Optimisation via Adiabatic Quantum Computing

Vivek Katial

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Introduction

Main Research Question

- What instances characteristics of optimisation problems make them predisposed to being solved on a Quantum Computer?

Background

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- $$i\hbar \frac{d}{dt} |\psi(t)\rangle = H(t) |\psi t\rangle$$
- $|\psi(t)\rangle$ is our state vector, $H(t)$ is the time dependent Hamiltonian. A Hamiltonian of an n-qubit system $H(t)$ is given by $2^n \times 2^n$ matrix.

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- Loosely speaking, the adiabatic theorem tells us that if we vary from H_B to H_P *slowly enough* the system will remain in its ground state. This fact is a direct result of the Adiabatic Theorem.

Adiabatic Quantum Computing (AQC)

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- Where $g(t)$ is the difference between the first two smallest eigenvalues of $H(t)$

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- A clause is an expression which the variables must satisfy. For example $z_1 \wedge z_2 \implies z_1 = z_2 = 1$

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- The 3-SAT problem in particular is a problem where each clause is comprised of 3 literals.

Mapping 3SAT to AQC

- How do we construct H_B and H_P so that we can solve our optimisation problem?

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- Finally, complete measurement

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    1. Minimum Energy Gap g_{min}
    2. Maximum Entropy of Entanglement

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- Ultimately, we can build a prediction model which maps instance features of different 3SAT instances to their inherent "Quantumness"

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- This methodology can then be further extended to other optmisation problems that can be mapped to Quadratic Unconstrained Binary Optimisation (QUBO) problems.

Instance Space Methodology

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- MATILDA Extension

Instance Features for 3SAT

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- SAT-Zilla Characteristics

Quantum Computing Research into Hard SAT problems

- Different Paths (Farhi 2009)
- Hamiltonian Parameters (Choi)
- Volks group research on SAT

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3. Developed working environment, set up GitHub, scripts, organizational tools enabling an efficient working environment.

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6. Developed a working implementation of Adiabatic Quantum Computing on different instances of 3SAT.

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2. Implement scripts to generate **hard** instances of 3SAT with respect to *instance characteristics*.
3. Apply Instance Space Methodology from MATILDA to find decision boundaries for *Entanglement Entropy* and *Minimum Energy Gap*

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8. Extend research to look at other optimisation problems

About Me

- Vivek Katial (vkatial@student.unimelb.edu.au)
    - PhD Candidate (Optimisation on Quantum Computers)
- Check out the slides at https://tinyurl.com/vkatial-preconfirmation

References

References

1. Cook SA (1971) The complexity of theorem-proving procedures, In, *Proceedings of the third annual acm symposium on theory of computing*, ACM, 151–158.