{P}_c^1$ is described here by the predicted basic maneuvers of the vehicle 2.

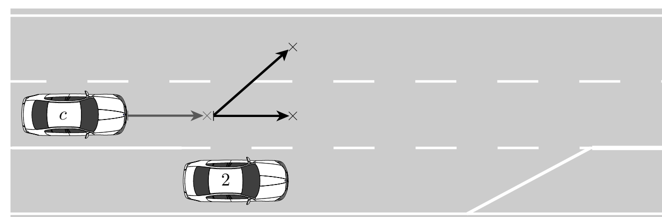


Fig. 4. The set of host-dependent prediction scenes $\mathcal{P}^2_{2,\pi_c^1}$. The set of scenes is described by the maneuver options following the previous maneuver sequence of the cognitive vehicle (e.g. here lane keeping maneuver).

scenes following a specific maneuver sequence π_c^{t-1} of the cognitive vehicle is defined as

$$\mathcal{P}_{x,\pi_c}^t := \pi_c^{t-1} \times \mathcal{M}_c^t \times \prod_{\substack{v \\ v\neq c,x}} \Pi_v^t, \tag{4}$$

where $\pi_c^{t-1} \times \mathcal{M}_c^t$ expresses all maneuver options after a specific maneuver sequence π_c^{t-1}. The Cartesian product, excluding the cognitive vehicle as well as the traffic participant x, represents the possible maneuver sequences of all other traffic participants. An exemplary set of host-dependent prediction scenes at the second time step $\mathcal{P}^2_{2,\pi_c^1}$ is visualized in Fig. 4. Each maneuver option, following the planned single maneuver sequence of lane keeping π_c^1 by the cognitive vehicle, expresses a host-dependent prediction scene by the vehicle 2.

Finally, the *set of planning scenes* $\mathcal{S}_{\pi_c}^t$ is defined as

$$\mathcal{S}_{\pi_c}^t := \Pi_c^t \times \mathcal{P}_c^t. \tag{5}$$

Here, any planning scene $s^t \in \mathcal{S}_{\pi_c}^t$ expresses a set of one planned maneuver sequence with the predicted maneuver sequences of surrounding traffic. The single initial scene s^0 is the set of observed maneuvers of all other vehicles at the starting point of the planning process.

As described in Sec I, the approach of interactive driving strategy will be reformulated in the following section as a problem which is solved by the tools of game theory, especially the extensive-form game formulation.

B. *Interactive Driving Strategy*

The planning (resp. predicting) of the most likely maneuver sequence of the cognitive vehicle π_c^{*T} will be discussed in detail in this section. As mentioned above, the approach is separated into the two modules of interactive planning and prediction. This serves only the purpose of clarity.

1) Replanning-aware Interactive Planning as Extensive-form Game: Besides the simple example given in Sec. I, the approach of extensive-form game allows explicit representation of a number of further important aspects, like the *imperfect information* each player has about the other players' moves when he/she makes a decision. This limited information is modeled by *information sets*. Applied to the presented planning problem, the cognitive vehicle uses a prediction model to estimate the likelihood of future scenes. The information sets can be used to model the uncertainties of the scene prediction, without settling for a certain prediction. Moreover, an extensive-form game allows representation of *incomplete information* in the form of probabilistic actions encoded as *moves by nature*. This feature will be used to model the other traffic participants.

The extensive-form game, which models the interactive driving strategy of the cognitive vehicle in this work, is defined by the quintuple $\Gamma = (K, P, I, \rho, U)$, where

- K is the game tree with the initial observed scene s^0 as the root node,
- P are the players:
  - P_0 is the *player nature* [4] representing other traffic,
  - P_1 represents the cognitive vehicle,
- I are the information sets, consisting of:
  - $\mathcal{P}_c^t$, set of prediction scenes from (3),
  - $\mathcal{S}_{\pi_c}^t$, set of planning scenes from (5),
- $\rho = P\left(p^t \in \mathcal{P}_c^t | \mathcal{S}_{\pi_c}^{t-1}\right)$ is the set of scene probabilities from the traffic,
- U^t is the set of payoffs belonging to each maneuver sequence of the cognitive vehicle.

Fig. 5 exemplarily shows a simple game tree cut after a single time step with two basic maneuver choices for the cognitive and two possible evolutions of a second vehicle. The nodes are labeled with the player whose turn it is, while the transitions are labeled with the chosen action. The maneuvers of other traffic, i.e. the probability distribution over possible future scenes ρ, are supplied by the prediction module which will be discussed in Sec. III-B2. The limited knowledge about the current state is represented by the information set (dashed box around the two nodes of the cognitive vehicle). Loosely speaking, this is the prediction of traffic from the perspective of the cognitive vehicle $\mathcal{P}_c^1$. Depending on the planned maneuver $m_{j,c}^1$, the cognitive vehicle arrives in the planning scene S_j^1. Consequently, the sets of planning scenes are also expressed as information sets for predicting the surrounding traffic in the next time step. The payoff $u^t \in U^t$ is expressed by the probability of the specified maneuver sequence, which represents a compromise between intention and risk assessment. The other traffic does not require a payoff function, since the exact strategy of other vehicles can not be determined (i.e. incomplete information).

The maneuver sequence with the maximum payoff is the most likely and, according to Assumption 1, is the expected decision of the cognitive vehicle. Thus, the performance of the interactive driving strategy is determined by the payoff function which will be derived in the following.

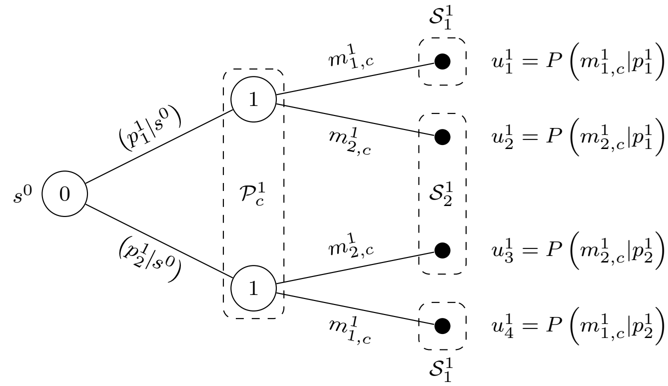


Fig. 5. Simple single time step extensive-form game tree.

