Instance Space Analysis on Quantum Algorithms

Vivek Katial

20/07/2020

Introduction

About Me

- Vivek Katial (vkatial@student.unimelb.edu.au)
  - PhD Candidate in School of Mathematics and Statistics

Talk Structure

- Background
- Overview of Literature
- Current Progress
- What's next?

Main Research Question

- What instances characteristics of optimisation problems make them predisposed to being solved on a Quantum Computer?

Adiabatic Quantum Computing (AQC)

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- $$i\hbar\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$
- $|\psi(t)\rangle$ is our state vector, $H(t)$ is the time dependent Hamiltonian. A Hamiltonian of an n-qubit system $H(t)$ is given by $2^n \times 2^n$ matrix.

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- Loosely speaking, the adiabatic theorem tells us that if we vary from H_B to H_P *slowly enough* the system will remain in its ground state. This fact is a direct result of the Adiabatic Theorem [1].

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- Where $g(t)$ is the difference between the first two smallest eigenvalues of $H(t)$

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- The satisfiability problem, abbreviated SAT, is a classic example of an NP-complete problem [2]
- The basic SAT formulation can be described as follows: Given a boolean formula (AND $\wedge$, OR $\vee$, NOT $\neg$) over n variables $(z_1, z_2, ..., z_n)$. Can one set z_i's in a manner such that the Boolean formula is true?
- A clause is an expression which the variables must satisfy. For example $z_1 \wedge z_2 \implies z_1 = z_2 = 1$

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- Exact Cover implies that clauses are "exclusive" and in the form of $z_i + z_j + z_k = 1$

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- We then evolve our system with the following interpolation:
- $$H(t) = [1 - \lambda(t)]\, H_B + \lambda(t) H_P, \qquad \lambda(t) = \frac{t}{T}$$
- Finally, complete measurement and our solution is our final state $|\psi(t = T)\rangle$

Instance Space Analysis

Algorithm Selection

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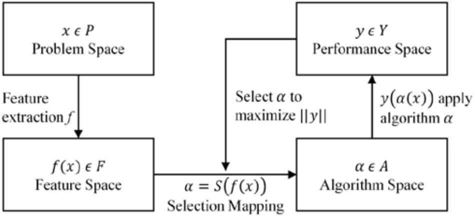

- However, what we are interested in is probing the strengths and weaknesses of AQC for different instances of SAT.

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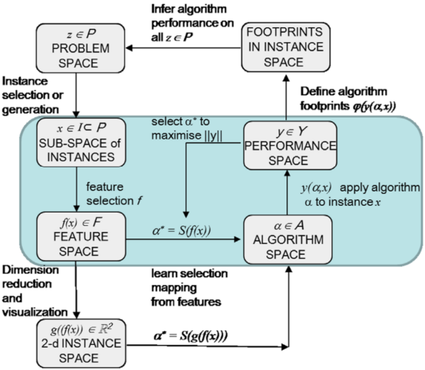

Current Literature

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- Their research indicated that as $N \to \infty$ the system is expected to lead to an exponentially small gap, and hence an exponential complexity.
- Farhi et al. [10] also investigated different evolution paths and their results suggested it is possible to overcome the exponentially small minimum gap by selecting random initial Hamiltonians

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- Hauke et al. [12] ran simulations of adiabatic quantum optimisation with $n = 16$. Their results indicated that large entanglement entropy has little significance for the success probability of the optimisation task.

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- Current approaches fail to investigate a suitable class of instances.

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  - Generalised USA Instances

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- The algorithm portfolio $\mathcal{A}$ includes 16 algorithm parameter configurations for the run time T and also the time step Δt. We also are using a single path function $\lambda(t)$
- The performance metric $y \in \mathcal{Y}$ is the probability of success for the algorithm.

Research Overview - Generating GUSA Instances

Algorithm 1: Generalised USA Instances

Fix number of bits to n;
$C = \{\}$;
$i = 0$;
while *While number of satisfying assignments* > 0 **do**
    $C_i =$ Three distinct bits randomly from a uniform distribution;
    $i = i + 1$;
    if *number of satisfying assignments* $= 1$ **then**
        return $(n, \mathbf{C})$;
    else if *number of satisfying assignments = 0* **then**
        restart;
    else if *number of satisfying assignments has decreased* **then**
        Add C_i into $\mathbf{C}$;
end
Result: $(n, \mathbf{C})$: n variables with a set of clauses $\mathbf{C}$

Research Overview - Generating RUSA Instances

Algorithm 2: Relaxed USA Instances

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    else if *number of satisfying assignments = 0* **then**
        restart;
    else
        Add C_i into $\mathbf{C}$;
    end
end
Result: $(n, \mathbf{C})$: n variables with a set of clauses $\mathbf{C}$

