

WITWERK

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200861

# \* Physics Assignment \*

## Reflection and Refraction of em wave :

When the em wave travelling in one medium reaches at the surface of another medium, some of its energy returns while the remainder transmitted some into second medium. the transmitted energy constitute the refracted wave and the energy of the incident wave which returns constitute reflection wave. the reflection and refraction of light at a plane surface between two media of different dielectric properties is a familiar example of refraction and reflection of the em wave.

The various aspect of the Phenomenon can divide themselves into two classes

- kinematic properties
- dynamic properties

(A)

kinematic properties (universal truth) :  
Following are kinematic properties of reflection and refraction

Frequency : The frequency of the

(iii) law of reflection :- In case of reflection the angle of reflection ( $\theta_R$ ) is equal to the angle of incident ( $\theta_i$ )

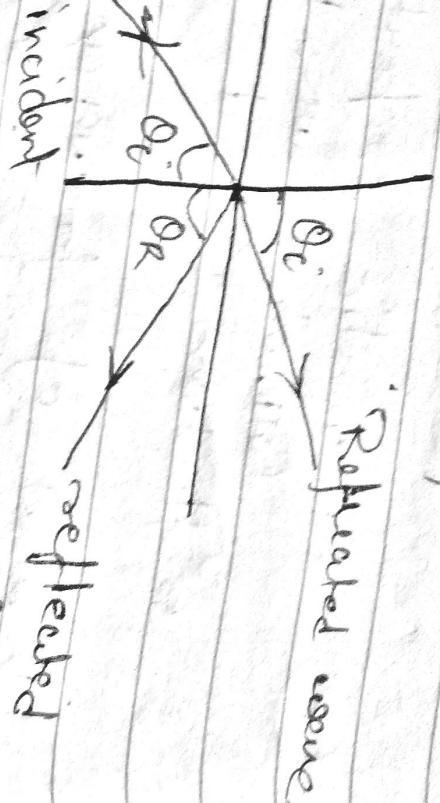
$$\theta_R = \theta_i$$

(iv) Snell's law :-

In case of refraction the ratio of the sines of the angle of refraction to the sines of the angle of incidence is equal to the ratio of the refractive indices of the two media i.e.

$$\frac{\sin \theta_R}{\sin \theta_i} = m_1$$

where  $m_1$  &  $m_2$  are the refractive indices of the two media respectively.



(iii) Law of reflection :- In case of reflection the angle of reflection ( $\theta_R$ ) is equal to the angle of incident ( $\theta_i$ )

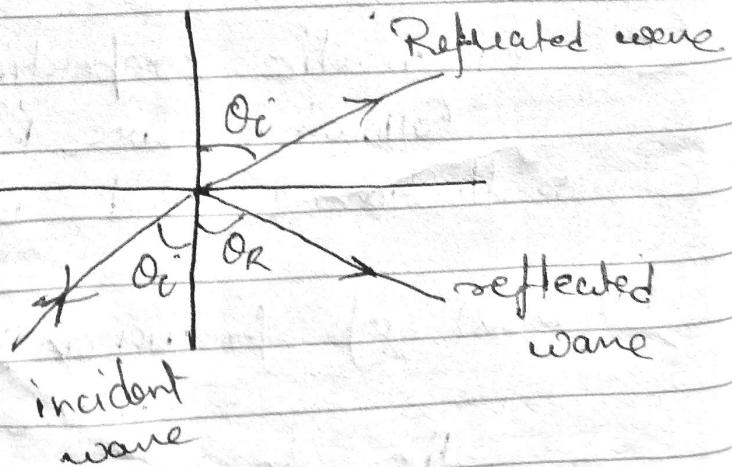
$$\theta_R = \theta_i$$

(iv) Snell's law :-

In case of refraction the ratio of the sum of the angle of refraction to the sine of the angle of incidence is equal to the ratio of refraction index of the two media i.e.,

$$\frac{\sin \theta_R}{\sin \theta_i} = \frac{n_1}{n_2}$$

where  $n_1$  &  $n_2$  are the refraction indices of the two media respectively.



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### (8) Dynamic Properties (changeable):

These properties are concerned with the:

- i) Intensities (amplitude) of reflected wave.
- ii) Phase change and polarisation of wave.