The payoff function for the single time step tree from Fig. 5 is given as

$$u^1 = P\left(m_{j,c}^1 | \mathcal{P}_c^1\right) = P_{\not\leftrightarrow}\left(m_{j,c}^1\right) P_{\leftrightarrow}\left(m_{j,c}^1 | \mathcal{P}_c^1\right), \quad (6)$$

where $P_{\not\leftrightarrow}\left(m_{j,c}^1\right)$ is the interaction-unaware a priori probability which is independent of other traffic. It models the intention of the cognitive vehicle. $P_{\leftrightarrow}\left(m_{j,c}^1 | \mathcal{P}_c^1\right)$ is the interaction-aware maneuver probability. It models the probability of performing a maneuver despite the collision risk associated with the set of predicted scenes $\mathcal{P}_c^1$. While the prediction module estimates the probability for each scene $p^1 \in \mathcal{P}_c^1$, the planning process can not assume that a certain scene will occur. In order to model this, the interaction-aware probability is conditioned on the full information set instead of only one predicted scene. The interaction-aware maneuver probability conditioned on the full prediction set is given as

$$\begin{aligned} P_{\leftrightarrow}\left(m_{j,c}^1 | \mathcal{P}_c^1\right) &= \sum_{p^1 \in \mathcal{P}_c^1} P_{\leftrightarrow}\left(m_{j,c}^1 | p^1\right) P\left(p^1 | s^0\right) \\ &= \sum_{p^1 \in \mathcal{P}_c^1} \left(1 - R\left(m_{j,c}^1, p^1\right)\right) P\left(p^1 | s^0\right). \end{aligned} \quad (7)$$

The term $R\left(m_{j,c}^1, p^1\right) \in [0, 1]$ represents the conditional collision risk of the maneuver $m_{j,c}^1$ and each predicted scene p^1. This can be modeled by different approaches, e.g. the ones presented in [31] or [32]. As per definition $\sum_{p^t \in \mathcal{P}_c^t} P\left(p^t | \mathcal{S}_{\pi_c}^{t-1}\right) = 1$, the last equation simplifies to

$$P_{\leftrightarrow}\left(m_{j,c}^1 | \mathcal{P}_c^1\right) = \sum_{p^1 \in \mathcal{P}_c^1} 1 - R\left(m_{j,c}^1, p^1\right) P\left(p^1 | s^0\right). \quad (8)$$

$P\left(p^1 | s^0\right)$ is the probability of each predicted scene $p^1 \in \mathcal{P}_c^1$ conditioned on the last set of planning scenes (i.e. in the single time step case, it is conditioned on the initial observed scene s^0). It follows from (8) that the payoff at the time step t is conditioned on the planning scenes from the previous time step $t-1$. Thus, (6) can be reformulated as

$$u^1 = P\left(m_{j,c}^1 | s^0\right) = P_{\not\leftrightarrow}\left(m_{j,c}^1\right) P_{\leftrightarrow}\left(m_{j,c}^1 | s^0\right). \quad (9)$$

The final step for evaluating maneuver sequences is to make the connection between consecutive maneuvers, extending the extensive-form from single to multiple time steps. Conditioning a basic maneuver $m_{j,c}^t$ on a previous maneuver sequence

π_c^{t-1} is analog to taking account of all possible outcomes of the last maneuver sequence of the cognitive vehicle when looking at the next basic maneuver. The scenes resulting from a cognitive maneuver sequence π_c^{t-1} have already been defined as the planning scenes $\mathcal{S}_{\pi_c}^{t-1}$ (5). According to (9), the payoff of each consecutive basic maneuver is calculated as

$$u^t = P\left(m_{j,c}^t|\mathcal{S}_{\pi_c}^{t-1}\right) = P_{\nleftrightarrow}\left(m_{j,c}^t\right) P_{\leftrightarrow}\left(m_{j,c}^t|\mathcal{S}_{\pi_c}^{t-1}\right). \quad (10)$$

For example, the payoff for a maneuver sequence of the cognitive vehicle at the second time step is recursively calculated as

$$u_{j\to k}^2 = P\big(m_{k,c}^2|\mathcal{S}_j^1\big) P\left(m_{j,c}^1|s^0\right), \quad (11)$$

where $\mathcal{S}_j^1$ is the set of planning scenes resulting from the basic maneuver $m_{j,c}^1$ at the first time step. The interaction-aware maneuver probability based on the previous maneuver sequence is defined, similar to (8) in the general form, as

$$P_{\leftrightarrow}\left(m_{j,c}^t|\mathcal{S}_{\pi_c}^{t-1}\right) = \sum_{p^t\in\mathcal{P}_c^t} 1-R\left(m_{j,c}^t,p^t\right) P\left(p^t|\mathcal{S}_{\pi_c}^{t-1}\right). \quad (12)$$

Finally, the payoff for a maneuver sequence over t time steps is generally formulated as[2]

$$\begin{aligned} P\left(m_{j,c}^t|\mathcal{S}_{\pi_c}^{t-1}\right) &= P_{\nleftrightarrow}\left(m_{j,c}^t\right) \cdot \\ &\qquad \sum_{p^t\in\mathcal{P}_c^t} 1-R\left(m_{j,c}^t,p^t\right) P\left(p^t|\mathcal{S}_{\pi_c}^{t-1}\right) \\ &= P\left(\mathcal{S}_{\pi_c}^t|\mathcal{S}_{\pi_c}^{t-1}\right). \end{aligned} \quad (13)$$

Recalling the problem formulation, the most likely maneuver sequence of the cognitive vehicle expresses the best trade-off between intention and interaction with traffic and is given by

$$\pi_c^{*T} = \underset{\mathcal{M}_c^t}{\arg\max} \prod_{t=1}^{T} P\left(m_{j,c}^t|\mathcal{S}_{\pi_c}^{t-1}\right). \quad (14)$$

In order to reason about maneuver decisions, the planning module requires the likelihood of all possible host-dependent future evolutions of traffic $\rho = P\left(p^t|\mathcal{S}_{\pi_c}^{t-1}\right)$. This incorporates the replanning ability of traffic participants which can react to the maneuvers of the cognitive vehicle over one time step.

The following prediction framework provides this required set of probability distributions. It combines different approaches of motion prediction [7] like physics- and maneuver-based prediction models with risk assessment in order to achieve an interactive prediction.

2) Interactive Scene Prediction: The task of the interactive scene prediction module is to calculate the likelihood of all scenes $p^t \in \mathcal{P}_c^t$, given the previously planned maneuver sequence of the cognitive vehicle, as

$$P\left(p^t|\pi_c^{t-1}\right) = P\left(p^t|\mathcal{S}_{\pi_c}^{t-1}\right) = \prod_{\substack{\pi_v^t\in p^t\\ v\neq c}} P\left(\pi_v^t|\pi_c^{t-1}\right). \quad (15)$$

The maneuver sequences of different vehicles at each single time step are regarded as independent from each other, as during the planning process traffic participants are not aware of a certain maneuver sequence of others. Regarding the prediction and planning loop in Fig. 2, each cycle considers one time step. The prediction in previous cycles has already been calculated and thus, out of a maneuver sequence, only the latest basic maneuver probability needs to be calculated. Eq. (15) can be rewritten with respect to this recursion as

$$P\left(p^t|\pi_c^{t-1}\right) = \prod_{\substack{\pi_v^t\in p^t\\ v\neq c}} P\left(m_{j,v}^t|\pi_c^{t-1}\right) P\left(\pi_v^{t-1}|\pi_c^{t-2}\right), \quad (16)$$

where the probability $P\left(\pi_v^{t-1}|\pi_c^{t-2}\right)$ has been calculated in the previous iteration.

The basic maneuver probabilities of other traffic participants combine the interaction-unaware with the interaction-aware probabilities, similar to the planning module of the interactive driving strategy. Equivalent to (10), the host-dependent probability of a maneuver $m_{j,v}^t$ of the v-th vehicle ($v \neq c$) is calculated as

$$P\left(m_{j,v}^t|\pi_c^{t-1}\right) = P_{\nleftrightarrow}\left(m_{j,v}^t\right) P_{\leftrightarrow}\left(m_{j,v}^t|\pi_c^{t-1}\right). \quad (17)$$

The interaction-unaware maneuver prediction $P_{\nleftrightarrow}\left(m_{j,v}^t\right)$ of each vehicle is the result of a combination of the following two different maneuver probabilities. The initial prediction is an intention-based maneuver probability, $P_I\left(m_{j,v}^t\right)$, which models the unobservable intention of each traffic participant as in [24]. This probability is refined here by an observation-based maneuver probability $P_O\left(s^0\right)$ which is based on the initially observed scene s^0.

