Optimisation via Adiabatic Quantum Computing

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Main Research Question

 What instances characteristics of optimisation problems make them predisposed to being solved on a Quantum Computer?

Background

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• Also, $|\alpha|^2$ and $|\beta|^2$ correspond to the probability associated with measuring either $|0\rangle$ or $|1\rangle$,

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$$|+\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

• This state corresponds to a 50% chance of being in $|0\rangle$ or $|1\rangle$ as $\alpha^2=\beta^2=0.5$

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- To extend this framework to n qubits, we must represent that state by a 2^n vector. A shorthand notation has been developed where $|0\rangle^{\otimes 2}$ corresponds to the state $|00\rangle$

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• $|\psi(t)\rangle$ is our state vector, H(t) is the time dependent Hamiltonian. A Hamiltonian of an n-qubit system H(t) is given by $2^n \times 2^n$ matrix.

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 Loosely speaking, the adiabatic theorem tells us that if we vary from H_B to H_P slowly enough the system will remain in its ground state. This fact is a direct result of the Adiabatic Theorem.

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• Where g(t) is the difference between the first two smallest eigenvalues of H(t)

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- The basic SAT formulation can be described as follows: Given a boolean formula (AND \land , OR \lor , NOT \neg) over n variables $(z_1, z_2, ..., z_n)$. Can one set z_i 's in a manner such that the Boolean formula is true?

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- The basic SAT formulation can be described as follows: Given a boolean formula (AND \land , OR \lor , NOT \neg) over n variables $(z_1, z_2, ..., z_n)$. Can one set z_i 's in a manner such that the Boolean formula is true?
- A clause is an expression which the variables must satisfy. For example $z_1 \wedge z_2 \implies z_1 = z_2 = 1$

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- The 3-SAT problem in particular is a problem where each clause is comprised of 3 literals.

Mapping 3SAT to AQC

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Note: We have applied a normalisation here.

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Finally, complete measurement

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 - 2. Maximum Entropy of Entanglement

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- Ultimately, we can build a prediction model which maps instance features of different 3SAT instances to their inherent "Quantumness"

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- More ideas likely to come...

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- Apply Instance Space Methodology from MATILDA to find decision boundaries for Entanglement Entropy and Minimum Energy Gap

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- 8. Extend research to look at other optimisation problems

About Me

- Vivek Katial (vkatial@student.unimelb.edu.au)
 - PhD Candidate (Optimisation on Quantum Computers)
- Check out the slides at https://tinyurl.com/vkatial-preconfirmation

References

References

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