AN EMPIRICAL STUDY AND COMPARATIVE ANALYSIS OF CONTENT BASED IMAGE RETRIEVAL (CBIR) TECHNIQUES WITH VARIOUS SIMILARITY MEASURES

*Nalini Pasumarthi, Lakshmi Malleswari

**Assistant Professor, ECE, Mahatma Gandhi Institute of Technology, India, pasumarthinalini@gmail.com*

Principal, Sri Devi Women's Engineering College, India, blmalleswari@gmail.com

***Index Terms*— Cumulative distances, Geometrical distances, Intensity Histograms, Statistical distances, Wavelet texture.**

***Abstract*— Content Based Image Retrieval (CBIR) is a process in which for a given query image similar images will be retrieved based on the image content similarity. Image content refers to its visual features, which are mathematical representations of a digital image. The image retrieval task primarily depends on image feature extraction and similarity measurement between the feature vectors. The performance of CBIR process not only decided by the optimum features extracted from the image but also on the proper choice of the similarity / dissimilarity measures (distance metrics). As the image features broadly diversified in terms of color, texture and shape based, using the same distance metric may not work well for all of these features. In this paper, first we presented overview of geometric and statistical distance metrics used in CBIR along with the comparative analysis of these measures on color and texture features. Color features extracted by computing color histograms in HSV space and texture features by wavelet decompositions. Geometrical distances such as Manhattan, Chebyshev, Euclidean, statistical distance metrics such as Cosine Similarity, Chi-square, Kullback- Leibler, Jeffrey and cumulative statistical distance metrics of Kolmogorov-Smirnov, Cramer von Mises and Earthmover's distances were analyzed for feature similarity. We gave certain conclusions on the performance of all these distance metrics in terms of Mean Average Precision (MAP) and Recall rates with color and texture features respectively.**

I. INTRODUCTION

DUE to rapid changes happening in multimedia and image processing applications and there is a huge increase in image data CBIR gained large scope in research community to work on. Applications of CBIR broadly diversified as general and domain specific. In general applications such as multimedia and photo archive managements, variance in content is very high, semantics are heterogeneous, content description is subjective and its performance is qualitatively measured and here the image databases are very large in size. On the other hand, In domain specific CBIR, variance in content is very low, semantics are homogeneous, content description is objective and quantitative evaluations were done for a specific object recognitions. These domains include medical images, satellite images, face and finger print databases and many more. In CBIR for a given query image, features are extracted and these features compared with feature set of database images and the images with more feature similarity or with least differences will be retrieved. In CBIR, both feature extraction and their similarity checking play significant roles.

Image features categorized as low-level visual features, scale, rotation and translational invariant features and high-level semantic features. Low-level features include color, texture and shape. For general CBIR applications, color features are most important and are the intensity values of a pixel obtained in any color plane representation of an image. Color features obtained with the help of histograms[3], correlograms[11] and color sets [12]. Texture features capture the granularity and repetitive patterns of image surfaces and play an important role in image retrieval. Texture analysis is widely used in interpretation and classification of terrain images, radiographic and microscopic cell images. Sixteen statistical textural features proposed by Haralick et al in [13] and six features like directionality, regularity, coarseness, roughness, line-likeness and contrast prescribed by Tamura et al [14] got more popularity as they are more visually perceptual. Transformation based texture calculation based on energy of subbands using DCT presented in [15]. Statistical textural features obtained with the help of discrete wavelet transforms presented in [16]. Gabor wavelet texture features compared with orthogonal wavelet texture features in [17]. In Operator or pattern based texture calculations, Local Binary Patterns (LBP), Local Ternary Patterns (LTP) and Local Derivative Patterns (LDP) are used. These are more powerful texture features as they are invariant to rotation and scaling proposed in [18]. Various visual features for image retrieval described in detail in [10] with experimental comparison and concluded same set of features will not work well for different datasets.