With the assumption of rational drivers, the task of calculating an interaction-aware maneuver probability is inevitably associated with risk assessment. The risk is modeled based on multiple possible future evolutions of traffic. Equivalent to (12), the interaction-aware maneuver probability of each evaluated vehicle is given by

$$\begin{aligned} P_{\leftrightarrow}\left(m_{j,v}^t|\pi_c^{t-1}\right) &= \sum_{p^t\in\mathcal{P}_{c\setminus v}^t} P_{\leftrightarrow}\left(m_{j,v}^t|p^t\right) P\left(p^t|\pi_c^{t-1}\right) \\ &= \sum_{p^t\in\mathcal{P}_{c\setminus v}^t} 1-R\left(m_{j,v}^t,p^t\right) P\left(p^t|\pi_c^{t-1}\right). \end{aligned} \quad (18)$$

$\mathcal{P}_{c\setminus v}^t$ is the set of predicted scenes from the perspective of the cognitive vehicle, excluding the evaluated vehicle v.

According to (18), the interactive prediction of the v-th vehicle depends on the prediction of the traffic (including the cognitive vehicle) from perspective of this evaluated vehicle $P_{\leftrightarrow}\left(m_{j,v}^t|p^t\right)$ and the planned maneuver of the cognitive vehicle from the previous step through $P\left(p^t|\pi_c^{t-1}\right)$. Note, that the last terms include the mutual dependence of maneuver probabilities of different vehicles at each time step on each other.

An intuitive solution to resolve this mutual dependency would be to use the interaction-unaware maneuver probabilities of other traffic to calculate the interaction-aware prediction

[2]The formulation $P\left(\mathcal{S}_{\pi_c}^t|\mathcal{S}_{\pi_c}^{t-1}\right)$ clearly shows the dependence of one set of planning scenes on the previous set. The set of planning scenes $\mathcal{S}_{\pi_c}^t$ at the time step t is a state while the maneuver probability from (13) is used for the transition probability. Note, that this satisfies the Markov property as the transition to each state only depends on the previous one.

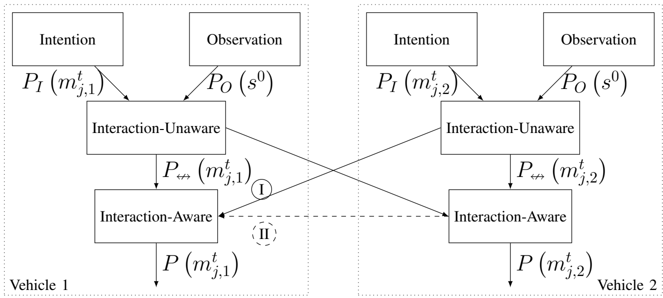


Fig. 6. The diagram of the developed interactive scene prediction framework. Depending on the hierarchy assumption, two possible data flows exist. The approach can be applied without restriction to any number of vehicles.

of the evaluated vehicle v', as in

$$P\left(p^t|\pi_c^{t-1}\right) = \prod_{\substack{v\\ v\neq v'}} P_{\leftrightarrow}\left(m_{j,v}^t\right). \tag{19}$$

If possible, extracting a hierarchy from traffic rules results in a solution that respects the structure of the traffic. With this solution, the mutual dependence between traffic participants can be resolved as well. In Fig. 6 the developed interactive scene prediction framework is presented exemplarily for two vehicles by its different modules and the two different data flows. Path I is not taken into account if a clear hierarchy exists. For example in the merge scenario from Fig. 3, vehicle 2 is supposed to yield the right-of-way of the cognitive vehicle when driving on the highway. In situations where a hierarchy could not be extracted, however, path II is neglected.

After the likelihood of all possible host-dependent scenes ρ is calculated, the most likely maneuver sequence of the cognitive vehicle is determined from (14).

The presented interactive driving strategy determines how mutual influences between vehicles are evaluated over multiple time steps. The next section outlines briefly an exemplary implementation of the presented approach, applied to highway entry ramp merging scenarios.

IV. IMPLEMENTATION

This section provides an overview of the most relevant implementation details.

Definition of Basic Maneuver: In this work, each basic maneuver $m_{j,v}^t \in \mathcal{M}_v$ is defined through the set of pairs of a *longitudinal motion* $\overset{\Rightarrow}{m}$ and a *lateral motion* $\overset{\Uparrow}{m}$,

$$m_{j,v}^t = \{\overset{\Rightarrow}{m}, \overset{\Uparrow}{m}\}. \tag{20}$$

The longitudinal motion is an element from the discrete set of feasible longitudinal accelerations (resp. decelerations) $\mathcal{M}_{\text{long}}$ given by

$$\overset{\Rightarrow}{m} \in \mathcal{M}_{\text{long}} := \{a_{\min}, \ldots, 0, \ldots, a_{\max}\}. \tag{21}$$

The lateral motion is an element from the discrete set of feasible lateral movements, $\mathcal{M}_{\text{lat}}$, given by

$$\overset{\Uparrow}{m} \in \mathcal{M}_{\text{lat}} := \{-1, 0, +1\} \mathrel{\widehat{=}} \{\text{LCL, LK, LCR}\}, \tag{22}$$

corresponding to a lane change to the left, keeping the lane or a lane change to the right. The basic maneuver set of the v-th vehicle at the time step t is thus defined by the Cartesian product of its motion sets as

$$\mathcal{M}_v^t = \{m_{1,v}^t, m_{2,v}^t, \ldots\} := \mathcal{M}_{\text{long}} \times \mathcal{M}_{\text{lat}}. \tag{23}$$

The driving strategy in the upcoming examples evaluates $|\mathcal{M}_c| = 15$ possible maneuvers. Five maneuvers per lane change intent cover constant velocity, maximal acceleration and three decelerating maneuvers. The maximal number of lane changes up to the planning horizon of $T = 6$ seconds is limited to one.

Intention-based Maneuver Probability: This maneuver probability models the intent of each driver (including the cognitive vehicle). The lateral and longitudinal motions in this work are assumed statistically independent for reason of simplification and combined to the intention-based maneuver probability, as[3]

$$P_I\left(m_{j,v}^t\right) = P_I\big(\{\overset{\Rightarrow}{m}, \overset{\Uparrow}{m}\}\big) = P_{I_1}\big(\overset{\Rightarrow}{m}\big)P_{I_2}\big(\overset{\Uparrow}{m}\big). \tag{24}$$

The intent of the cognitive vehicle is based on preferring a given desired velocity. The desired velocity of other vehicles is assumed as their highest velocity since the first observation. The cognitive vehicle prefers keeping the current lane, while it assumes a merging vehicle will tend to change to the left hand lane proportional to the distance to the end of the highway entry ramp (the information is provided by the prior digital maps). These assumptions specify different distributions for the cognitive and merge vehicle for this example.

