Computer Vision Assignment 4

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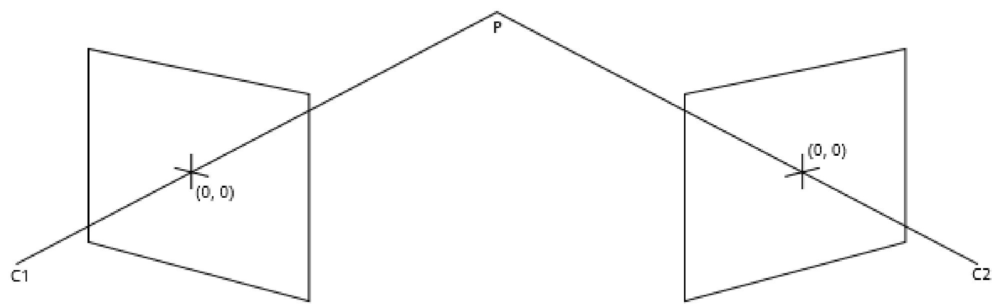
Q1.1

We know that the image coordinates are normalized and the coordinate origin (0,0) coincides with the principal point.



$$[u \quad v \quad 1] \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 0$$

$$[0 \quad 0 \quad 1] \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$F_{33} = 0$$

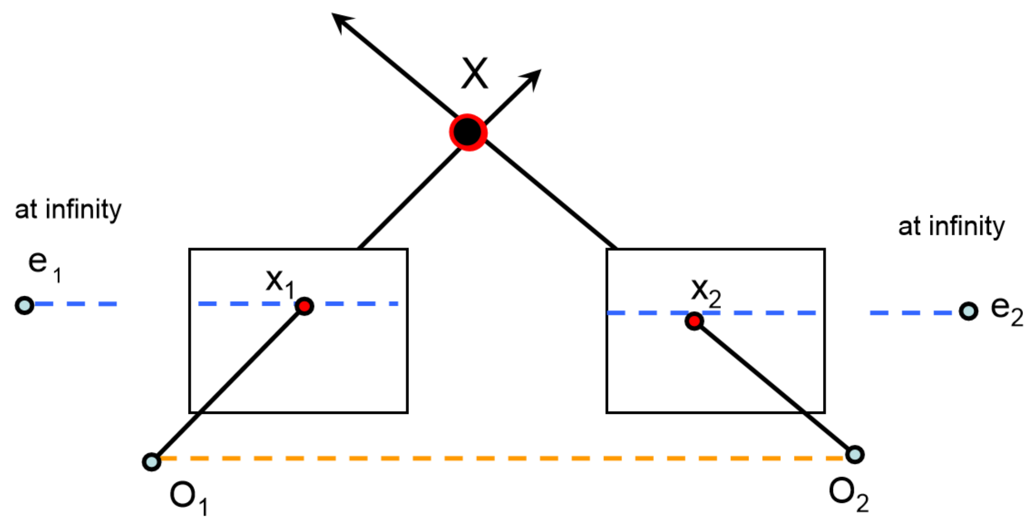
Q1.2

We know the equation,

$$p^{l'} E p^r = 0$$

For canonical cameras, K1=K2=I

$R = I, t = [\,T\ 0\ 0]$



We also know that,

$E = [t]_x R$

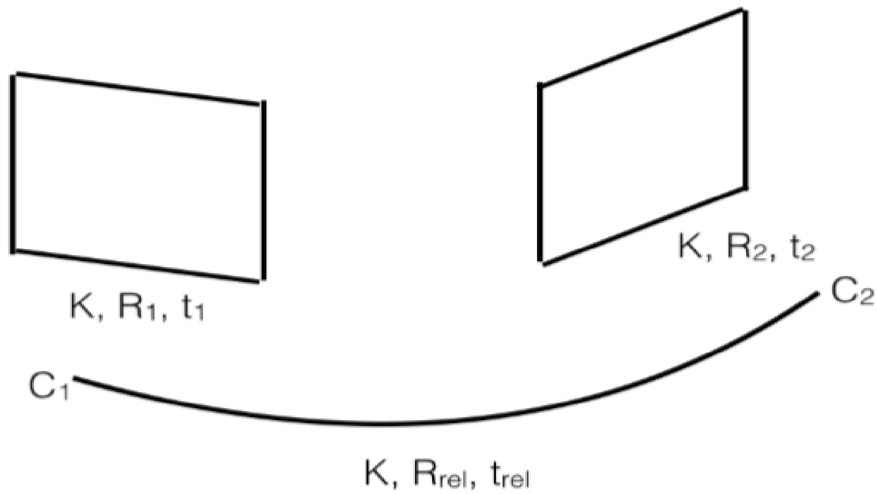
$$[u \quad v \quad 1]\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -T\\ 0 & T & 0\end{bmatrix}\begin{bmatrix}u'\\ v'\\ 1\end{bmatrix} = 0$$

Multiplying the above, we get

$$Tv = Tv'$$

So, from the above expression it means that, the y axis is same and therefore the lines are parallel to x axis.