To some extent, performance of CBIR decided by the correct usage of distance metrics. Similarity measurement in CBIR can be rank based or distance based [29]. According to [2], dissimilarity measures used in CBIR broadly classified as geometric measures and statistical measures [30]. Dissimilarity measures for interval data are Minkowski distances. Similarity measures for interval data are Cosine and Pearson correlation distance metrics. Minkowski is a p^{th} power to the p^{th} root distance. This considered as multiple of the power mean of the component wise differences between two vectors. It is also a generalized form of geometric distances, its L^2 metric is Euclidean distance (ED), L^1 metric is Manhattan distance (MHD) and L^∞ metric is Chebyshev distance (CD). Squared Euclidean distance gives faster response as it excludes sqrt operation. Bray Curtis distance is the ratio of Manhattan distances. Canberra distance is the sum of scaled absolute differences. Cosine similarity generates a value that says how similar images based on the angle between them, not like in Euclidean distance, which works on the magnitude differences. Cosine distance vectors shifted by their mean values called as correlation distance. Pearson correlation and Cosine Similarity measures used for shape similarity matching. Chi-square and Phi-square are used for finding count data dissimilarity. Chi-square measures the line likeliness of one distribution from the other.

Statistical distance measures find the distance between random variables or probability distributions or between an individual samples to group of sample points. CBIR using probabilistic architecture detailed in [41]. They are Kullback Leibler (KL), Jeffrey (JD), Bhattacharya, Kolmogorov-Smirnov (KS), Cramer von Mises (CvM) and Earthmover's (EMD) distances. All of these, measure distance between two probability distributions X and Y. Kullback belongs to f-divergence group. KL distance is a measure of extent to which two probability density functions agree. JD is the symmetric distribution of KL distance. KS, CvM and EMD are cumulative statistical distance measures. KS represents a distance between two probability distributions defined on a single real variable. KS quantifies a distance between the empirical distribution function of a sample with CDF of the reference distribution. This is sensitive t differences in location as well as shape. CvM is a criterion for finding the goodness of fit of a cumulative distribution function. Harold Cramer and Richard Edler von Mises proposed this in 1929. It is an alternative to KS test. EMD considered as 1st Wasserstein or 1st Mallows distance metric [41]. This metric originally introduced by Leonid Wasserstein in 1961 and later modified by R.L. Dobrushin in 1970. This metric at level one i.e W_1 widely used to compare color histograms. The name Earthmover's distance, proposed by J. Stolfi in 1994. The representation of a distribution by a set of clusters is called as signature.