Observation-based Maneuver Probability: The observation-based maneuver probability $P_0\left(s^0\right)$ is calculated based on the ongoing observed trajectory of the surrounding vehicles. The lateral movement is predicted in this work by a Bayes classifier, using a Gaussian mixture model [19]. It is based on lateral distance to the center of the lane and lateral velocity in the lane-relative coordinate system of each vehicle. The required a priori probability is provided by the previously introduced intention-based maneuver probability. The longitudinal movement is predicted by the physics-based approach of the constant velocity assumption.

Risk Assessment: The interaction-aware maneuver probabilities from (12) and (18) require calculating collision risks. As proposed in [31], the collision risk in this work is a heuristic of time to collision (TTC) and intervehicular time (TIV) values. TTC is the time it takes two vehicles to collide under a constant velocity assumption. TIV is the time it takes a following vehicle to travel the current distance to a leading vehicle. In this work, the overall risk is approximated as the maximum risks between the specific maneuver $m_{j,v}^t$ and every maneuver sequence belonging to the predicted scene p^t as

$$R\left(m_{j,v}^t, p^t\right) = \max_{\pi_{v'}^t \in p^t} r_{\text{TTC}}\left(m_{j,v}^t, \pi_{v'}^t\right) \max_{\pi_{v'}^t \in p^t} r_{\text{TIV}}\left(m_{j,v}^t, \pi_{v'}^t\right). \tag{25}$$

With this setup, the potential of the developed replanning-aware interactive scene prediction and planning is evaluated in the following section.

[3]The multivariate distribution will be learned in future work from real test data.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2015.2508009, IEEE Transactions on Vehicular Technology

TABLE I
RELEVANT MANEUVER SEQUENCES FOR THE FIRST SCENARIO. THE SEQUENCE PROBABILITIES OF INCREASING TIME STEPS ARE DEPICTED TOP TO BOTTOM. EACH TIME STEP REPRESENTS AN INTERVAL OF 2 s IN THE CURRENT IMPLEMENTATION. THE TABLE ALSO SHOWS THE INTERACTION-WARE MANEUVER PROBABILITIES. THE MOST LIKELY MANEUVER SEQUENCE IS HIGHLIGHTED AT EACH TIME STEP.

Time Step	Context Relevant Maneuver Sequence Probabilities					
$t=1$	$P\left(m_4^1\right)$	13.89%	$P\left(m_9^1\right)$	**18.51%**		
	$P_{\leftrightarrow}\left(m_4^1\right)$	100%	$P_{\leftrightarrow}\left(m_9^1\right)$	99.99%		
$t=2$	$P\left(m_4^1, m_4^2\right)$	3.21%	$P\left(m_9^1, m_4^2\right)$	3.17%	$P\left(m_9^1, m_9^2\right)$	**4.12%**
	$P_{\leftrightarrow}\left(m_4^2 \| m_4^1\right)$	100%	$P_{\leftrightarrow}\left(m_4^2 \| m_9^1\right)$	98.24%	$P_{\leftrightarrow}\left(m_9^2 \| m_9^1\right)$	95.06%
$t=3$	$P\left(m_4^1, m_4^2, m_4^3\right)$	**0.96%**	$P\left(m_9^1, m_4^2, m_4^3\right)$	0.91%	$P\left(m_9^1, m_9^2, m_9^3\right)$	0.73%
	$P_{\leftrightarrow}\left(m_4^3 \| m_4^1, m_4^2\right)$	100%	$P_{\leftrightarrow}\left(m_4^3 \| m_9^1, m_4^2\right)$	96.53%	$P_{\leftrightarrow}\left(m_9^3 \| m_9^1, m_9^2\right)$	60.38%

V. RESULTS

The performance of the presented approach has been evaluated in a MATLAB® simulated environment which is developed for simulating arbitrary highway scenarios.

The advantages of the approach are demonstrated by three scenarios that demand different cooperative behaviors. The first scenario focuses on the benefits of planning over multiple time steps. It compares the developed approach with two other driving strategies. The second scenario shows the effects of the replanning-aware prediction of the traffic. The last scenario shows another cooperative reaction. Here, the cognitive vehicle yields by decelerating, to improve the overall safety of the traffic scene. The merging vehicle is simulated with a similar implementation of the introduced interactive driving strategy, but planning just the next maneuver. The behavioral parameters are different and unknown to the cognitive vehicle. The result is a maneuver sequence over three time steps ($T = 6\,\mathrm{s}$), of which only the first maneuver is executed before replanning of the cognitive vehicle.

1) Yield by Lane Change Scenario: The cognitive vehicle using the developed interactive driving strategy will be referred in the following as c_{AI1}. Its behavior is compared to a driving strategy c_{AI2} which considers interaction solely over one time step (similar to [24]). Finally, the driving strategy c_{AI3} is introduced to show the difference to a naïve approach which uses a host-independent and interaction-unaware prediction of the traffic. This driving strategy uses a cost optimization approach over multiple time steps ($T = 6\,\mathrm{s}$), as described in [19].

The performed trajectories of the three driving strategies during the entire simulation time are given in Fig. 7. It is important to note that the merging vehicle has not even started to merge, i.e. the observation-based prediction does not predict a lane change maneuver. The vehicle c_{AI1} yields early to the merging vehicle by changing to the left hand lane. In comparison, the vehicle c_{AI2} changes the lane at a later point in time and vehicle c_{AI3} keeps the lane and thus does not show any cooperative behavior.

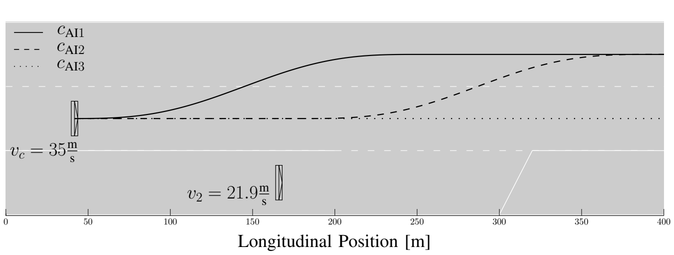


Fig. 7. The yield by lane change scenario driven with the three different driving strategies.

This scenario clearly shows how planning over multiple time steps results in an earlier reaction than one-step planning despite the same behavioral parameters. It also indicates how the performance of the planning depends strongly on the prediction approach. Thus, the cognitive vehicle c_{AI1} yields early to the merging vehicle to reduce the risk for its consecutive maneuvers at a later point in time. In contrast, c_{AI3} which uses a simple host-independent prediction does not even consider interaction with the merging vehicle. This shows that using intention and interaction for the prediction module instead of only an observation-based approach is essential.

In order to present the reasoning of the replanning-aware interactive driving strategy, a context relevant selection of maneuver sequences and the associated probabilities is listed in Table I. The sequence probabilities of increasing time steps are depicted top to bottom, with the probabilities of previous sequence sections above the probabilities at a later time step. Each time step represents an interval of $2\,\mathrm{s}$ in the current implementation. Note, that the probabilities decrease over time because they represent the probabilites of maneuver sequences. As such, with multiple time steps they are a multiplication of multiple basic maneuver probabilities. The table also shows the interaction-aware maneuver probabilities $P_{\leftrightarrow}()$. As discussed in Sec. III, the latter denotes the probability of a maneuver despite the assessed risks (25). It is finally multiplied with the intention-based probability, in order to calculate the total maneuver probability $P()$ (17). The most likely maneuver sequence is highlighted at each time step.