Q1.3



Transformation with respect to the world frame

$H_1^0 \quad = \; R_1^0 \; + t_1^0$

$H_2^0 \quad = \; R_2^0 \; + t_2^0$

$H_2^1 \quad = \; H_0^1 \; . \; H_2^0$

But in actual it is

$H_2^1 \quad = \; (H_1^0\,)^{-1}. \; H_2^0$

$$\begin{bmatrix} (R_1^0\,)^{-1} & (R_1^0\,)^{-1}t_1^0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} R_2^0 & t_2^0 \\ 0 & 1 \end{bmatrix}$$

Multiplying we get

$R_{rel} = (R_1)^{-1}R_2$

t_{rel}=$(R_1)^{-1}t_2 - (R_1)^{-1}t_1$

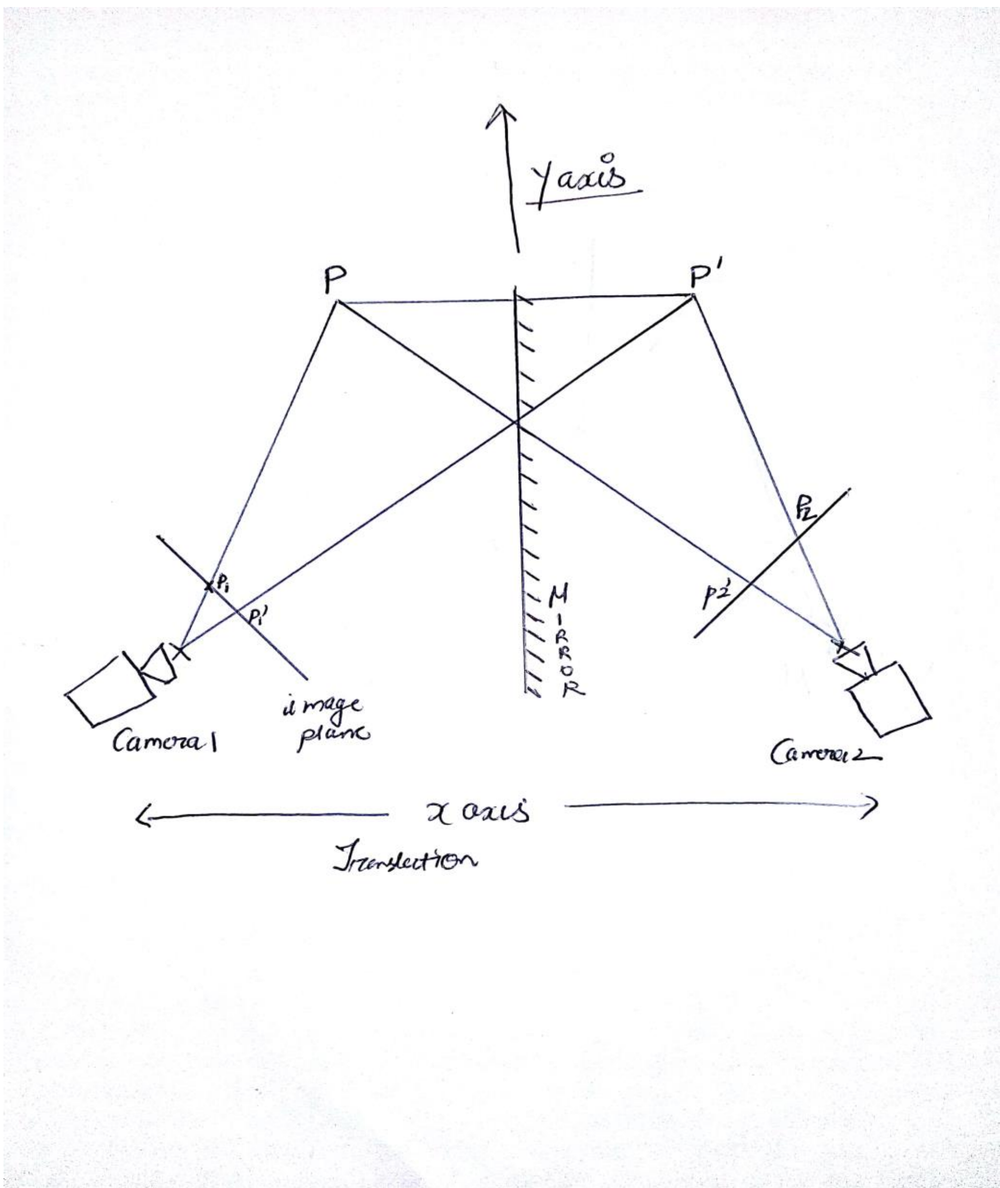
Substituting the value of R_{rel} and t_{rel} in the below equation we get