Research Overview - $f(x)$ Instance Characterstics

Feature Group	Feature
Problem Size	Number of variables: n Number of clauses: m Clause-to-Variable Ratio: $\frac{n}{m}$, $\frac{n}{m}^2$,$\frac{n}{m}^3$ Inverse Clause-to-Variable Ratio: $\frac{m}{n}$, $\frac{m}{n}^2$,$\frac{m}{n}^3$ Linearised Clause to Variable Ratio: $\|4.26 - \frac{n}{m}\|, \|4.26 - \frac{n}{m}\|^2, \|4.26 - \frac{n}{m}\|^3$
Variable Clause Graph	Variable Node Degree: mean, median, min, max Clause Node Degree : mean, median, min, max
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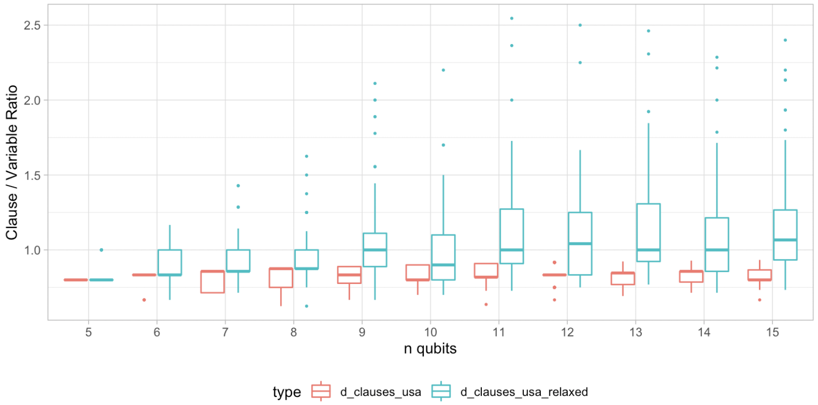
Table 1: Instance Features for 3SAT Exact Cover

Research Overview - Distribution of Features



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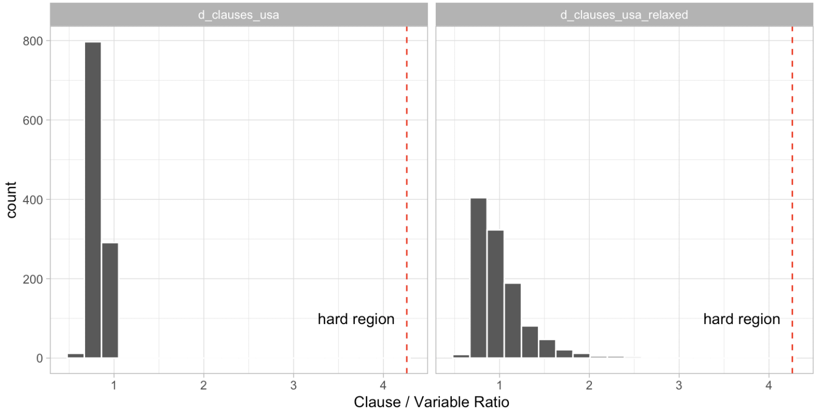

Results

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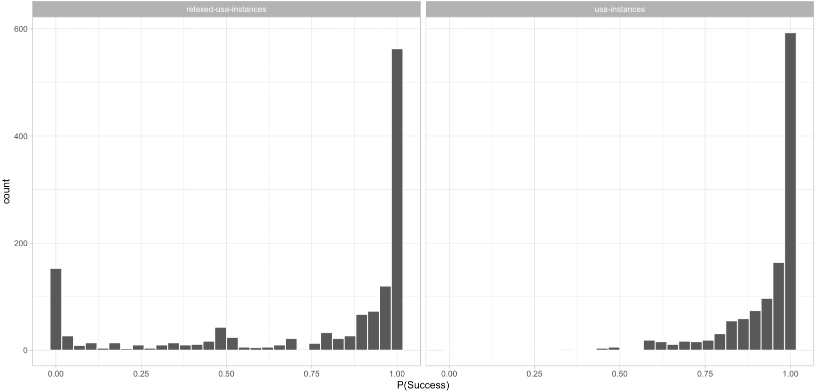

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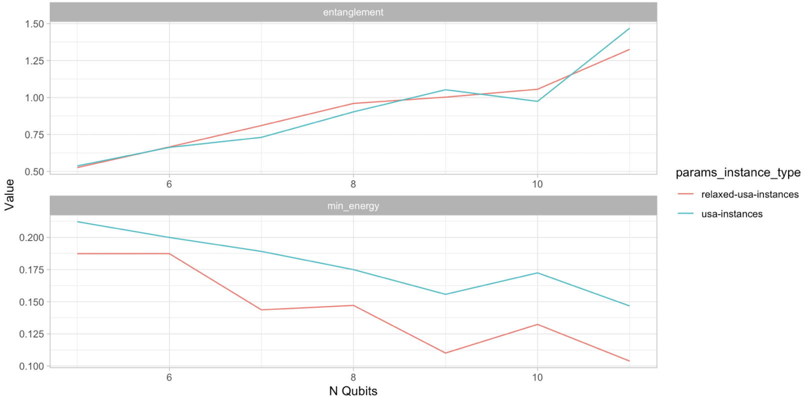