Proof: Assuming that the electric vector of an electromagnetic wave is given by

$$E = E_0 e^{-i(\omega t - kr)}$$

and is crossing a boundary the tangential component of electric intensity is continuous then prove the necessary law of reflection and refraction

Sol: Let the medium below the plane:

$\epsilon_1 = \epsilon_0 (\text{ie } x-z \text{ plane})$  have permittivity  $\epsilon_1$  and permeability  $\mu_1$  respectively.

while above it is  $\epsilon_2, \mu_2$ . If the plane wave with wave vector  $\vec{k}$

$$(E_{oi})_x e^{-i(\omega_i t - k_i z)} + (E_{or})_x e^{-i(\omega_R t - k_R z)} \\ = E_{iT} e^{-i(\omega_i t + k_T z)} \quad \text{--- (2)}$$

Equation (2) can only be satisfied if time and space varying component of the phases are equal therefore:

$$\omega_i t = \omega_R t = \omega_T t$$

$$\omega_i = \omega_R = \omega_T \quad \text{--- (3)}$$

The equations (3) show that the frequency of the wave remain unchanged by reflection and refraction

Also

$$(k_i z)_{z=0} = (k_T z)_{z=0} = (k_R z)_{z=0} \quad \text{--- (4)}$$

As the incident became in  $x-z$

$$plane \quad (k_i z) = k_i (x \sin \theta_i + z \cos \theta_i)$$

$$(k_R z) = k_R (x \sin \theta_R + z \cos \theta_R) \quad \text{--- (5)}$$

$$(k_T z) = k_T (x \sin \theta_T + z \cos \theta_T) \quad \text{--- (6)}$$

From equation (4), solving (6) we get

$$k_i x \sin \theta_i = k_R x \sin \theta_R$$

$$k_i \sin \theta_i = k_R \sin \theta_R$$

$$As \quad \omega_i = \omega_R \quad \& \quad v_i = v_R \cdot (\text{medium is})$$

$$\sin \theta_i = \sin \theta_R$$

Also from equations (i), (ii) & (iv)

$$K_c \sin \theta_i = K_t \sin \theta_t$$

$$\omega_i \sin \theta_i = \frac{\omega_t}{v_1} \sin \theta_t$$

$$= v_i = v_t$$

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{v_t}{v_i} = \frac{c v_1}{c v_2} = \frac{m_1}{m_2}$$

Fresnel's formula. (Dynamical theory)

These formulae relates the amplitude of the reflected and transmitted wave with that of incident wave. Here we will consider the case of reflection & refraction at the interface of two dielectric media.

Case (i). E parallel to the plane of incidence.

In this case the magnetic number are all to the bound any way.

$$\left. \begin{aligned} (H_i)_t &= H_i \\ (H_r)_t &= H_R \\ (H_t)_t &= H_T \end{aligned} \right\} \rightarrow (i)$$

$$\left. \begin{aligned} (E_i)_t &= E_i \cos \theta_i \\ (E_R)_t &= E_R \cos \theta_R \\ (E_t)_t &= E_T \cos \theta_T \end{aligned} \right\} - (b)$$

Now using boundary condition

$$(E_1)_t = (E_2)_c$$

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$$(H_1)_e = (H_2)_e, \text{ we get}$$

$$E_i \cos \theta_c - E_R \cos \theta_R = E_T \cos \theta_T$$

$$(E_i - E_R) \cos \theta_c = E_T \cos \theta_T - 2(a)$$

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$$H_i + H_R = H_T - 2(b)$$

But we know that

$$H = \sqrt{\mu} \cdot E$$

Now  $H_i = \sqrt{\mu_i} E_i$ ,  $H_R = \sqrt{\mu_r} E_R$

8  $H_T = \sqrt{\mu_2} E_T$

for non magnetic media  $\approx$

$$\frac{\mu_2}{\mu_1} = \mu_r = 1 \Rightarrow \mu_2 = \mu_1$$

Relative permeability

Also

$$\frac{E_2}{E_1} = \frac{E_{T2} G_0}{G_1 E_0} = \frac{E_2}{E_1} = \frac{n_2^2}{n_1^2}$$

for non-joule media

Eqn 2 (b) gives

$$\sqrt{\mu_1} (E_i + E_R) = \sqrt{\mu_2} E_T$$

$$E_i + E_R = \frac{n_2}{n_1} E_T \quad \text{--- (3)}$$

Putting:  $E_T = n_1/n_2 (E_i + E_R)$  in 2(a) we get

$$(E_i - E_R) \cos \theta_i = n_1/n_2 (E_i + E_R) \cos \theta_T$$

$$\frac{E_R}{E_i} / H = \frac{\cos \theta_i - n_1/n_2 \cos \theta_T}{n_1 \cos \theta_T + \cos \theta_R}$$

