Optimisation via Quantum Computing

Vivek Katial

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Introduction

About Me

- Vivek Katial (vkatial@student.unimelb.edu.au)
  - PhD Candidate (Optimisation on Quantum Computers)

Talk Structure

- Background
- Proposed Research
- Overview of Literature
- Current Progress
- What's next?

Main Research Question

- What instances characteristics of optimisation problems make them predisposed to being solved on a Quantum Computer?

Background

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- $|\psi(t)\rangle$ is our state vector, $H(t)$ is the time dependent Hamiltonian. A Hamiltonian of an n-qubit system $H(t)$ is given by $2^n \times 2^n$ matrix.

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- Loosely speaking, the adiabatic theorem tells us that if we vary from H_B to H_P *slowly enough* the system will remain in its ground state. This fact is a direct result of the Adiabatic Theorem [1].

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- Where $g(t)$ is the difference between the first two smallest eigenvalues of $H(t)$

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- A clause is an expression which the variables must satisfy. For example $z_1 \wedge z_2 \implies z_1 = z_2 = 1$

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- However, our **satisfying assignment** is only $\vec{z} = 1111$

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- We then evolve our system with the following interpolation:
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- Finally, complete measurement and our solution is our final state $|\psi(t = T)\rangle$

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    2. Entropy of Entanglement

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- Ultimately, we can build a prediction model which maps instance features of different 3SAT instances to their inherent "Quantumness"

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- This methodology can then be further extended to other optimisation problems that can be mapped to Quadratic Unconstrained Binary Optimisation (QUBO) problems.

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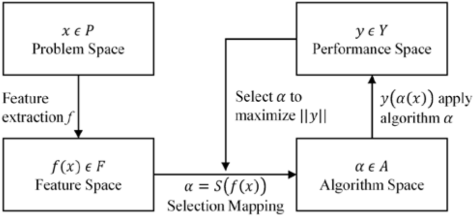

- However, what we are interested in is probing the strengths and weaknesses of AQC for different instances of SAT.

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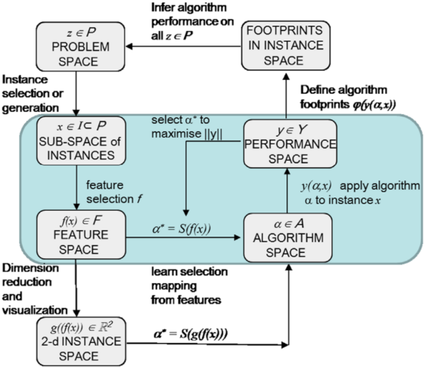

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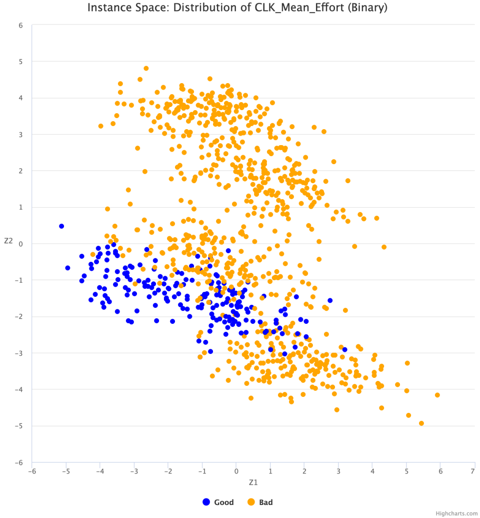

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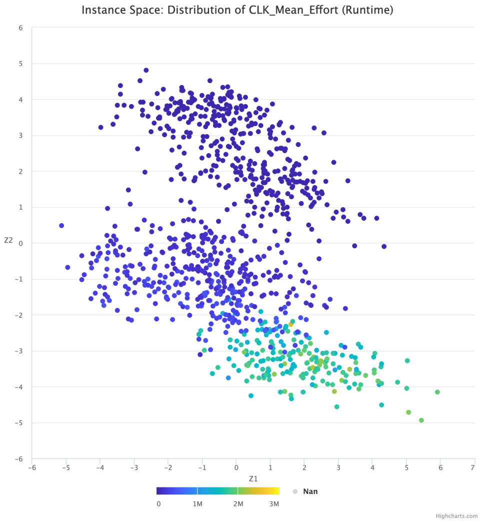

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2. Variable-Clause Graph Features
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5. Proximity to Horn Formula
6. Balance Features
7. DPLL Probing Features
8. Local Search Probing Features
9. LP-Based Features

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- Their research indicated that as $N \to \infty$ the system is expected to lead to an exponentially small gap, and hence an exponential complexity.
- Farhi et al. [13] also investigated different evolution paths and their results suggested it is possible to overcome the exponentially small minimum gap by selecting random initial Hamiltonians

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- However, their results also showed that for large N, the minimum gap scaling is exponential and can be associated with a *quantum phase transition* [14].
- Hauke et al. [15] ran simulations of adiabatic quantum optimisation with $n = 16$. Their results indicated that large entanglement entropy has little significance for the success probability of the optimisation task.

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- Gabor et al. [17] shows that the phase transition from 3SAT persists in some form (but possibly to a lesser extent) in AQC via **real experimental results** on D-WAVE.
- Most signficant for us is that none of this work approaches instances in a robust manner such as the MATILDA framework

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3. Developed working environment, set up GitHub, scripts, organizational tools enabling an efficient working environment.

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2. Implement scripts to generate **hard** instances of 3SAT with respect to *instance characteristics*.
3. Apply Instance Space Methodology from MATILDA to find decision boundaries for *Entanglement Entropy* and *Minimum Energy Gap*

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8. Extend research to look at other optimisation problems

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