Instance Space Analysis on Quantum Algorithms

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Introduction

Background

- Developing a framework to robustly evaluate quantum algorithms using Instance Space Analysis
- Explore the tools needed to develop Quantum Algorithms:
  - Adiabatic Quantum Algorithms (year 1)
  - Universal Gate Based Quantum Algorithms (year 2)
  - Explore various optimisation problems, initialisation techniques and instance classes

Agenda

- Explore the Optimisation Problems

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- Quantum Algorithms

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- Methodology

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- Research Plan

Optimisation Problems

- Traveling Salesperson Problem (TSP) and Vehicle Routing (VRP)
- MAXCUT
- 3SAT - Exact Cover

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- $$\min_{\pi} \sum_{i=1}^{n} d(c_{\pi_i}, c_{\pi_{i+1}})$$

TSP Example

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  - To convert this problem to a QUBO express constraints as penalty terms in the objective function.

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- This QUBO can then solved by a quantum algorithm

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Assign a qubit to each decision variable, make a the following substitution

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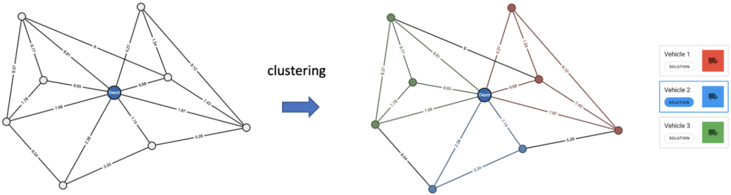

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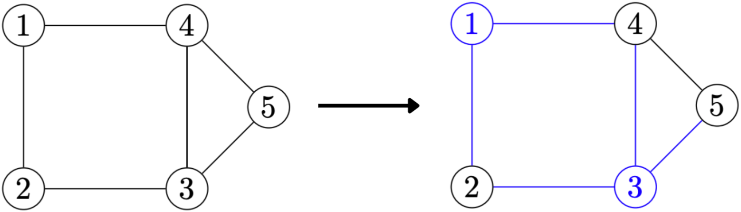
- This problem is NP-hard [1]

MAXCUT Formulation



Solution: Two partitions are $S = \{1, 2\}$ and $T = \{2, 4, 5\}$. The size of the cut is 5.

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- A clause is an expression which the variables must satisfy. For example $z_1 \wedge z_2 \implies z_1 = z_2 = 1$

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- Exact Cover implies that clauses are "exclusive" and in the form of $z_i + z_j + z_k = 1$

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- Finally, complete measurement and our solution is our final state $|\psi(t = T)\rangle$

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  - Potential for faster performance, as it can use advanced techniques such as error correction and quantum parallelism [5].

Quantum Algorithms

Quantum Algorithm	**Optimisation Problem
AQC	3SAT
QAOA	MAXCUT
	TSP
VQE	MAXCUT
	TSP
F-VQE	MAXCUT
	TSP

Types of instances investigated (MAXCUT only)

- Regular Graphs
- Unifrom Random Graphs
- Watts-Strotgatz Small World
- Nearly Complete Bipartite Graphs
- Power Law Tree Graphs
- Nearly Complete Bipartite Graphs
- Geometric Graphs

Classical Optimisers Explored

- SPSA, COBYLA, NELDER_MEAD
- 

Initialisation Techniques Explored (QAOA)

- Random Initialisation
- Perturb from previous layer
- Ramped up Initalisation
- Fourier Transform
- Trotterized Quantum Annelaing

Instance Space Analysis

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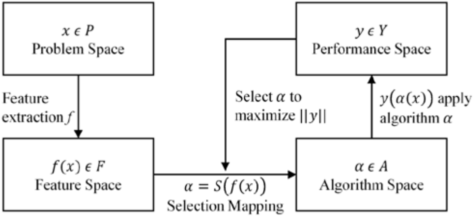

- However, what we are interested in is probing the strengths and weaknesses of AQC for different instances of SAT.

Instance Space Methodology

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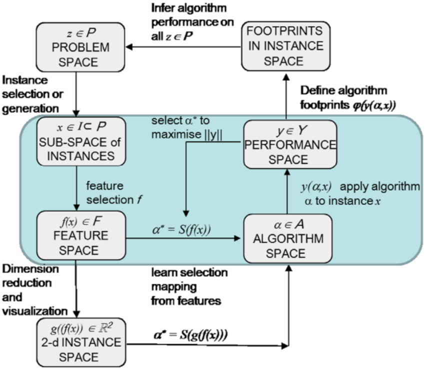

