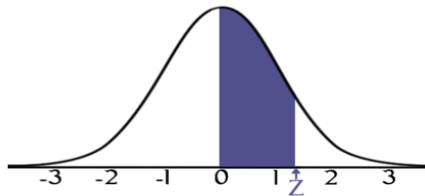# Test 2

Total Time : 45 min

Total marks : 25

STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

1) A number of customers arrive at a grocery store. All the customers are equally likely to come at any time of the day. On an average 120 customers arrive per day. Let X be the number of customers arriving from 10am to 11:30am. What is $P(6 \leq X \leq 8)$? [2 Marks]

2) Probability density function f(x) of a random variable X is given as follows:

  f(x) = 1/6 for 0 < x < 2,
       = 1/3 for 2 < x < 4,
       = 0 otherwise
 a) Compute E(X) and V(X) [3 Marks]
 b) Compute Median of X.
    (Hint: Median M is defined s.t. $P(X < M) = .5$) [2 Marks]

3) A new billing system is to be established in a consulting firm. The manager thinks that the new system is cost effective only if the average monthly account is more than $170. Using the new billing system, a random sample of monthly accounts of 400 customers is drawn, for which the sample mean is $178 and standard deviation is $65.
a) Can the manager conclude that the new system is cost effective at level of significance 0.05? Find the Z-score to show your result. [1.5 marks]
b) The old billing system of the firm had previously produced a mean monthly account of $167 in a random sample of 300 customers, where the standard deviation was $82. Is there a significant difference between the old and the new system? Find the p-value.
  [2 marks]

 4) In a petrol station, the daily demand for regular petrol is normally distributed with mean of 4000 liters and standard deviation of 400 liters.
a) Today, there is 4500 liters of petrol in storage. What is the probability that the storage is sufficient for today's demand? [1 marks]
b) What is the probability that the average demand for the month of June is less than 3900 liters? [1.5 marks]

5) Nate is taking a math class, an english class, and a social studies class during his senior year of high school. Next year, he will be attending the University of North Carolina and studying to become an engineer. The 4-year tuition for The University of North Carolina is $120,000
If Nate can earn an "A" in his math class, he will win a one-time $10,000 math scholarship. If he can earn an "A" in all three classes, he will win a $50,000 one-time general scholarship in addition to the math scholarship.
His estimated probability of earning an "A" for each class is in the table below.

What is the expected amount of tuition that Nate will pay to attend The University of North Carolina for 4 years?   [2 Marks]

Class	Probability of an "A"
Math	70
English	40
Social Science	10

6) What is the smallest sample size required to provide a 95% confidence interval for a mean, if it is important that the interval be no longer than 1cm? You may assume that the population is normal with variance 9cm2.   [1.5 Marks]
   a) 417
   b) 34
   c) 139
   d) 102

7) Suppose a hypothesis test results in a p-value of .03. What can you say about the null and alternative hypotheses? Assume significance level 1%.   [1 Mark]

8) Suppose 2 people are to arrive at a certain location at some randomly chosen time between 1:00 PM and 2:00 PM, and each person will wait a certain amount of time before leaving. Let's assume person one can wait for w_1 and the other can wait for w_2 (both expressed as fractions of the total time interval). What is the probability that they will meet?   [3 Marks]

9) Tom is currently applying for graduate job positions. For each application he submits, the probability that it gets short-listed and he gets invited for an interview is 0.1.
A) If he applies for 5 positions, what is the probability that he will get at least one interview?
                                                                                [1 Mark]
B) find the minimum number of applications he will have to submit to ensure that the probability that he gets at least one interview is greater than 0.9.   [1.5 Marks]

10) A process is considered to be performing acceptably if its mean is 200. Because it is expensive to shut down and reconfigure this process, such measures are undertaken only if there is compelling evidence that the process mean is not 200. Experience has shown that the process is normally distributed with a standard deviation of 40.
A. What is an appropriate alternate hypothesis in this setting?   [1 Mark]
        A)   H1: μ = 200

B)   H1: $\mu \neq 200$
C)   H1: $\mu \geq 200$
D)   H1: $\mu \leq 200$
E)   None of the above

B. In this situation, a Type I error would be made when it is concluded that μ is _____ 200 when in fact μ _____ 200.   [1 mark]

         A)   Greater than; less than or equal to
         B)   Less than; greater than or equal to
         C)   Equal to; greater than
         D)   Not equal to; equals
         E)   Equals; not equal to