II. RELATED WORK

Minkowski distance metric at different levels presented in [26]. These metrics are preferred when each dimension holds equal importance in retrieval process. Minkowski metric used for feature vector comparison by An et al [8]. ED was used in [8] for feature vector comparison. Manhattan distance or City block distance or Minkowski L_1 depends on the rotation of the coordinate system, rather its translation L1 metric Minkowski distance to compare LBP histograms of query and database images [19]. Similarity of color histograms using L_1 metric which is also known as city block metric proposed by Swain et al [3]. Similarity of color histograms using L2 metric also known as Euclidean proposed in [4]. It was used in [8] for feature vector comparison. L_∞ metric or Chebyshev metric used as a similarity measure for image retrieval in [22]. Minkowski L_1, L_2, L_∞ metrics, Cosine Angle Distance (CAD), Chi-square and Mahalanobis distance measures for shape databases evaluated in [20]. Kullback - Leibler distance for texture features discussed in [7]. Papers [21] discuss the effect of cumulative statistical distance metrics for similar image retrieval. Three dimension analogues of the Kolmogorov-Smirnov test (L1-metric), the Cramer - von Mises test (L2-metric), or the L1-metric or EMD used for cumulative histogram similarity check in [2]. An efficient Earth Mover Distance metric and its effect on image retrieval presented in [24]. EMD can also be considered as Mallows distance according to [25]. Image histogram comparison using Kolmogorov-Simirnov described in [37]. Earthmover's distance and hybrid feature including color, texture and shape as feature vector to match image in [9]. Performances of various distance measures including Euclidean, Manhattan and Cosine similarity for CBIR evaluation presented in [23] and [27]. Shape based image retrieval using CAD discussed in [34] and shown significant improvement over ED based retrieval. The cosine similarity between spatial weighted visual word vectors, by using a Gaussian Mixture Model (GMM), used as distance measurement between regions in [35]. Texture retrieval by joint modeling of feature extraction and Kullback - Leibler distance discussed in Do et al [7]. The effect of this distance on statistical texture features obtained from a DWT presented in [16]. Chi - Squared distance is used in [28] to compare two binned data sets and determine if they are drawn from the same distribution function. According to [29], ED and CAD are more similar in large databases. CAD got the normalization advantage over ED when different features combined. Distance between color layout features and texture features obtained by using correlation distance in [33]. CAD metric shown better retrieval efficiency over geometric distance based techniques. Comprehensive Performance Comparison of Cosine, Walsh, Haar, Kekre, Sine, Slant and Hartley Transforms for CBIR with Fractional Coefficients of transformed Image analyzed in [36]. Wavelet based texture features such as standard deviation, energy and mean compared with Manhattan, and Canberra distances in [31]. Effect of Kullback-Leibler divergence between Gaussian Mixture Models (GMM) presented in [39]. Wavelet based color texture retrieval using the KL divergence between bivariate generalized Gaussian models described in [40]. Stricker et al [2] presented color-indexing schemes by using cumulative color histograms that has complete color distributions and the color distributions compared using L_1,(EMD) L_2 (Cramer von Mises)and L_∞ (Kolmogorov Smirnov) distances. According to Siegel et al [5] and Breiman [6] the discrete, three dimensional analogous of the Kolmogorov-Simirnov test L_∞, Cramer -von Mises test (L_2) and the Earth Mover Distance (L_1) are the statistical distance metrics. The EMD based on the minimal cost that must paid to transform one distribution into the other, in a precise sense. Rubner et al [1] presented image retrieval framework by using Earthmover's Distance EMD as a similarity measure to achieve perceptual similarity. Advantage of EMD in texture similarity measures EMD does not require segmentation of multiple textures. Earthmover’s distance and hybrid feature including color, texture and shape as feature vector to match image in Zeng et al [9]. Multiple step image retrieval using various filters for fast and accurate process, EMD was used by [32] for multimedia databases. The computational cost of EMD is super cubic to the number of bins.

III. MATHEMATICAL BACKGROUND

A. Minkowski Distance

The Minkowski distance of order *p* between two points X and Y belongs to feature space R, X = (x_1, x_2, x_3..x_i....x_n) and Y = (y_1, y_2, y_3..y_i.....y_n) € R is given in equation (1)

$$d(x,y) = \sum_{i=1}^{n}((|x_i - y_i|)^p)^{\frac{1}{p}} \quad (1)$$

In which p represent the order of the distance metric. When p < 1, it is not considered as a distance metric as it does not satisfy the triangular inequality. When p = 1, the distance turns into Manhattan distance and when p = 2 it become Euclidean distance. In the limiting case of *p* reaching infinity, the Chebyshev distance is obtained. Minkowski distance metric is generalized form of Euclidean distance and city block distances, preferred when each dimension holds equal importance in retrieval process.

B. Manhattan or City Block Distance

This metric0 measures the direct grid distance along the pixels and diagonal movements not allowed. Manhattan distance metric retrieve images at a faster rate when compared with Euclidean distance. The metric shown good MAP in both feature similarity measures, but worked well for HSV histogram comparison over texture feature similarity. This distance is shown in equation (2).

$$d(x,y) = \sum_{i=1}^{n}|x_i - y_i| \quad (2)$$

C. Euclidean Distance

This distance represent length of the line segment connecting two points in a feature set. In image processing and retrieval, the images are with n dimensional feature vectors and hence n- dimensional Euclidean distance is used. If x and y represent two RGB images then the Euclidean distance between them obtained as shown in equation (3).

$$d(x,y) = \sum_{i=1}^{n}\sqrt{(x_i - y_i)^2} \quad (3)$$

In this paper, we used the Euclidean distance for color and texture feature comparisons.