$$= \frac{\cos \theta_i - \sin \theta_T \cos \theta_T}{\sin \theta_i}$$

$$\cos \theta_T = \frac{\sin \theta_T + \cos \theta_i}{\sin \theta_i}$$

$$\frac{E_R}{E_i} / H = \frac{\sin \theta_i \cos \theta_i - \sin \theta_T \cos \theta_T}{\sin \theta_i \cos \theta_i + \sin \theta_T \cos \theta_T}$$

$$\frac{E_R}{E_i} / H = \frac{\sin 2\theta_i - \sin 2\theta_T}{\sin 2\theta_i + \sin 2\theta_T}$$

$$\frac{E_R}{E_i} / H = \frac{\operatorname{Jem}(\theta_i - \theta_T)}{\operatorname{Jem}(\theta_i + \theta_T)} \quad @$$

Similarly eliminating  $E_R$  from Eq 2(a) using, we get

$$E_T = 2 \cos \theta_i \sin \theta_T \quad B$$

$$\left. \begin{array}{l} (E_i)e = E_i \\ (E_R)e = E_R \\ (E_T)e = E_T \end{array} \right\} - 1(a)$$

$$(H_i)e = H_i \cos \theta_i$$

$$(H_R)e = H_R \cos \theta_R \quad \left. \right\} - 1(b)$$

$$(H_T)e = H_T \cos \theta_T$$

$$(E_1)e = (E_2)e$$

$$\therefore (H_1)e = (H_2)e$$

we have

$$E_i + E_R = E_T \quad \left. \right\} - 2(a)$$

$$H_i \cos \theta_i + H_R \cos \theta_R = H_T \cos \theta_T \quad \left. \right\} - 2(b)$$

using H & E relations we have 2(b) as

$$(E_i - E_R) \cos \theta_i = \frac{n_2}{n_1} \cdot E_T \cos \theta_T \quad (3)$$

similarly putting  $E_R = E_T - E_i$  from eqn  
2(a). in (3) we get

$$\frac{E_T}{E_i} e = \frac{2 \cos \theta_i \sin \theta_T}{\sin(\theta_i + \theta_T)} \quad (2)$$

Relations A; B; C and are known  
eg - fresnel formulae.

Reflection and transmission coefficient

$$\frac{E_R}{E_i} = \frac{n_2/n_1 \cos \theta_i - \cos \theta_f}{n_2/n_1 \cos \theta_i + \cos \theta_f}$$

$$= \frac{n_2/n_1 \cos \theta_i - \sqrt{1 - \sin^2 \theta_f}}{n_2/n_1 \cos \theta_i + \sqrt{1 - \sin^2 \theta_f}}$$

$$= \frac{n_2/n_1 \cos \theta_i - \sqrt{1 - (n_1/n_2)^2 \sin^2 \theta_i}}{n_2/n_1 \cos \theta_i + \sqrt{1 - (n_1/n_2)^2 \sin^2 \theta_i}}$$

for normal incidence.  $\theta_i = 0$

$$\frac{E_R}{E_i} / \frac{1}{\sqrt{\rho}} = \frac{n_2/n_1 - 1}{n_2/n_1 + 1} = \frac{n_2 - n_1}{n_2 + n_1}$$

The reflection coefficient ( $R$ ) = Reflected flux  
incident flux

$$= \frac{\text{Intensity of reflected beam}}{\text{Intensity of incident beam}}$$

$$R = \frac{(E_R)^2 \times \rho}{(E_i)^2 \times \rho} = \left( \frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

Transmission Coefficient  $T = 1 - R$

$$T = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

Prove Prove that the glass air interface  
 $n_1 = 1$   $n_2 = 1$  for normal incidence  
the reflection & transmission coefficient

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Here  $n_1 = 1.5$  &  $n_2 = 1$

Glass . Air

$$R = \left( \frac{n_2 - n_1}{n_2 + n_1} \right)^2 = 0.04$$

$$T \Rightarrow 1 - R$$

$$\Rightarrow 1 - 0.04$$

Ans