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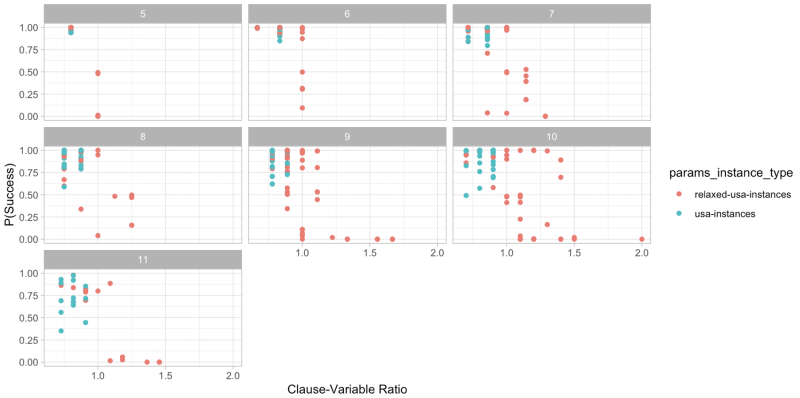
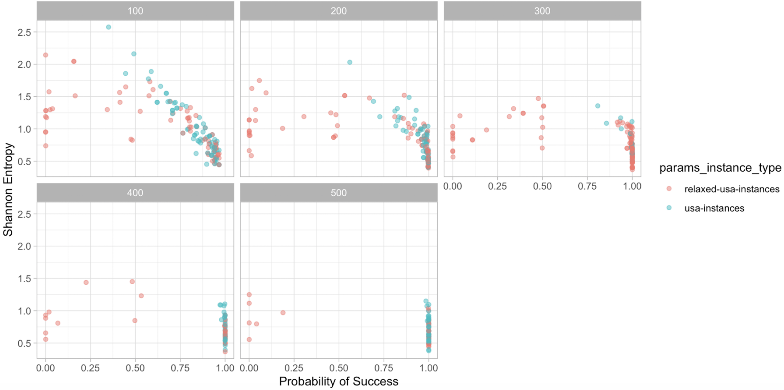
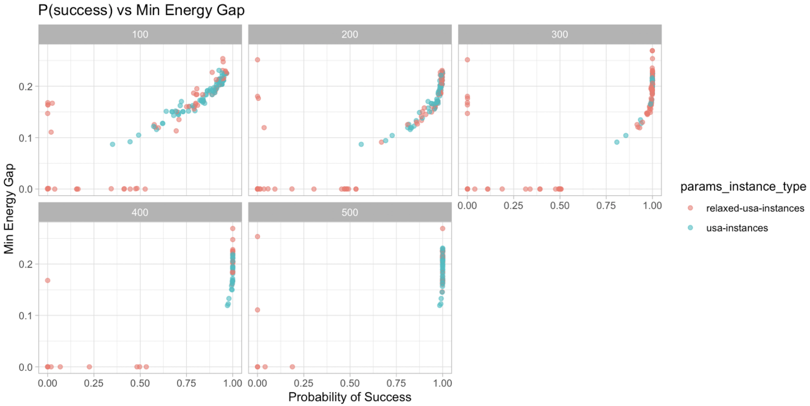


Research Overview - Entropy



Research Overview - Minimum Energy Gap



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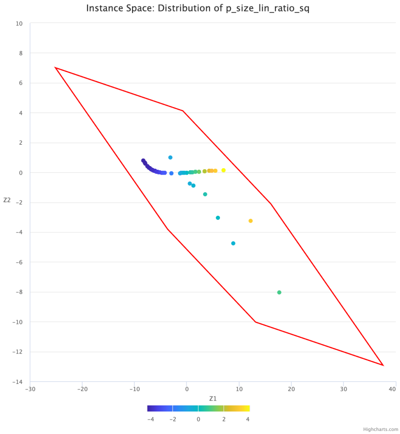
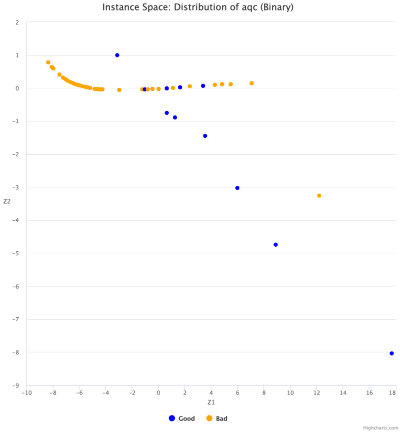


Research Overview - Results

- We set a threshold $\tau = 0.95$ for “good” or easy instances.



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8. Extend research to look at other QUBO problems

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