D. Chebyshev / Chessboard or Infinity Distance

This distance also known as chessboard obtained when limiting value reaches to infinity. This distance between two points (x, y) is expressed as shown in equation (4).

$$d(x,y) = max_{i=1}^{n}|x_i - y_i| \quad (4)$$

E. Cosine Angle Distance

Cosine Angle Distance (CAD) does not follow the triangular similarity. The cosine distance metric normalizes all feature vectors to unit length and makes it invariant against relative in-plane scaling transformation of the image content. This measure is best suited to find the orthogonality between two vectors. If the cosine angle computed between the Eigen values of vectors, it works much better. CAD is shown in equation (5).

$$d(x,y) = \frac{\sum_i x_i y_i}{\sqrt{\sum_i x_i^2 \sum_i y_i^2}} = \frac{x_i \cdot y_i}{||x_i||||y_i||} \quad (5)$$

F. Chi-Square Distance:

The chi-square distance measure is used in correspondence analysis and related ordination techniques. Chi-squared distance does not reach a constant, maximal value for sample pairs with no species in common, but fluctuates according to variations in the representation of species with high or low total abundances. Chi-squared tests are often constructed from a sum of squared errors or through the sample variance as shown in equation (6).

$$d_{chisquare}(x,y) = \sum_{i=1}^{N}\frac{(x_i - \mu_i)^2}{\mu_i} \quad (6)$$

G. Kullback-Leibler Distance

Solomon Kullback and Richard Leibler introduced Kullback Leibler divergence in 1951[38]. It does not obey triangular inequality hence it is not a valid distance metric. This also considered as relative entropy of two distributions and is shown in equation (7).

$$d_{kullback}(x,y) = \sum_{i=1}^{N} x_i log\frac{y_i}{x_i} \quad (7)$$

H. Jeffrey Distance

Jeffrey Divergence is the symmetric version of Kullback-Leibler distance with respect to samples x and y. The divergence equation is shown in equation (8).

$$d_{jeffray}(x,y) = \sum_{i=1}^{N} x_i log\frac{y_i}{\mu_i} + y_i log\frac{x_i}{\mu_i} \quad (8)$$

I. Kolmogorov Smirnov Divergence

This is a non-parametric test and calculates distance between empirical distribution function of the sample with cumulative distribution function of the reference. KS distance shown in equation (9).

$$D_{L_\infty}(x,y) = max_{i=1}^{N}|(X_n - Y_n)| \quad (9)$$

J. Cramer von Mises Divergence

This metric tests the goodness of fit for cumulative distribution function and to compare two empirical distributions. Harald Cramer and Richard Edler von Mises propose this metric [42] [43]. It is an alternative for Kolmogorov Smirnov test.

$$D_{L_1}(x,y) = \sum_{i=1}^{N}|(X_i - Y_i)| \quad (10)$$

K. Earth Mover's Distance

It is a measure of distance between two probability distributions. It works to find the minimum distance function. EMD is shown in equation (11).

$$D_{L_2}(x,y) = \sum_{i=1}^{N}\sqrt{}(X_i - Y_i)^2 \quad (11)$$

IV. METHODOLOGY

In this paper we presented image retrieval task based on HSV color histogram features and Haar wavelet based texture features for analyzing the performance of standard geometrical, statistical and cumulative distance measures consisting Euclidean, Manhattan, Chebyshev, Cosine angle, Chi-square, Kullback- Leibler, Jeffrey, Kolmogorov-Smirnov, Cramer von Mises and Earthmover's distances. Experiments are done in Matlab to analyze the effect of distance metrics on different types of images.

1. Image database is loaded into Matlab workspace.
2. Query Image is selected from the database.
3. Color feature extraction is done by computing color histograms for query and database images in HSV Color space.
4. Using Discrete Haar Wavelet transform (DWT), texture features extracted.
5. Geometrical distance measures Manhattan, Euclidean and Chebyshev and statistical distance metrics Chi-square, Kullback and Jeffrey and cumulative statistical measures Cramer, Kolmogorov and Earthmover's and Cosine angle distance were applied for feature similarity measurement.
6. Performance measures, precision (P), Mean average precision MAP and recall (R) evaluated for the retrieved images.
7. Comparative analysis is done for image retrieval based on effect of similarity measures for color and texture features.