Interpreting the numbers of Table I shows that keeping the lane with constant velocity (m_9) is the most likely maneuver sequence choice for the first two time steps. The lane keeping maneuver sequence (m_9^1, m_9^2, m_9^3) becomes unlikely in the third time step due to a comparatively low interaction-aware probability. In the last time step the maneuver sequence of consecutive left lane change maneuvers with constant velocity (m_4^1, m_4^2, m_4^3) becomes most likely. Starting the lane change process at a later time step than the first time step (m_9^1, m_4^2, m_4^3) comes with a lower interaction-aware probability.

2) No-Yield Scenario: The second scenario explains the advantage of combining planning and prediction in order to model the mutual dependence between maneuvers of different

TABLE II
RELEVANT MANEUVER SEQUENCES FOR THE SECOND SCENARIO. NOTATIONS SIMILAR TO TABLE I.

Time Step	Context Relevant Maneuver Sequence Probabilities					
$t=1$	$P\left(m_7^1\right)$ $P_{\leftrightarrow}\left(m_7^1\right)$	13.46% 99.99%			$P\left(m_9^1\right)$ $P_{\leftrightarrow}\left(m_9^1\right)$	**28.85%** 99.98%
$t=2$	$P\left(m_7^1, m_8^2\right)$ $P_{\leftrightarrow}\left(m_8^2 \mid m_7^1\right)$	2.67% 95.43%	$P\left(m_7^1, m_{10}^2\right)$ $P_{\leftrightarrow}\left(m_{10}^2 \mid m_7^1\right)$	4.39% *82.23%*	$P\left(m_9^1, m_9^2\right)$ $P_{\leftrightarrow}\left(m_9^2 \mid m_9^1\right)$	**8.87%** *95.76%*
$t=3$	$P\left(m_7^1, m_8^2, m_9^3\right)$ $P_{\leftrightarrow}\left(m_9^3 \mid m_7^1, m_8^2\right)$	1.01% 58.44%	$P\left(m_7^1, m_{10}^2, m_9^3\right)$ $P_{\leftrightarrow}\left(m_9^3 \mid m_7^1, m_{10}^2\right)$	0.75% 22.76%	$P\left(m_9^1, m_9^2, m_9^3\right)$ $P_{\leftrightarrow}\left(m_9^3 \mid m_9^1, m_9^2\right)$	**1.14%** 15.73%

TABLE III
PROBABILITY OF THE START OF MERGING, CONDITIONED ON THE LAST MANEUVER OF THE COGNITIVE VEHICLE FOR THE SECOND SCENARIO.

Cognitive maneuver (t=1)	m_6^1	m_7^1	m_8^1	m_9^1	m_{10}^1
Merge probability (t=2)	37.53%	32.71%	25.32%	17.51%	11.29%

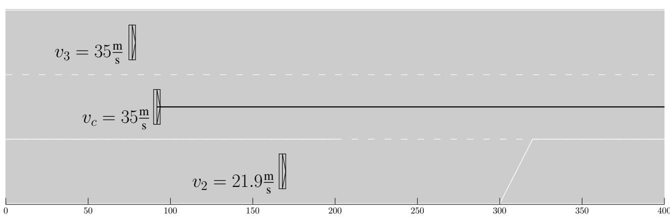


Fig. 8. The second merge scenario. Due to the host-dependent prediction over multiple time steps, the host vehicle prefers the current lane and velocity.

vehicles over time. The cognitive vehicle is again approaching a highway entry ramp with a merging vehicle while the adjacent left lane is blocked by other traffic. Changing to the left lane in order to let the merging vehicle merge onto the highway is thus not a safe option. Consequently, the cognitive vehicle faces the choice of letting the merging vehicle merge in front or behind itself by decelerating or just continuing on the lane. Fig. 8 visualizes the situation while Table II shows selected maneuver sequence probabilities related to the depicted situation. The maneuver m_9 represents keeping constant velocity while a higher index represents an accelerating maneuver and lower indices represent decelerating maneuvers. The driving strategy favors overtaking the merging vehicle by keeping constant velocity (m_9^1, m_9^2, m_9^3) over decelerating (m_7^1, m_8^2, m_9^3) and letting the vehicle merge in front of it.

In order to better understand this reasoning, the risk and host-dependent prediction of the merging vehicle requires a closer look. Table III shows the probabilities of the merging vehicle starting a merge maneuver at the second time step, conditioned on the latest maneuver choice of the cognitive vehicle. The values are the sum of the probabilities of all scenes, following the previous maneuver choice of the cognitive vehicle. The probabilities show, that decelerating in the first time step causes a higher merge probability in the second time step. This has an effect on the interaction-aware maneuver probability of the cognitive vehicle, which is based on the collision risk as well as the probability of the scene itself (see (12)).

For example, the velocity after the maneuver sequence of m_7^1 and m_{10}^2 is lower than after the one of m_9^1 and m_9^2. This does not directly imply a higher interaction-aware maneuver probability for the former sequence (italic values in Table II). In contrast, the probability of performing the maneuver despite the risk is still lower for the first maneuver sequence. The reason is that the probability of the merging vehicle starting a merge maneuver conditioned on m_7^1 is about double as high as for m_9^1. In summary, the effect of the cognitive vehicle decreasing the risk by decreasing its velocity is overcome by the higher chance of the merging vehicle changing the lane.

This effect cannot be observed anymore in the last time step. The maneuver sequence (m_9^1, m_9^2, m_9^3) has a lower interaction-aware maneuver probability than the sequence (m_7^1, m_8^2, m_9^3) because this maneuver sequence results in an overall riskier situation. However, due to the intention of the cognitive vehicle for keeping the lane with constant velocity, this maneuver sequence is still preferred.

3) Yield by Deceleration Scenario: This scenario shows the third cooperative solution. Compared to the previous scenario, the distance to the merging vehicle is now increased. Fig. 9 visualizes the relevant situation while Table IV shows that this scenario is resolved by decelerating and letting the vehicle merge in front. Afterwards, the cognitive vehicle chooses to change the lane in order to reduce risk and accelerate to its desired velocity. This is possible because the distance to the vehicle on the left hand lane increases as the cognitive vehicle decelerates.

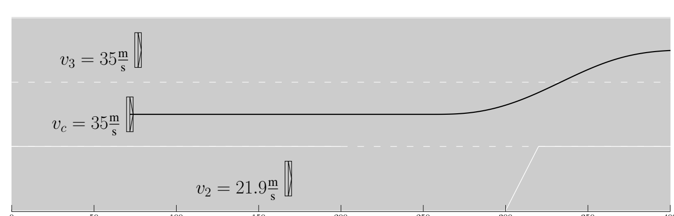


Fig. 9. The yield by deceleration scenario. The situation is resolved by decelerating and letting the vehicle to merge in front.

The three scenarios clearly show the benefits of the interactive planning framework over multiple time steps and using a host-dependent maneuver prediction of traffic. Furthermore, it is shown that the same driving strategy leads to different cooperative behaviors depending on the initial traffic situation. In cases where the mutual dependence between maneuver

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2015.2508009, IEEE Transactions on Vehicular Technology

TABLE IV
RELEVANT MANEUVER SEQUENCES FOR THE LAST SCENARIO. NOTATIONS SIMILAR TO TABLE I.

