

Who is calling the shots...All or None?

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Abstract	This lesson discusses how patterns emerge in natural and engineered systems. These concepts explain the self-similar structures found in snowflakes, coastlines, river basins, lightning and are actively studied to explain stellar pulsation, epidemics, opinion formation, traffic jams, turbulence etc. These patterns are not evident at small scale or among the units constituting the larger unit but the interactions between the smaller units result in patterns at large scale. Patterns like fractals and synchronization are discussed with examples. Students can work on googlesheet to mathematically generate some of these patterns.
List of Standards Addressed Common Core, NC Essential Science, Next Gen, etc.	Some relevant standards PSc 1.2 Bio 2.1.4, 5.E.1, 5.L.2 EEen2.6, Math 4.NF.1, 5.NF.6
Appropriate Grade Levels	K-5, Grade 12
Group Size/# of students activities are designed for	5-6
Setting	Indoors
Approximate Time of Lesson	40 minutes
Resources Needed for	Some printouts of Norway coastline, Sketch pen/Pencil/pen (preferably different colors), ruler For Grade 12 only: Internet enabled device (Access to googledocs)
Resources Needed for Educators	Internet enabled device (Access to YouTube) 2 or more metronomes, skateboard or a simple board and two empty cola cans
Apps/Websites Needed	Youtube.com, googledocs
Lesson Activity	Introduction To engage the students, the lesson can be started by showing a video of metronome synchronization to the students. The video is available at: https://www.youtube.com/watch?v=5v5eBf2KwF8 After/during the video following questions can be asked: Q: What is the device shown in the video? A: The devices are called metronomes. They are similar to pendulum wall clocks. The needle/pendulum oscillates back and forth with a fixed time period and makes a sound every time it reaches some specific location. So the sound comes at regular intervals like 1 second. The user can adjust these regular intervals or frequency. The units are typically bpm, which is beats per minute i.e. how many sounds will be produced in 1 minute. In a wall clock the frequency of pendulums fixed at 1 Hz or it swings back and forth once per second. (However its bpm is 1/60 because it produces

sound only one time after 60 minutes or 1 hour.)

More information: <https://en.wikipedia.org/wiki/Metronome>

Q: What do we observe in the video?

A: The metronomes start at different times, so they produce sounds at different times. After some time, they all produce sound simultaneously. This is called **Synchronization**.

Background

Q: How do they synchronize?

A: The oscillations of metronomes create small vibrations in the board on which they are placed as well as in the surrounding air. These vibrations are then transmitted to other metronomes such that they are coupled with each other. This small coupling between them causes them to synchronize. This synchronization occurs only when there is some coupling. For example, if the surface on which they are placed is very hard then they will not synchronize.

Q: Is the synchronization the only pattern?

A: No. There are many other patterns possible. It all depends on the system, the type and strength of coupling between the units and the units themselves. They can be in phase synchronization or even anti synchronized.

Synchronization is a pattern in time. That is their behavior is similar with time. Other patterns can be observed in space where similar patterns repeat at different length scales as in fractals.

Fractals are the structures that repeat themselves at different length scales. Many natural entities are fractals like snowflakes, coastlines, and leaves. Natural fractals are self-similar for finite length scales only but mathematical fractals generated from simple math tricks can be self-similar at infinite number of length scales. Following simple mathematical rules at small scales generates these patterns.

Furthermore, the patterns can be spatiotemporal, i.e. the patterns repeat in space as well as time.

Step by Step Activity

This can be replicated in class with 2 or more metronomes. When metronomes are on a hard surface they don't synchronize but when they are placed on two coke cans then the surface can move freely due to force of metronomes and they are synchronized.

<https://www.youtube.com/watch?v=Aaxw4zbULMs>

Activity 1: Demonstration by Educator

1. Take 2 or more metronomes.
2. Place them about 1 meter away from each other on ground.
3. Give a little push to the needle of both and observe them for about 5 minutes. Do they synchronize?
4. Now place them on a small smooth skateboard and observe for 5 minutes to check if they synchronize or not. If they synchronize, check whether they remain synchronized up to five minutes. If the metronomes come to rest, give them a little push before after placing them on skateboard. Two empty cola cans and a small board measuring roughly 18 inch by 10 inch can replace the skateboard.

Placing magnets on the metronomes can change the interaction between the metronomes. We know that the force between the magnets is proportional to inverse of the square of the distance between them. So they will interact strongly if they are placed closely.