V. EXPERIMENTS & RESULTS

We used COREL database for comparing our CBIR system consisting ten different groups of images including flowers, animals, beach scenes, food, people, busses and mountains. Sample query images are shown in Fig 1. We presented the comparative analysis of what distance metric performed well for a particular type of query for its color and texture features.

Fig.1. Sample Query Images

Through this experimentation it is observed that in HSV color space Chi-square, Manhattan, Cosine angle and Kullback Leibler performed well with more than 70% MAP as shown in table I. and recall among all other distance metrics. Image retrieval performed using HSV color histograms shown in Fig.2. Cumulative distance metrics outperformed well for texture feature similarity measurement as shown in table II. Retrieval of bus images using texture feature shown in fig.3.

Fig.2. Mountain images with HSV color features

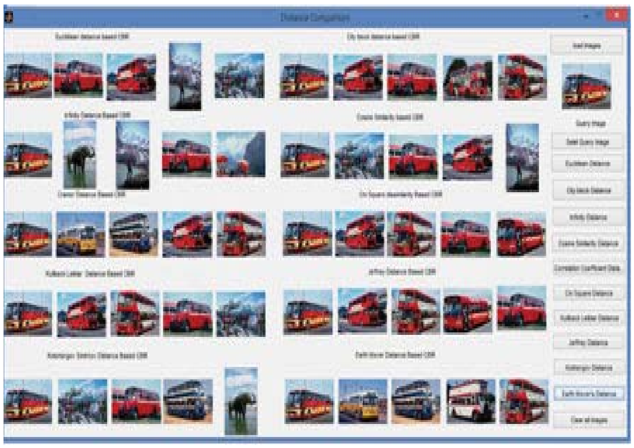

Fig.3. Busses images with DWT texture features

TABLE I
PRECISION AND MAP FOR HSV HISTOGRAM

Class	Distances									
	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
C1	0.8	0.8	0.8	0.9	1	0.9	0.8	0.6	0.7	0.7
C2	0.4	0.2	0.2	0.3	0.3	0.4	0.2	0.2	0.4	0.5
C3	0.7	0.6	0.5	0.6	0.7	0.7	0.7	0.3	0.4	0.5
C4	0.9	0.7	0.3	0.8	1	0.9	0.9	0.9	0.9	0.8
C5	1	1	1	1	1	1	1	1	1	1
C6	0.3	0.3	0.2	0.4	0.2	0.4	0.3	0.5	0.6	0.6
C7	0.7	0.4	0.5	0.9	0.7	0.7	0.7	0.5	0.5	0.4
C8	0.8	0.9	0.9	0.8	0.9	1	1	0.7	0.7	0.9
C9	0.3	0.3	0.3	0.3	0.4	0.4	0.3	0.8	0.5	0.6
C10	0.8	0.7	0.5	0.8	0.8	0.7	0.7	0.8	0.9	0.7
MAP	0.7	0.6	0.5	0.7	0.7	0.7	0.6	0.6	0.6	0.7

D1: MANHATTAN, D2: EUCLIDEAN, D3: CHEBYSHEV, D4: COSINE ANGLE, D5: CHI-SQUARE, D6: KULLBACK- LEIBLER, D7: JEFFREY, D8: KOLMOGOROV SMIRNOV, D9: CRAMER VON MISES AND D10: EARTHMOVER'S DISTANCE.
C1: PEOPLE, C2: BEACH, C3: BUILDINGS, C4: BUS, C5: DINOSAUR, C6: ELEPHANTS, C7: FLOWERS, C8: MOUNTAINS, C9: HORSES, C10: FOOD

