

# Spectral scaling analysis of RR Lyrae stars in OGLE-IV Galactic bulge fields

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**Abstract** Recent studies of variable stars have uncovered characteristic nonlinear features in flux of these stars and indicate the presence of quasiperiodicity. A common technique to study quasiperiodic systems is spectral scaling analysis which relies on the fact that different dynamical behaviors can be identified on the basis of distribution of peaks in the periodogram. Here we apply the spectral scaling technique to the OGLE-IV photometry of the RR Lyrae stars in the Galactic bulge. We find that spectra of the fundamental mode (RRab) and first overtone RR Lyrae stars (RRc) scales differently and thus the spectral scaling can be used to distinguish between different RR Lyrae sub classes. Furthermore, goodness of fit for RRc stars with multiple modes is better than other stars. The scaling exponent for stars observed in high cadence is close to the values reported using Kepler photometry. This analysis can help us to reclassify the stars based on their dynamical characteristics.

## 1 Introduction

Recent high resolution photometry of RR Lyrae variable stars has shown that their dynamics is much more complex than previously thought and the Blazhko effect is not the only unsolved puzzle about their complex behavior [1, 2, 3, 4, 5, 6, 7]. Spectral analysis of the Kepler and Optical Gravitational Lensing Experiment (OGLE) photometry has shown that there are many additional excited modes. Blazhko effect and period doubling was observed in many RR Lyrae stars in Kepler field of view [8]. Molnar et. al [9] report similar findings in variable stars observed with K2 mis-

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sion [9, 10]. The precise origin and nature of these additional modulations and their possible connection to Blazhko effect is yet to be fully understood.

Switching of dominant pulsation mode has been reported in some RR Lyrae stars [11]. Bryant [12] has proposed that Blazhko effect is due to interaction of near resonant modes [12, 13]. Lindner et. al [14] proposed that the dynamics of RRc stars in Kepler field of view is strange nonchaotic implying that their light intensity has a fractal attractor. All these studies highlight that nonlinearity plays a significant role in the dynamics of these stars and is being incorporated in hydrodynamic models of stellar pulsation [15].

The classification of RR Lyrae stars in sub classes plays an important role in asteroseismology and numerous methods have been proposed for the same [16, 17, 18, 19, 20]. Our method of spectral scaling complements these approaches and can be used to distinguish other subtle characteristics which may be same or different in these subclasses.

## 2 Data and methods

We used the publicly available data from OGLE-IV project <sup>1</sup> [21]. Soszynski et al. [22] reported more than 38257 RR Lyrae stars in the Galactic bulge as observed during OGLE-IV<sup>2</sup>. These include about 27258 RRab, 10825 RRc stars and 174 RRd stars. Few of these stars have been reclassified. Most of the observations in OGLE IV were in I-band and the number of collected epochs varies from 80 to more than 8000 [22]. Hence we used the I-band data for our analysis.

### 2.1 Methodology

Accurate periodogram computation for astronomical time series is difficult as the data points are noisy and unevenly sampled. For more details, refer to [23] and references therein. Many phenomenon in nature exhibit scaling indicating an underlying relationship between the variables [e.g., see [24, 25, 14, 26, 27]]. Lindner et. al [26] reported that the spectra of RRc stars in Kepler field of view scales with an exponent close to  $-1.5$  and we perform similar analysis for this database.

We calculated the Lomb-Scargle periodogram using FNPEAKS <sup>3</sup> for frequencies between  $10^{-4}$  to 24 with a resolution of  $10^{-4}$ . Then we identified the peaks in the periodogram and sorted them in descending order of amplitude (SNR) of the peaks. We selected a fixed number of the peaks starting from the highest peak and plotted the peak height vs peak number. Then we fitted a linear polynomial on log-log scale

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<sup>1</sup> <http://ogle.astrouw.edu.pl>

<sup>2</sup> <ftp://ftp.astrouw.edu.pl/ogle4>

<sup>3</sup> <http://helas.astro.uni.wroc.pl>

and called it a "scaling exponent". We also calculated the 95 percent confidence intervals and adjusted  $r$ -square indicating the goodness of fit. We performed this analysis for all the 38000 stars for different number of peaks and SNR values. A good scaling does not necessarily imply a power law [28] and our primary goal in this manuscript is to classify the RR Lyrae variables based on scaling exponent. The scaling exponent and shape of folded light curves are closely related. As the shape of the curve distorts from sinusoidal, more and more frequency components are needed to generate the curve.

## 2.2 Generality

To evaluate the generality of our results we tested the results of a small sample of stars by calculating the periodogram with PERIOD04 and obtained similar results. We also verified the results by taking a small interval of frequencies from  $10^{-4}$  to 10 days.