Here is a video that demonstrates the importance of interactions:

https://www.youtube.com/watch?v=gFnVmuU8_Lg

In the beginning there are 3 metronomes, which are synchronized, as the coupling is sufficiently strong. When the metronome in the middle is removed, the coupling becomes too weak and they desynchronize. When the middle metronome is placed between them, they synchronize again.

Activity 2:

This activity demonstrates how even complex oscillations can get synchronized.

Please visit the following link and follow the instructions given there.

https://docs.google.com/spreadsheets/d/1sIKVDtPoAWL_4tvPceqgOaiQ2wtm9waxFLq3l9pS1W4/edit?usp=sharing

Activity 3:

1. Divide the class in 5-10 groups.
2. Give a ruler, pen/pencil/sketch pen and one copy of the map of Norway included in the lesson plan to each group.
3. Now ask the first group to mark on the coastline of Norway by drawing lines measuring 5cm.
4. Similarly ask other groups to measure the distances with scales d , of length 3cm, 2cm, 1cm, 0.5 cm, 0.2cm respectively.
5. Ask each group how many line segments they needed to cover the coastline.
6. Calculate the total coastline of Norway measured by each group. This is equal to the line segments calculated in step 5 multiplied by the scale of that group. For example, if a group measures coastlines using 0.5cm as the scale and needs 50 line segments to cover the coastline then the coastline is $50 \times 0.5 = 25\text{cm}$.
7. Are all measurements of coastlines same? If not, Why?

Coastlines are generally fractal and the length of coastline depends on the scale chosen. For Norway, researchers have found out that the number of line segments as calculated in step 5, is proportional to scale d , as

$$N = C * d^{-1.5}$$

For a normal line of length C , the relation is

$$N = C * d^{-1}$$

This is interesting because the area covered by coastline is same.

Reflection/Assessment

Q: What do we learn?

A: Little cooperation among small units can yield huge benefits in form of synchronization of all the units. There is no central command structure but still patterns emerge at large scale.

Q: Where else do we observe synchronization?

A; Firefly synchronization <https://www.youtube.com/watch?v=ZGvtnE1Wy6U>

Even very complex oscillations like chaotic oscillations can get synchronized. In the attached gif files the complex oscillations of Rossler oscillator are shown. The blue dot gives the position of oscillator in x, y, z coordinates at a particular instant and the blue line is the location of the points visited previously. When 100 of these oscillators are coupled then they all get synchronized (shown in the other gif file). Here each red dot is the instantaneous location of a single oscillator (compare with the blue dot in the other gif file).

Q: What are the advantages of synchronization?

A: The coupling between the metronomes ensures that they all produce sound at the same time. It is very important in engineering applications where all units have to operate simultaneously. Importantly, it ensures same behavior even if there are some differences between the units. For example, the perturbations given to metronomes were not exactly identical but they were still able

	<p>to synchronize. Additionally there might be minor differences between them due to manufacturing tolerances but still they perform similarly. This phenomenon when the individual units have slightly different frequencies but they still oscillate with same frequency is called Entrainment. This is possible only when the differences are small.</p> <p>Similar behavior is observed during marching, dancing or singing where we are able to perform in unison without a prompter/conductor.</p> <p>Q: Are there any disadvantages of synchronization?</p> <p>A: Yes, as the metronomes started at different times and finally synchronized, this means that many or may be all of them did not maintain their frequency. So, their bpm changed!</p> <p>Synchronization is not always desirable. For example, synchronization can result in huge vibrations on the bridges. For example, it happened on London Millennium Bridge where due to small vibrations in the bridge, the steps of people walking on bridge got synchronized resulting in even bigger oscillations. For this reason, march past is not allowed on many bridges.</p>
Final Product/Assessment (e.g. quiz, blog, presentation, essay, etc.)	Updates and other materials: https://vivekkohar.github.io/lessonplans.html
Feedback Form for Teachers	http://goo.gl/tLw7sE

Background Reading for Teachers:

https://en.wikipedia.org/wiki/Patterns_in_nature

https://en.wikipedia.org/wiki/Injection_locking

https://en.wikipedia.org/wiki/Barnsley_fern

Images:



Image taken from Wikipedia: https://commons.wikimedia.org/wiki/File:Map_Norway_political-geo.png



Image from Wikipedia https://en.wikipedia.org/wiki/Fractal#/media/File:Fractal_Broccoli.jpg



Image from Wikipedia: https://en.wikipedia.org/wiki/Fractal#/media/File:Frost_patterns_2.jpg