TABLE II
PRECISION AND MAP FOR DWT TEXTURE

Class	Distances									
	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
C1	0.4	0.3	0.2	0.1	0.4	0	0.1	0.6	0.5	0.6
C2	0.8	0.7	0.5	0.3	0.5	0	0.5	0.3	0.3	0.3
C3	0.2	0.2	0.2	0.1	0.2	0.1	0.1	0.2	0.2	0.2
C4	0.5	0.5	0.3	0.1	0.4	0	0.3	0.5	0.5	0.5
C5	1	1	1	1	1	1	1	1	1	1
C6	0.8	0.7	0.4	0.2	0.5	0	0.5	0.6	0.7	0.6
C7	0.9	0.9	0.5	0.3	0.4	0.7	0.3	0.8	0.8	0.7
C8	0.8	0.7	0.5	0.1	0.6	0.6	0.5	0.5	0.5	0.5
C9	0.7	0.7	0.5	0.3	0.4	0.1	0.4	0.5	0.4	0.4
C10	0.7	0.6	0.5	0.2	0.6	0.3	0.5	0.7	0.7	0.7
MAP	0.7	0.6	0.4	0.2	0.5	0.3	0.4	0.6	0.6	0.6

D1: MANHATTAN, D2: EUCLIDEAN, D3: CHEBYSHEV, D4: COSINE ANGLE, D5: CHI-SQUARE, D6: KULLBACK- LEIBLER, D7: JEFFREY, D8: KOLMOGOROV SMIRNOV, D9: CRAMER VON MISES AND D10: EARTHMOVER'S DISTANCE.
C1: PEOPLE, C2: BEACH, C3: BUILDINGS, C4: BUS, C5: DINOSAUR, C6: ELEPHANTS, C7: FLOWERS, C8: MOUNTAINS, C9: HORSES, C10: FOOD

VI. CONCLUSIONS

In this paper, first we presented an overview of geometric and statistical distance metrics used in image retrieval applications and performed the comparative analysis of ten diversified distance measures including standard Euclidean, Manhattan, Infinity, Cosine Angle, Chi-square, Kullback Leibler, Kolmogorov Smirnov, Cramer von Mises and Earthmover's distance on color and texture features. Color features extracted by computing color histograms in HSV space and texture features with wavelet decompositions. Among geometrical distances Manhattan distance shown outstanding performance for color features and in statistical distance metrics Cosine Similarity, Chi-square, Kullback-Leibler, shown good MAP score for all the types of queries and among cumulative statistical distance metrics Cramer von Mises and Earthmover's distances were shown more than 70% MAP for texture feature similarity. Finally it is observed that geometrical and statistical distance measures - Manhattan, Cosine Angle, Chi-Square, shown good MAP in color based retrieval, cumulative distance measures - Cramer von Mises and Earthmover's distances outperformed well for texture based image retrieval. It is verified that in CBIR, retrieval not only depends on the image content representation but also on the similarity/ distance measures used.

REFERENCES

[1] Rubner, Yossi, Carlo Tomasi, and Leonidas J. Guibas. "The earth mover's distance as a metric for image retrieval." *International journal of computer vision* 40.2 (2000): 99-121.

[2] Stricker, Markus A., and Markus Orengo. "Similarity of color images."*IS&T/SPIE's Symposium on Electronic Imaging: Science & Technology*. International Society for Optics and Photonics, 1995.

[3] Swain, Michael J., and Dana H. Ballard. "Color indexing. *International journal of computer vision* 7.1 (1991): 11-32.

[4] Niblack, W., R. Barber, and W. Equitz. *The QBIC Project: Query Images by Content Using Color Texture*. and Shape. Research Report, IBM Research Division, RJ 9203 (81511), 1993.

[5] Siegel, Sidney. "Nonparametric statistics for the behavioral sciences." (1956).

[6] Breiman, Leo. "Statistics. With a view toward applications." *Boston: Houghton Mifflin Co., 1973* 1 (1973).

[7] Do, Minh N., and Martin Vetterli. "Wavelet-based texture retrieval using generalized Gaussian density and Kullback-Leibler distance." *Image Processing, IEEE Transactions on* 11.2 (2002): 146-158.

[8] Youngeun An, Muhammad Riaz, and Jongan Park. Cbir based on adaptive segmentation of hsv color space. In Computer Modelling and Simulation (UKSim), 2010 12th International Conference on, pages 248–251. IEEE, 2010.

