Optimisation via Adiabatic Quantum Computing

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Introduction

Main Research Question

- What instances characteristics of optimisation problems make them predisposed to being solved on a Quantum Computer?

Background

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- $|\psi(t)\rangle$ is our state vector, $H(t)$ is the time dependent Hamiltonian. A Hamiltonian of an n-qubit system $H(t)$ is given by $2^n \times 2^n$ matrix.

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- Loosely speaking, the adiabatic theorem tells us that if we vary from H_B to H_P *slowly enough* the system will remain in its ground state. This fact is a direct result of the Adiabatic Theorem.

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- Where $g(t)$ is the difference between the first two smallest eigenvalues of $H(t)$

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- The satisfiability problem, abbreviated SAT, is a classic example of an NP-complete problem [1]
- The basic SAT formulation can be described as follows: Given a boolean formula (AND $\wedge$, OR $\vee$, NOT $\neg$) over n variables $(z_1, z_2, ..., z_n)$. Can one set z_i's in a manner such that the Boolean formula is true?
- A clause is an expression which the variables must satisfy. For example $z_1 \wedge z_2 \implies z_1 = z_2 = 1$

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- The 3-SAT problem in particular is a problem where each clause is comprised of 3 literals.

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- Finally, complete measurement and our solution is our final state $|\psi(t = T)\rangle$

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    2. Maximum Entropy of Entanglement

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- Ultimately, we can build a prediction model which maps instance features of different 3SAT instances to their inherent "Quantumness"

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- Given that we have access to IBM's Quantum Computer, we can also use this approach on QAOA and produce results from a *universal quantum computer*.
- This methodology can then be further extended to other optmisation problems that can be mapped to Quadratic Unconstrained Binary Optimisation (QUBO) problems.

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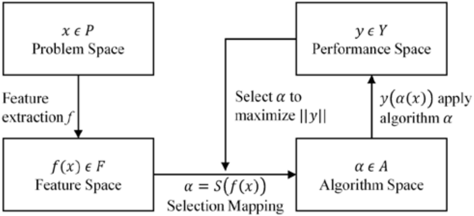

- However, what we are interested in is probing the strengths and weaknesses of AQC for different instances of SAT.

Instance Space Methodology

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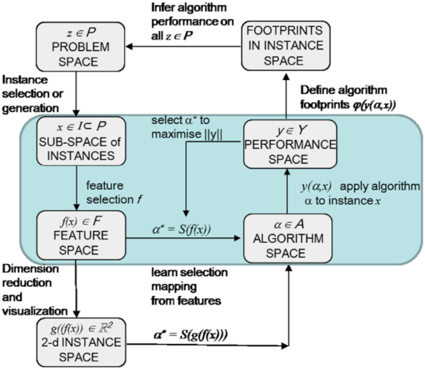

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5. Proximity to Horn Formula
6. Balance Features
7. DPLL Probing Features
8. Local Search Probing Features
9. LP-Based Features

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- Farhi et al. [13] also investigated different evolution paths and their results suggested it is possible to overcome the exponentially small minimum gap by selecting random initial Hamiltonians

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- Gabor et al. [15] shows that the phase transition from 3SAT persists in some form (but possibly to a lesser extent) in AQC via **real experimental results** on D-WAVE.
- Most signficant for us is that none of this work approaches instances in a robust manner such as the MATILDA framework

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3. Developed working environment, set up GitHub, scripts, organizational tools enabling an efficient working environment.

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5. Provisioned all infrastructure required for above on Melbourne Research Cloud and SPARTAN
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Next Steps (till Confirmation)

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2. Implement scripts to generate **hard** instances of 3SAT with respect to *instance characteristics*.
3. Apply Instance Space Methodology from MATILDA to find decision boundaries for *Entanglement Entropy* and *Minimum Energy Gap*

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8. Extend research to look at other optimisation problems

About Me

- Vivek Katial (vkatial@student.unimelb.edu.au)
  - PhD Candidate (Optimisation on Quantum Computers)
- Check out the slides at https://tinyurl.com/vkatial-preconfirmation

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