E=$[t]_{rel} R_{rel}$

$F = K^{-1T} [t]_{rel} R_{rel} K^{-1}$

Q1.4

For our convenience we assume that there is a second camera with a projection matrix M2 which is the reflection of the first camera with a projection matrix M1.



Considering a pure translation between the cameras along the x axis ,

We can write,

M2=T.M1

Where t is the pure translation by some amount.

Equations for 2D points from camera 1 and camera 2 is

$$\tilde{p}_1 = M_1\tilde{P}$$
$$\tilde{p}_1' = M_1\tilde{P}'$$
$$\tilde{p}_2 = M_2\tilde{P}$$
$$\tilde{p}_2' = M_2\tilde{P}'$$

The relationship between the 2D points from camera 1 and 2D points of camera 2 is given by the following equation.

$$\tilde{p}_2 F\tilde{p}_1' = 0$$
$$\tilde{p}_2' F\tilde{p}_1 = 0$$
$$\tilde{p}_1 F^T\tilde{p}_2' = 0$$
$$\tilde{p}_1' F^T\tilde{p}_2 = 0$$

$$\begin{aligned}
&\tilde{p}_1' F^T\tilde{p}_2 + \tilde{p}_2 F\tilde{p}_1' = 0\\
&\Rightarrow M_1\tilde{P}' F^T M_2\tilde{P} + M_2\tilde{P} F M_2\tilde{P} = 0\\
&\Rightarrow M_1 T\tilde{P} F^T M_1 T\tilde{P} + M_1 T\tilde{P} F M_1 T\tilde{P} = 0\\
&\Rightarrow \left(M_1 T\tilde{P}\right)\left(F^T + F\right)\left(M_1 T\tilde{P}\right) = 0
\end{aligned}$$

$$\left(F^T + F\right) = 0$$

From this equation, it is evident that F is a fundamental skew symmetric matrix.

Q2.1 Eight-point Algorithm

Recovered F matrix=

-1.44439172037979e-09	-7.86268842605896e-08	0.00113268843790934
-1.12046048382129e-07	1.26181700323091e-09	4.15431319598230e-06
-0.00108810353337907	1.53773111548100e-05	-0.00464231304074352

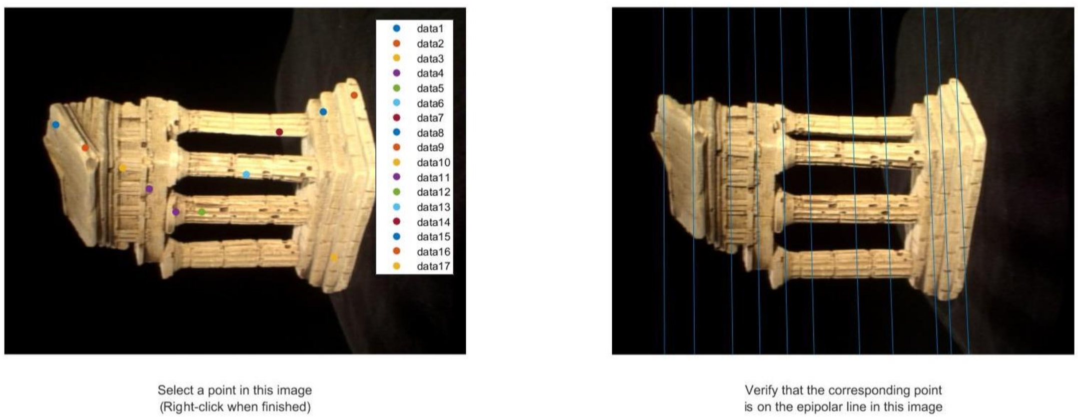


Fig 1: Correspondences with 8-point algorithm

Q2.2 Seven Point Algorithm

Recovered best F matrix=

0.0000	-0.0000	0.0042
-0.0000	0.0000	-0.0000
-0.0041	0.0002	-0.0364

Output for the 7-point algorithm for different set of input points.