[9] Zhiyong Zeng, Shengzhen Cai, and Shigang Liu. A novel image representation and learning method using SVM for region-based image retrieval. In Industrial Electronics and Applications (ICIEA), 2010 the 5th IEEE Conference on, pages 1622–1626. IEEE, 2010.

[10] Thomas Deselaers, Daniel Keysers, and Hermann Ney. Features for image retrieval: an experimental comparison. Information Retrieval, 11(2):77–107, 2008.

[11] Jing Huang, S Ravi Kumar, Mandar Mitra, Wei-Jing Zhu, and Ramin Zabih. Image indexing using color correlograms. In Computer Vision and Pattern Recognition, 1997. Proceedings., 1997 IEEE Computer Society Conference on, pages 762–768. IEEE, 1997.

[12] John R Smith and Shih-Fu Chang. Tools and techniques for color image retrieval. In Electronic Imaging: Science & Technology, pages 426–437. International Society for Optics and Photonics, 1996.

[13] Robert M Haralick, Karthikeyan Shanmugam, and Its' Hak Dinstein. Textural features for image classification. Systems, Man and Cybernetics, IEEE Transactions on, (6):610–621, 1973.

[14] Hideyuki Tamura, Shunji Mori, and Takashi Yamawaki. Textural features corresponding to visual perception. Systems, Man and Cybernetics, IEEE Transactions on, 8(6):460–473, 1978.

[15] John R Smith and Shih-Fu Chang. Transform features for texture classification and discrimination in large image databases. In Image Processing, 1994. Proceedings. ICIP-94., IEEE International Conference, volume 3, pages 407–411. IEEE, 1994.

[16] Minh N Do and Martin Vetterli. Wavelet-based texture retrieval using generalized gaussian density and kullback-leibler distance. Image Processing, IEEE Transactions on, 11(2):146–158, 2002.

[17] Wei-Ying Ma and BS Manjunath. A comparison of wavelet transform features for texture image annotation. In icip, page 2256. IEEE, 1995.

[18] Timo Ojala, Matti Pietik¨ainen, and Topi M¨aenp¨ a¨a. Gray scale and rotation invariant texture classification with local binary patterns. In Computer Vision-ECCV 2000, pages 404–420. Springer, 2000.

[19] Valtteri Takala, Timo Ahonen, and Matti Pietik¨ainen. Block-based methods for image retrieval using local binary patterns. In Image analysis, pages 882–891. Springer, 2005.

[20] Zhang, Dengsheng, and Guojun Lu. "Evaluation of similarity measurement for image retrieval." *Neural Networks and Signal Processing, 2003. Proceedings of the 2003 International Conference on*. Vol. 2. IEEE, 2003.

[21] Puzicha, Jan, et al. "Empirical evaluation of dissimilarity measures for color and texture." *Computer Vision, 1999. The Proceedings of the Seventh IEEE International Conference on*. Vol. 2. IEEE, 1999.

[22] Liu, Haiming, Dawei Song, Stefan Rüger, Rui Hu, and Victoria Uren. "Comparing dissimilarity measures for content-based image retrieval." In *Information Retrieval Technology*, pp. 44-50. Springer Berlin Heidelberg, 2008.

[23] Kokare, Manesh, B. N. Chatterji, and P. K. Biswas. "Comparison of similarity metrics for texture image retrieval." *TENCON 2003. Conference on Convergent Technologies for the Asia-Pacific Region*. Vol. 2. IEEE, 2003.

[24] Ling, Haibin, and Kazunori Okada. "An efficient earth mover's distance algorithm for robust histogram comparison." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 29.5 (2007): 840-853.

[25] Levina, Elizaveta, and Peter Bickel. "The earth mover's distance is the Mallows distance: some insights from statistics." *Computer Vision,*

2001. ICCV 2001. Proceedings. Eighth IEEE International Conference on. Vol. 2. IEEE, 2001.

[26] Long, Fuhui, Hongjiang Zhang, and David Dagan Feng. " Fundamentals of content-based image retrieval." *Multimedia Information Retrieval and Management*. Springer Berlin Heidelberg, 2003. 1-26.