Time Step	Context Relevant Maneuver Sequence Probabilities					
$t=1$	$P\left(m_8^1\right)$ $P_{\leftrightarrow}\left(m_8^1\right)$	19.99% 99.99%			$P\left(m_9^1\right)$ $P_{\leftrightarrow}\left(m_9^1\right)$	**27.26%** 99.99%
$t=2$	$P\left(m_8^1, m_8^2\right)$ $P_{\leftrightarrow}\left(m_8^2 \vert m_8^1\right)$	3.75% 95.54%			$P\left(m_9^1, m_9^2\right)$ $P_{\leftrightarrow}\left(m_9^2 \vert m_9^1\right)$	**7.49%** 97.48%
$t=3$	$P\left(m_8^1, m_8^2, m_9^3\right)$ $P_{\leftrightarrow}\left(m_9^3 \vert m_8^1, m_8^2\right)$	0.86% 60.29%	$P\left(m_8^1, m_8^2, m_{10}^3\right)$ $P_{\leftrightarrow}\left(m_{10}^3 \vert m_8^1, m_8^2\right)$	**0.87%** 56.78%	$P\left(m_9^1, m_9^2, m_9^3\right)$ $P_{\leftrightarrow}\left(m_9^3 \vert m_9^1, m_9^2\right)$	0.86% 25.16%

choices is negligible (e.g. in regular highway scenarios), the interactive driving strategy will be simplified to the interaction-unaware cost optimization approach over multiple time steps presented in previous work [19].

The presented approach was developed and evaluated in a simulated environment. The next section analyzes the complexity of the interactive driving strategy and introduces an algorithm from game theory to reduce the computational time and to achieve on-line capability of the algorithm.

VI. COMPLEXITY ANALYSIS

The approach described in this work evaluates a multitude of possibilities. This section will outline the complexity of the developed interactive driving strategy.

The number of possible maneuver sequences increases exponentially with time steps,

$$N_{m,\text{planning}}^t = |\Pi_c^t| = |\mathcal{M}_c|^t. \tag{26}$$

The interaction-unaware probabilities have to be evaluated for each maneuver. However, the more relevant factor are the interaction-aware probabilities as they require risk assessment with the predicted maneuvers of other traffic. The risk assessment for each maneuver has to be evaluated with respect to $|p_t|$ maneuvers of $|\mathcal{P}_c^t|$ predicted scenes (25). Based on the definition of $\mathcal{P}_c^t$, the number of risks to be evaluated for each single basic maneuver is

$$\begin{aligned} N_{R,\text{planning}}^t &= |p_t| \cdot |\mathcal{P}_c^t|, \text{ with} \\ |\mathcal{P}_c^t| &= |\Pi_v^t|^{V-1}, \\ |p_t| &= V-1, \end{aligned} \tag{27}$$

where V is the number of vehicles. The number of maneuver sequences of other vehicles $|\Pi_v^t|$ is calculated just as (26), but the differentiation allows a different number of basic maneuver options for the cognitive vehicle and other traffic participants. In order to determine the full complexity of the driving strategy, the complexity of calculating the prediction probabilities of $\mathcal{P}_c^t$ needs to be examined.

To calculate the probability of the predicted scenes, the prediction modules multiplies $|p_t|$ maneuver probabilities for all $|\mathcal{P}_c^t|$ prediction scenes. The number of calculated maneuver probabilities for one set of prediction scenes thus equals $N_{R,\text{planning}}^t$, the number of collision to be evaluated in the planning process. However, the whole idea of the regarding mutual dependence of vehicles is to have more than one set of prediction scenes, depending on the previously evaluated maneuver sequence of the cognitive vehicle. It means that $|\Pi_c^{t-1}|$ sets of prediction scenes have to be considered at time step t. The number of basic maneuvers to be evaluated for prediction thus is

$$\begin{aligned} N_{m,\text{prediction}}^t &= |\Pi_c^{t-1}| \cdot |p_t| \cdot |\mathcal{P}_c^t|, \text{ with} \\ |\mathcal{P}_c^t| &= |\Pi_v^t|^{V-1}, \\ |p_t| &= V-1. \end{aligned} \tag{28}$$

Similar to planning, this requires the calculation of interaction-unaware and interaction-aware maneuver probabilities. For the latter, each maneuver has to be evaluated with respect to $|p_t|$ maneuvers of $\mathcal{P}_{c\setminus v}^t$ predicted scenes (18). Thus, the number of risks to be determined for each single maneuver is

$$\begin{aligned} N_{R,\text{prediction}}^t &= |p_t| \cdot |\mathcal{P}_{c\setminus v}^t|, \text{ with} \\ |\mathcal{P}_{c\setminus v}^t| &= |\mathcal{M}_c|^t \cdot |\Pi_v^t|^{V-2}, \\ |p_t| &= V-1, \end{aligned} \tag{29}$$

where $V-2$ is the number of all vehicles excluding the cognitive vehicle and the currently evaluated prediction candidate v.

The total number of evaluated maneuvers is finally calculated as

$$\begin{aligned} N_m^t &= N_{m,\text{planning}}^t + N_{m,\text{prediction}}^t \\ &= |\mathcal{M}_c|^t + |\mathcal{M}_c|^{t-1} \cdot (V-1) \cdot |\mathcal{M}_v|^{t(V-1)}. \end{aligned} \tag{30}$$

Likewise, the total number of evaluated risks is

$$\begin{aligned} N_R^t &= N_{m,\text{planning}}^t \cdot N_{R,\text{planning}}^t + N_{m,\text{prediction}}^t \cdot N_{R,\text{prediction}}^t \\ &= |\mathcal{M}_c|^t \cdot (V-1) \cdot |\mathcal{M}_v|^{t(V-1)} + \\ &\quad (V-1)^2 \cdot |\mathcal{M}_c|^t \cdot |\mathcal{M}_v|^{t(2V-3)}. \end{aligned} \tag{31}$$

Eq. (31) shows an exponential dependence on time as well as number of traffic participants. However, in most cases where a cooperative behavior should be performed, it is sufficient to consider only the most relevant traffic participant (i.e. $V=2$). This reduces the complexity of the algorithm to $\mathcal{O}(|\mathcal{M}_c|^{2t})$. In order to further reduce the computational complexity and thus the computing time, alpha-beta pruning [5] is used. This concept from game theory eliminates the maneuver possibilities from the search-tree which are guaranteed to not be the best solution.

Fig. 10 shows the computing time results for the three scenarios from the last section. The exemplary single-threaded MATLAB® implementation was run on an Intel® Core i5-2540M@2.6 GHz. The time values were averaged over 10 executions that led to the resulting maneuver sequences previously presented for the scenarios. The pruning algorithm was able to eliminate more than 70% of the maneuver options in the last time step. The computing time does not increase for the latter two scenarios even though the number of vehicles increases. This can be explained by the cognitive vehicle using a constant velocity prediction for the non-merging vehicle (i.e. neglecting explicit modeling of the mutual dependence to non-merging vehicle), since the focus of the exemplary implementation was on the interaction between the cognitive and the merging vehicle. Put simply, the computational complexity does not increase with the number of vehicles since the mutual dependence of other traffic besides the interesting one (here the merging vehicle) will be neglected. The parallel computing nature of the proposed algorithm enables a native GPGPU, e.g. CUDA C implementation [33] in order to minimize the required computing time.