a)

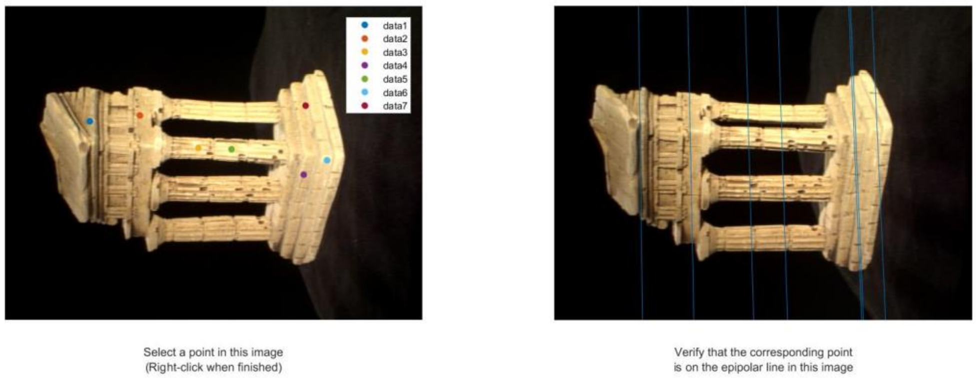


Fig 2: Correspondences with 7-point algorithm

Q3.1 Essential matrix

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E=K2'*F*K1
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Essential Matrix=

-0.00333887924220011	-0.182412669998792	1.69196368520047
-0.259944407570051	0.00293797787788453	-0.0448735808147042
-1.69707201537301	-0.0123318106875355	-0.000627378815906576

Q3.2 Triangulate

We know that x=CX where X is the homogenous 3D point, and x is the 2D image point. C is a 3 X4 camera projection matrix .

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} X \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = 0$$

Here x and y are the 2D points of image 1 and x' and y' are the image coordinates for image2.

$$A = \begin{bmatrix} x(C_1 31 + C_1 32 + C_1 33 + C_1 34) - (C_1 11 + C_1 12 + C_1 13 + C_1 14) \\ y(C_1 31 + C_1 32 + C_1 33 + C_1 34) - (C_1 21 + C_1 22 + C_1 23 + C_1 24) \\ x'(C_2 31 + C_2 32 + C_2 33 + C_2 34) - (C_2 11 + C_2 12 + C_2 13 + C_2 14) \\ y'(C_2 31 + C_2 32 + C_2 33 + C_2 34) - (C_2 21 + C_2 22 + C_2 23 + C_2 24) \end{bmatrix}$$