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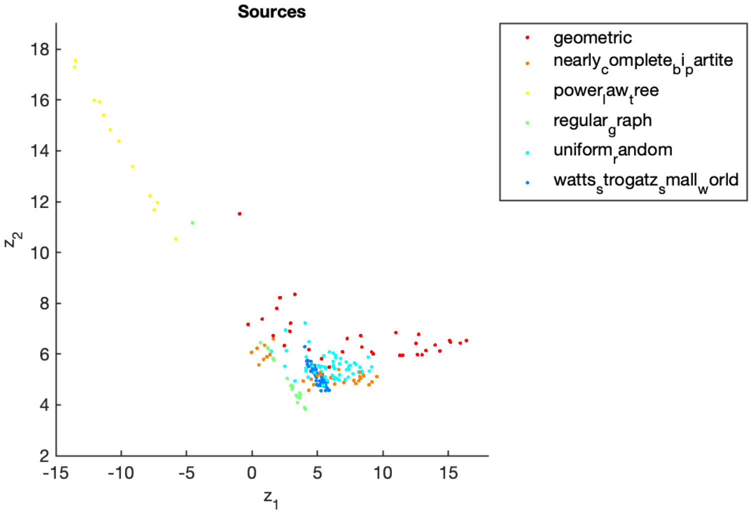
- The feature space $\mathcal{F}$ consists of 21 different instance features generated
- The algorithm portfolio $\mathcal{A}$ includes $F - VQE$, VQE and $QAOA$
- The performance metric $y \in \mathcal{Y}$ is the probability of success for the algorithm.

Graph Features

21 features were considered

Graph Based Features	**Boolean Features**
Average Distance	Bipartite
Clique Number	Connected
Algebraic Connectivity	Acyclic
Diameter	Eulerian
Edge Connectivity	Regularity
Vertex Connectivity	Planar
Maximum Node Degree	**Laplacian (Spectral Features)**
Minimum Node Degree	Largest Eigenvalue
Cardinality of minimal dominating set	Laplacian Second Largest Eigenvalue
Number of Components	Smallest Eigenvalue
Number of vertices	
Radius	

Sources


Sources
geometric
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power_law_tree
regular_graph
uniform_random
watts_strogatz_small_world
18
16
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8
6
4
2
-15
-10
-5
0
5
10
15
z_1
z_2

Feature Distribution

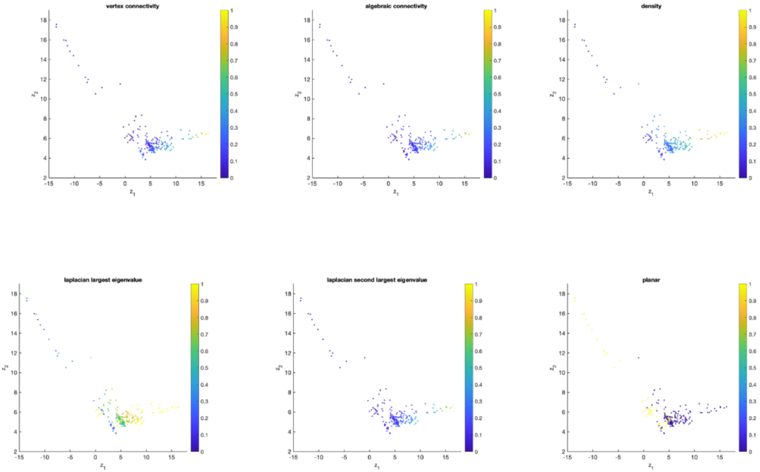

Figure 2: Features

Performance Distribution

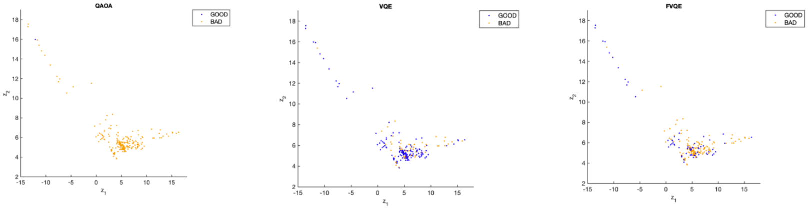

Figure 3: Performance

Thesis Outline

- Thesis Structure
  - Introduction
  - Solving Optimisation Problems using a Quantum Computer
  - Features and Algorithm Performance of Quantum Algorithms
  - Instance Space Analysis
  - Frameworks for Evaulating Quantum Algorithms / Discussion
  - Evaulation of Other Optimisation Algorithms
  - Conclusion

Discuss thesis outline

- Here is a link to the thesis outline

Next Steps

- Formalise findings into a paper (currently in progress)
- Solve all existing models on Quantum Hardware
- Investigate generalised Hamiltonian features

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