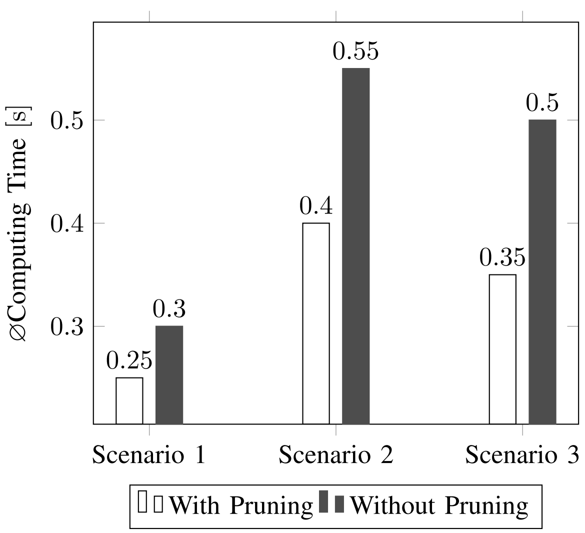


Fig. 10. The average computing time for each execution of the interactive driving strategy with and without pruning for the scenarios.

As explained before the algorithm presented in this paper predicts the future behavior of other traffic participants at a high-level base. Concerning execution time, the algorithm in its current state offers an on-line prediction and will be sped up further in future works. The current state serves as a proof-of-concept, demonstrating its prediction capabilities with a compared the complex problem low computational demand.

The merging traffic scenario has been chosen for demonstration because it offers the clearest isolation of a strong interaction in a everyday environment. Further test, however, also show similar benefits at other scenes.

VII. CONCLUSION

This work presented a novel on-line capable approach to the decision-making process in highly-automated driving based on game theory. The introduced prediction and planning loop of the cognitive vehicle captures the mutual dependence between maneuver choices of all traffic participants over multiple time steps. The replanning ability of other vehicles was thus integrated into the planning of a reasonable interactive maneuver sequence for the cognitive vehicle. It was shown that the approach is able to realize different proactive and cooperative driving behaviors in various simulated highway scenarios. Furthermore, this approach can be used as a sophisticated prediction module in other advanced driver assistance systems. It evaluates the effects of own maneuver on surrounding traffic and predicts their motion over multiple time steps.

For the mathematical modeling of the problem and its solution, methods from game theory have been applied. The planning and prediction framework regards the future evolution of traffic in such an extent that analysis of the problem complexity was also necessary. Nevertheless, the on-line capability of the presented approach has been shown.

The precision of the proposed prediction and planning framework can certainly be improved if relevant information such as intended maneuvers of other traffic participants are a priori known to the framework. This can be achieved in the future through inter-vehicle communication.

Future work will focus on improving the computing time. Moreover, evaluating the approach with real data and its benefits in more complex interactive situations will be investigated.

REFERENCES

[1] M. Benmimoun, A. Pütz, A. Zlocki, and L. Eckstein, “Effects of ACC and FCW on speed, fuel consumption, and driving safety,” in *Vehicular Technology Conference (VTC Fall), 2012 IEEE*, 2012, pp. 1–6.

[2] P. Trautman and A. Krause, “Unfreezing the robot: Navigation in dense, interacting crowds,” in *Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on*, Oct 2010, pp. 797–803.

[3] R. Myerson, *GAME THEORY*. Harvard University Press, 1997.

[4] D. Fudenberg and J. Tirole, *Game Theory*. MIT Press, 1991.

[5] S. J. Russell and P. Norvig, *Artificial Intelligence: A Modern Approach*, 2nd ed. Pearson Education, 2003.

[6] H. Zhang, Y. Su, L. Peng, and D. Yao, “A review of game theory applications in transportation analysis,” in *Computer and Information Application (ICCIA), 2010 International Conference on*, Dec 2010, pp. 152–157.

[7] S. Lefèvre, D. Vasquez, and C. Laugier, “A survey on motion prediction and risk assessment for intelligent vehicles,” *Robomech Journal*, 2014.

[8] D. Vasquez, F. Large, T. Fraichard, and C. Laugier, “High-speed autonomous navigation with motion prediction for unknown moving obstacles,” in *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, vol. 1, 2004, pp. 82–87.

[9] D. Vasquez, *Incremental Learning for Motion Prediction of Pedestrians and Vehicles*, ser. Springer Tracts in Advanced Robotics. Springer, 2010, vol. 64.

[10] P. Kumar, M. Perrollaz, S. Lefèvre, and C. Laugier, “Learning-based approach for online lane change intention prediction,” in *IEEE Intelligent Vehicles Symposium*, 2013.

[11] M. Aeberhard, S. Rauch, M. Bahram, G. Tanzmeister, J. Thomas, Y. Pilat, F. Homm, W. Huber, and N. Kaempchen, “Experience, results and lessons learned from automated driving on germanys highways,” *IEEE Magazine on intelligent transporation systems*, 2014, accepted.

[12] D. D. Salvucci, “Modeling driver behavior in a cognitive architecture,” *Human Factors: The Journal of the Human Factors and Ergonomics Society*, vol. 48, no. 2, pp. 362–380, 2006.

[13] C. Frese, *Planung kooperativer Fahrmanöver für kognitive Automobile*, ser. Karlsruher Schriften zur Anthropomatik. KIT Scientific Publishing, 2012, vol. 10.

[14] J. Wei, J. M. Dolan, and B. Litkouhi, “A prediction- and cost function-based algorithm for robust autonomous freeway driving,” in *IEEE Intelligent Vehicles Symposium*, Juni 2010, pp. 512–517.

[15] D. C. K. Ngai and N. H. C. Yung, “A multiple-goal reinforcement learning method for complex vehicle overtaking maneuvers,” *Intelligent Transportation Systems, IEEE Transactions on*, vol. 12, no. 2, pp. 509–522, 2011.

[16] M. Ardelt, C. Coester, and N. Kämpchen, “Highly automated driving on freeways in real traffic using a probabilistic framework,” *Intelligent Transportation Systems, IEEE Transactions on*, vol. 13, pp. 1576–1585, 2012.

[17] J. Wei, J. Dolan, and B. Litkouhi, “Autonomous vehicle social behavior for highway entrance ramp management,” in *Intelligent Vehicles Symposium (IV), 2013 IEEE*, June 2013, pp. 201–207.

[18] J. Nilsson and J. Sjberg, “Strategic decision making for automated driving on two-lane, one way roads using model predictive control,” in *Intelligent Vehicles Symposium*, 2013.

[19] M. Bahram, W. Anton, M. Aeberhard, and D. Wollherr, “A prediction-based reactive driving strategy for highly automated driving function on freeways,” in *IEEE Intelligent Vehicles Symposium*, 2014.

[20] W. Schwarting and P. Pascheka, “Recursive conflict resolution for cooperative motion planning in dynamic highway traffic,” in *Intelligent Transportation Systems (ITSC), 2014 IEEE 17th International Conference on*, 2014.