Q4.1 Epipolar Correspondences

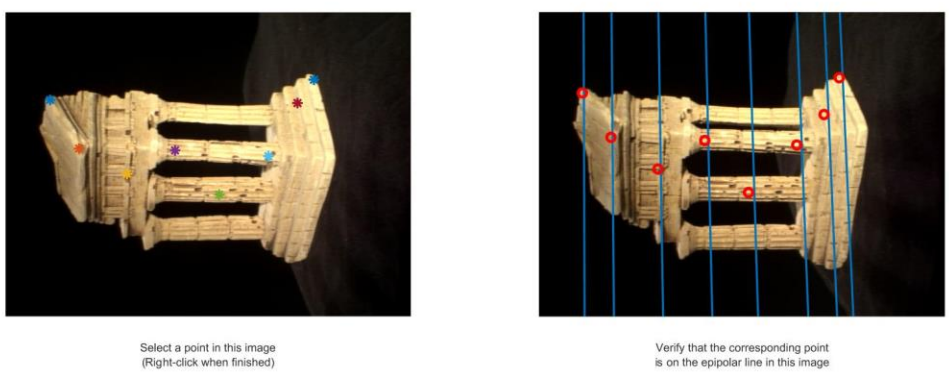
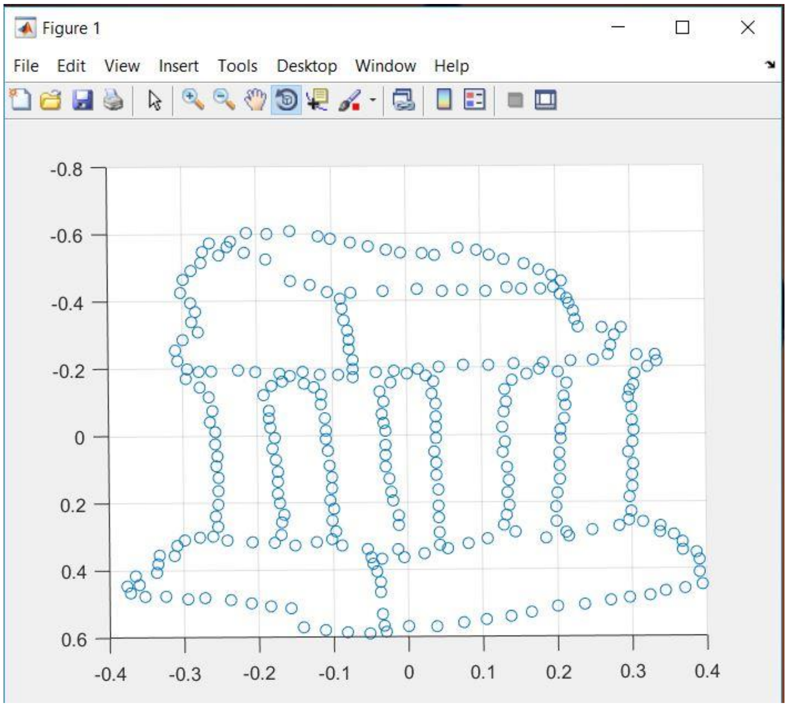
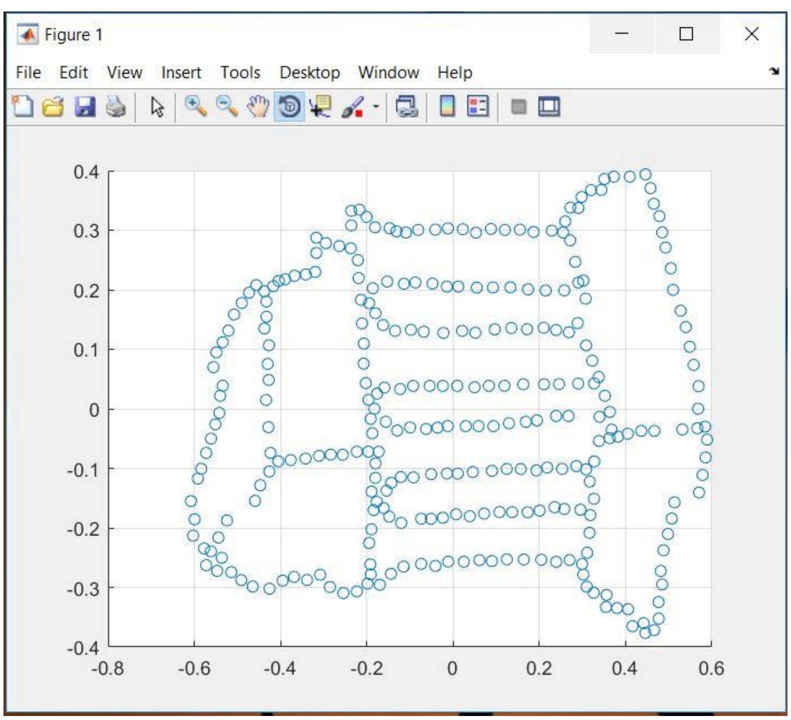


Fig 4: Epipolar correspondences

Q4.2 Visualization

Temple:





