Optimisation via Adiabatic Quantum Computing

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Introduction

Main Research Question

- What instances characteristics of optimisation problems make them predisposed to being solved on a Quantum Computer?

Background

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- $|\psi(t)\rangle$ is our state vector, $H(t)$ is the time dependent Hamiltonian. A Hamiltonian of an n-qubit system $H(t)$ is given by $2^n \times 2^n$ matrix.

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- Loosely speaking, the adiabatic theorem tells us that if we vary from H_B to H_P *slowly enough* the system will remain in its ground state. This fact is a direct result of the Adiabatic Theorem.

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- Where $g(t)$ is the difference between the first two smallest eigenvalues of $H(t)$

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- A clause is an expression which the variables must satisfy. For example $z_1 \wedge z_2 \implies z_1 = z_2 = 1$

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- The 3-SAT problem in particular is a problem where each clause is comprised of 3 literals.

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- Farhi et al describes it very well.

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- Finally, complete measurement

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    2. Maximum Entropy of Entanglement

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- Ultimately, we can build a prediction model which maps instance features of different 3SAT instances to their inherent "Quantumness"

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- This methodology can then be further extended to other optmisation problems that can be mapped to Quadratic Unconstrained Binary Optimisation (QUBO) problems.

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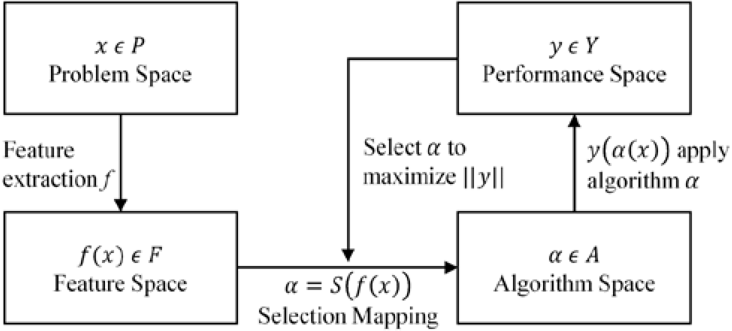
- 

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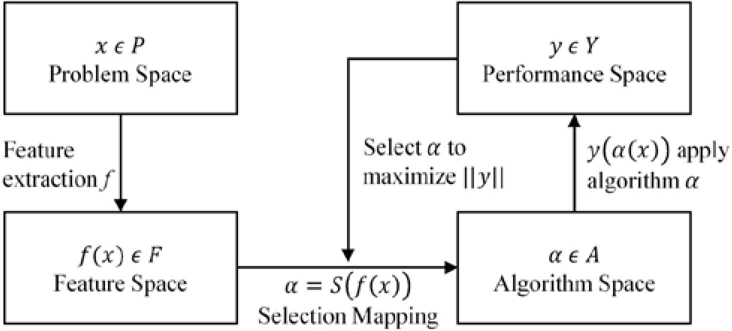
- 

- However, what we are interested in is probing the strengths and weaknesses of AQC for different instances of SAT.

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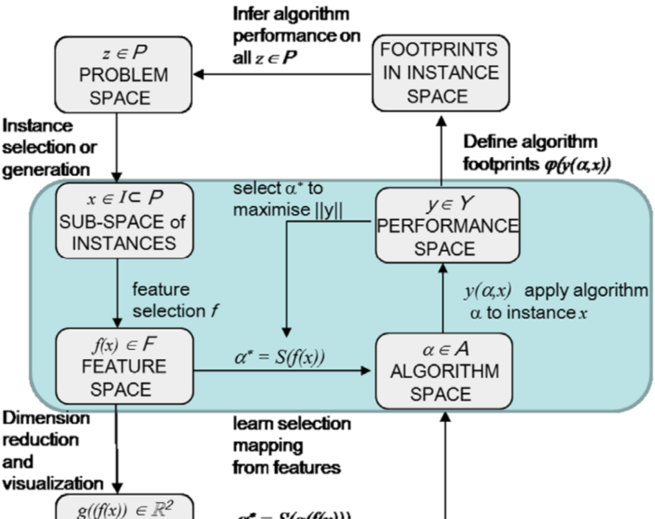

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1. Problem Size Features
2. Variable-Clause Graph Features
3. Variable Graph Features
4. Clause Graph Features
5. Proximity to Horn Formula
6. Balance Features
7. DPLL Probing Features
8. Local Search Probing Features
9. LP-Based Features

Quantum Computing Research into Hard SAT problems

- Different Paths (Farhi 2009)
- Hamiltonian Parameters (Choi)
- Volks group research on SAT

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3. Developed working environment, set up GitHub, scripts, organizational tools enabling an efficient working environment.

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6. Developed a working implementation of Adiabatic Quantum Computing on different instances of 3SAT.

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2. Implement scripts to generate **hard** instances of 3SAT with respect to *instance characteristics*.
3. Apply Instance Space Methodology from MATILDA to find decision boundaries for *Entanglement Entropy* and *Minimum Energy Gap*

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8. Extend research to look at other optimisation problems

About Me

- Vivek Katial (vkatial@student.unimelb.edu.au)
  - PhD Candidate (Optimisation on Quantum Computers)
- Check out the slides at https://tinyurl.com/vkatial-preconfirmation

References

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