[27] Vadivel, A. K. M. S. S. A., A. K. Majumdar, and Shamik Sural. "Performance comparison of distance metrics in content-based image retrieval applications." *International Conference on Information Technology (CIT), Bhubaneswar, India*. 2003.

[28] Kamel Belloulata, Lakhdar Belallouche, Amina Belalia, and Kidiyo Kpalma. Region based image retrieval using shape-adaptive dct. In Signal and Information Processing (ChinaSIP), 2014 IEEE China Summit & International Conference on, pages 470–474. IEEE, 2014.

[29] Qian, Gang, et al. "Similarity between Euclidean and cosine angle distance for nearest neighbor queries." *Proceedings of the 2004 ACM symposium on Applied computing*. ACM, 2004.

[30] Blumenthal, Leonard Mascot. *Theory and applications of distance geometry*. Vol. 347. Oxford, 1953.

[31] Reddy, P. V. N., and K. Satya Prasad. "Multiwavelet Based Texture Features for Content Based Image Retrieval." *Int J Comput Appl* 17.1 (2011): 39-44.

[32] Assent, Ira, Andrea Wenning, and Thomas Seidl. "Approximation techniques for indexing the earth mover's distance in multimedia databases." *Data Engineering, 2006. ICDE'06. Proceedings of the 22nd International Conference on*. IEEE, 2006.

[33] Tahoun, Mohamed A., Khaled A. Nagaty, and Taha I. El-Arief. "A robust content-based image retrieval system using multiple features representations." *Networking, Sensing and Control, 2005. Proceedings. 2005 IEEE*. IEEE, 2005.

[34] Zou, Bei-ji, and Marie Providence Umugwaneza. "Shape-based trademark retrieval using cosine distance method." *Intelligent Systems Design and Applications, 2008. ISDA'08. Eighth International Conference on*. Vol. 2. IEEE, 2008.

[35] Chen, Xin, Xiaohua Hu, and Xiajiong Shen. "Spatial weighting for bag-of-visual-words and its application in content-based image retrieval." *Advances in Knowledge Discovery and Data Mining*. Springer Berlin Heidelberg, 2009. 867-874.

[36] Kekre, H. B., Dr Sudeep D. Thepade, and Akshay Maloo. "Comprehensive performance comparison of Cosine, Walsh, Haar, Kekre, Sine, slant and Hartley transforms for CBIR with fractional coefficients of transformed image." *International Journal of Image Processing (IJIP)* 5.3 (2011): 336.

[37] Brunelli, Roberto, and Ornella Mich. "Histograms analysis for image retrieval." *Pattern Recognition* 34.8 (2001): 1625-1637.

[38] Kullback, Solomon, and Richard A. Leibler. "On information and sufficiency."*The annals of mathematical statistics* 22.1 (1951): 79-86.

[39] Hershey, John R., and Peder A. Olsen. "Approximating the Kullback Leibler divergence between Gaussian mixture models." *Acoustics, Speech and Signal Processing, 2007. ICASSP 2007. IEEE International Conference on*. Vol. 4. IEEE, 2007.

[40] Verdoolaege, Geert, et al. "Wavelet-based colour texture retrieval using the Kullback-Leibler divergence between bivariate generalized Gaussian models." *Image Processing (ICIP), 2009 16th IEEE International Conference on*. IEEE, 2009.

[41] C. L. Mallows (1972). "A note on asymptotic joint normality".Annals of Mathematical Statistics **43** (2): 508–515.

[42] Cramér, H. (1928). "On the Composition of Elementary Errors". *Scandinavian Actuarial Journal* **1928** (1): 13–74

[43] von Mises, R. E. (1928). *Wahrscheinlichkeit, Statistik und Wahrheit*. Julius Springer.W.-K. Chen, *Linear Networks and Systems* (Book style). Belmont, CA: Wadsworth, 1993, pp. 123–135.