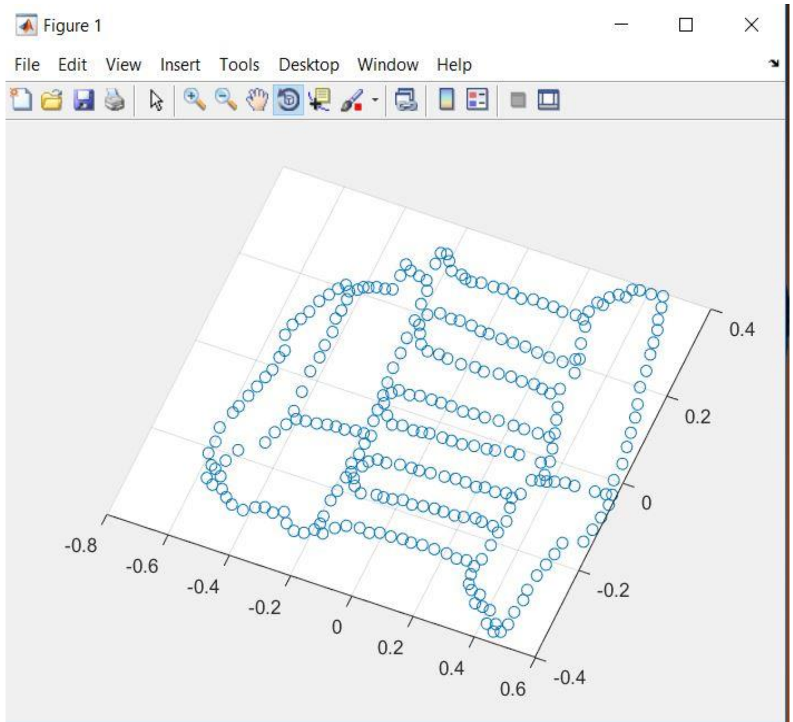


Figure 1
File Edit View Insert Tools Desktop Window Help
-0.8
-0.6
-0.4
-0.2
0
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0.4
0.6
-0.4
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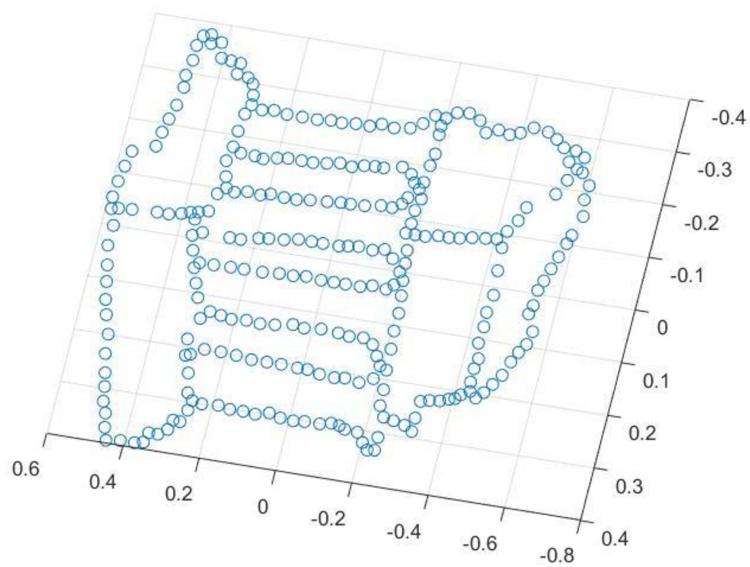
0.4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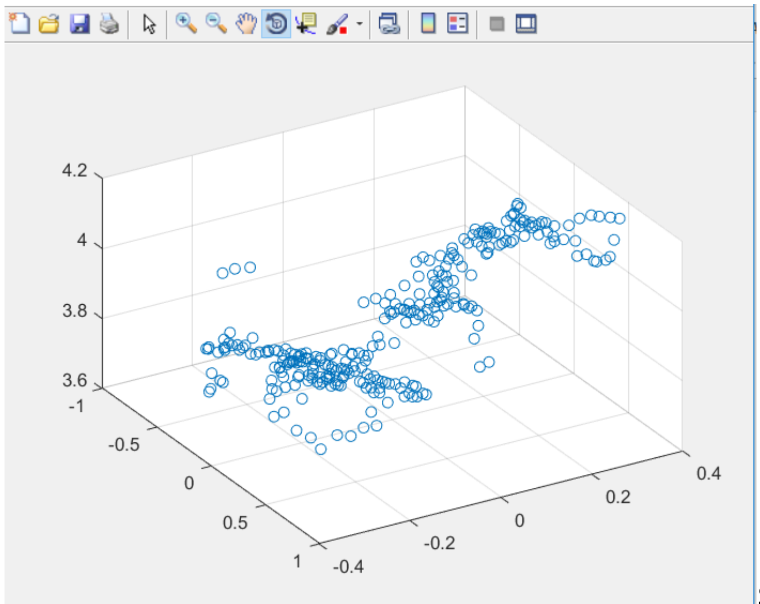

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