[21] A. Turnwald, W. Olszowy, D. Wollherr, and M. Buss, “Interactive navigation of humans from a game theoretic perspective,” in *Intelligent Robots and Systems (IROS 2014), 2014 IEEE/RSJ International Conference on*, 2014.

[22] S. Arora, A. Raina, and A. Mittal, “Collision avoidance among agvs at junctions,” in *Intelligent Vehicles Symposium, 2000. IV 2000. Proceedings of the IEEE*, 2000, pp. 585–589.

[23] S. Lefèvre, C. Laugier, and J. Ibanez-Guzman, “Evaluating risk at road intersections by detecting conflicting intentions,” in *Intelligent Robots and Systems (IROS), 2012 IEEE/RSJ International Conference on*, Oct 2012, pp. 4841–4846.

[24] A. Lawitzky, D. Althoff, C. F. Passenberg, G. Tanzmeister, D. Wollherr, and M. Buss, “Interactive scene prediction for automotive applications,” in *Intelligent Vehicles Symposium (IV), 2013 IEEE*. IEEE, 2013, pp. 1028–1033.

[25] L. Li, F.-Y. Wang, and H. Kim, “Cooperative driving and lane changing at blind crossings,” in *Intelligent Vehicles Symposium, 2005. Proceedings. IEEE*, June 2005, pp. 435–440.

[26] L. Li and F.-Y. Wang, *Advanced Motion Control and Sensing for Intelligent Vehicles*. Springer US, 2007.

[27] L. Li, D. Wen, and D. Yao, “A survey of traffic control with vehicular communications,” *Intelligent Transportation Systems, IEEE Transactions on*, vol. 15, no. 1, pp. 425–432, Feb 2014.

[28] M. L. Sichitiu and M. Kihl, “Inter-vehicle communication systems: A survey,” *Communications Surveys Tutorials, IEEE*, vol. 10, no. 2, pp. 88–105, Feb 2008.

[29] V. Paruchuri, “Inter-vehicular communications: Security and reliability issues,” in *ICT Convergence (ICTC), 2011 International Conference on*, Sept 2011, pp. 737–741.

[30] M. Werling, S. Kammel, J. Ziegler, and L. Gröll, “Optimal trajectories for time-critical street scenarios using discretized terminal manifolds,” *The International Journal of Robotics Research*, 2011.

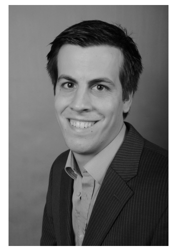
[31] S. Glaser, B. Vanholme, S. Mammar, D. Gruyer, and L. Nouveliere, “Maneuver-based trajectory planning for highly autonomous vehicles on real road with traffic and driver interaction,” *Intelligent Transportation Systems, IEEE Transactions on*, vol. 11, no. 3, pp. 589–606, 2010.

[32] A. Lawitzky, D. Wollherr, and M. Buss, “Maneuver-based risk assessment for high-speed automotive scenarios,” in *Intelligent Robots and Systems (IROS), 2012 IEEE/RSJ International Conference on*. IEEE, 2012, pp. 1186–1191.

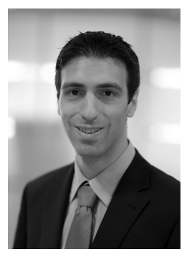
[33] J. Sanders and E. Kandrot, *CUDA by Example: An Introduction to General-Purpose GPU Programming*. Addison Wesley, 2010.

Mohammad Bahram was born in Karaj, Iran on August 29, 1985. He received the diploma engineer of electrical engineering and information technology (Diplom-Ingenieur) from the RWTH Aachen University in 2012. Since 2012, he has been working towards the Doctoral degree at the Chair of Automatic Control Engineering at Technische Universität München, Munich, Germany in conjunction with the BMW Group Research and Technology in Munich, Germany. Since 2015 he is working on driving simulation technologies at the BMW Group Research and Technology. His research interests include driving strategy, motion planning and maneuver prediction for the highly automated driving as well as driving simulation technologies.

Andreas Lawitzky received the B.Sc. degree and the Diplom-Ingenieur degree from the department of electrical engineering and information technology of Technische Universität München, Munich, Germany in 2009 and 2011, respectively. The latter thesis was carried out at the Autonomous Robotics Laboratory of the CSIRO ICT Centre at the Queensland Centre for Advanced Technologies (QCAT), Brisbane, Australia. Since 2011 he is a doctoral candidate at the Chair of Automatic Control Engineering at Technische Universität München, Munich, Germany working in the field of highly-automated driving in cooperation with BMW Group Research and Technology in Munich, Germany. His research interests include safety assessment, collision avoidance and safety verification for mobile robots.



Jasper Friedrichs received the Master’s degree in electrical engineering and information technology from the Technische Universität München, Munich, Germany, in 2014. He wrote his master’s thesis about the presented prediction and planning framework in collaboration with the BMW Group Research and Technoloy, Munich. Afterwards he relocated to California for research assignment with the Big Data Analytics unit of Infosys Limited. Along this path his research interest has shifted from autonomous systems and game theory to natural language processing.



Michael Aeberhard was born in Lausanne, Switzerland on December 5, 1983. He received a Bachelor of Science in Computer Engineering from the Georgia Institute of Technology in Atlanta (GA), USA in 2007, a Master of Science in Electrical and Computer Engineering from the Georgia Institute of Technology in Metz, France in 2009 and the Master Professionnel degree from the Ecole supérieure d’électricité (Supélec) in Metz, France in 2009.

Since 2010, he has been working towards the Doctoral degree in Engineering (Dr.-Ing.) at the Technische Universität Dortmund in Dortmund, Germany in conjunction with the BMW Group Research and Technology in Munich, Germany. Since 2012 he is a sub-project leader for highly automated driving at the BMW Group Research and Technology. His research interests include sensor data fusion, multi-object tracking, environment perception and functional development for advanced driver assistance systems and highly automated driving.

Dirk Wollherr is currently Professor (since 2014) at the Chair of Automatic Control Engineering, Department of Electrical, Electronic and Computer Engineering, Technische Universität München, Munich, Germany. He received the diploma engineer degree in Electrical Engineering in 2000 and the Doctor of Engineering degree in Electrical Engineering in 2005 and the Habilitation degree in 2013 from the Technische Universität München, Munich, Germany. From 2001-2004 he has been research assistant at the Control Systems Group, Technische Universität Berlin, Germany. In 2004 he has been granted a research fellowship from the Japanese Society for the Promotion of Science (JSPS) at the Yoshihiko-Nakamura-Lab, The University of Tokyo, Japan. From 2006-2008 Dr. Wollherr has been General Manager of the Cluster of Excellence ”Cognition for Technical Systems (CoTeSys)” and 2005-2014 he has been principal investigator, independent junior research group leader, and research area leader within CoTeSys. He has been TUM Carl-von-Linde Junior Fellow, Institute of Advanced Studies from 2010 until 2013. Furthermore, he has been active in dissemination, e.g. General Chair of the German ”Robotik 2008” conference and Finance Chair of RO-MAN 2008, as well as in several EU projects, e.g. Robot@CWE, CyberWalk, Movement, and IURO. His research interests include automatic control, robotics,autonomous mobile robots, human-robot-interaction, and humanoid walking.