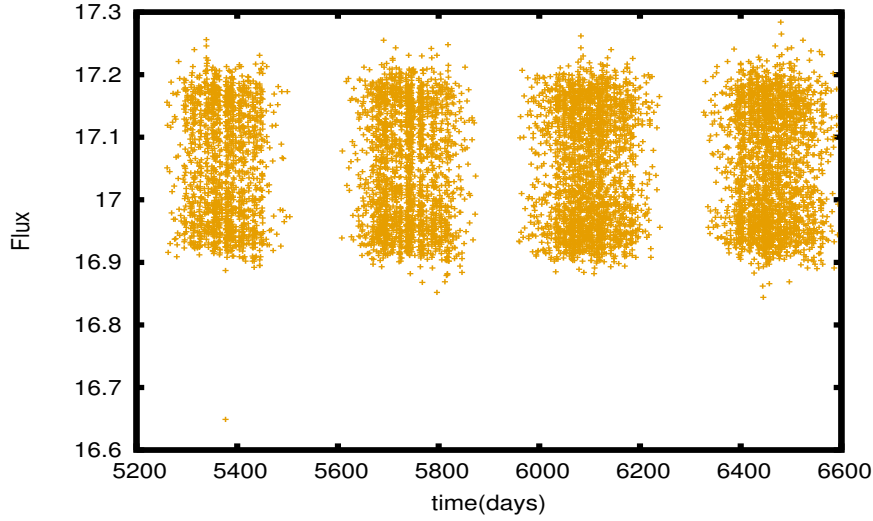
## 3 Results

Large number of data points is vital to the study of variable stars and extraction of nonlinear features. So we first identified stars with more than 8000 data points in the  $I - band$ . First we present the analysis of the stars for which high cadence data is available followed by stars with lesser number of data points. The number of data points affects the scaling exponent. The typical data for a star is shown in Fig 1. We observe that there are large gaps in the time series which correspond to active and inactive observation periods. This leads to strong aliases at one year period. For stars with few data points, these aliases dominate the periodogram.

The corresponding LS periodogram for the time series shown in Fig. 1 is shown in Fig. 2. We observe that periodogram is quite rough and there are many peaks corresponding to higher harmonics as well as other frequency combinations. Presence of multiple harmonics suggests non sinusoidal light curves. Other frequencies indicate aperiodic behavior. The lower panel plots the scaling of the periodogram. We observe that it scales with an exponent close to  $-1.5$  which is similar to what was observed in Kepler data for RRc stars.

Next we plot the periodogram of a RRab star in Fig. 3. Similar to RRc stars, there is significant power in other harmonics and daily aliases. The slope of line fitted on log-log scale shown in the bottom panel of Fig. 3 indicates a spectral scaling exponent close to  $-2$ . This indicates that power at other frequencies fall off rapidly in RRab stars as compared to RRc stars or the periodogram of RRc stars is more rough compared to RRab stars.

We repeated the same analysis for all the RR Lyrae stars and the scaling exponent for RRab and RRc stars with more than 8000 data points is plotted in Fig. 4. We

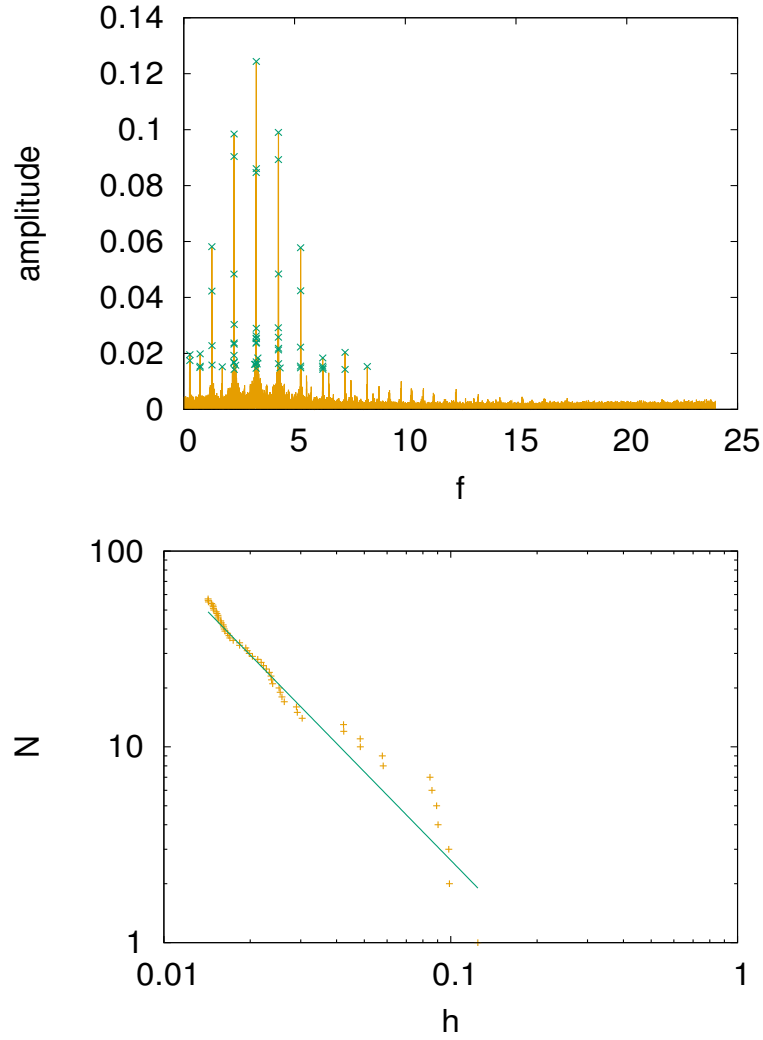


**Fig. 1** The time series of RRc star OGLE-BLG-RRLYR-05301. Note the seasonality in the availability of data points.

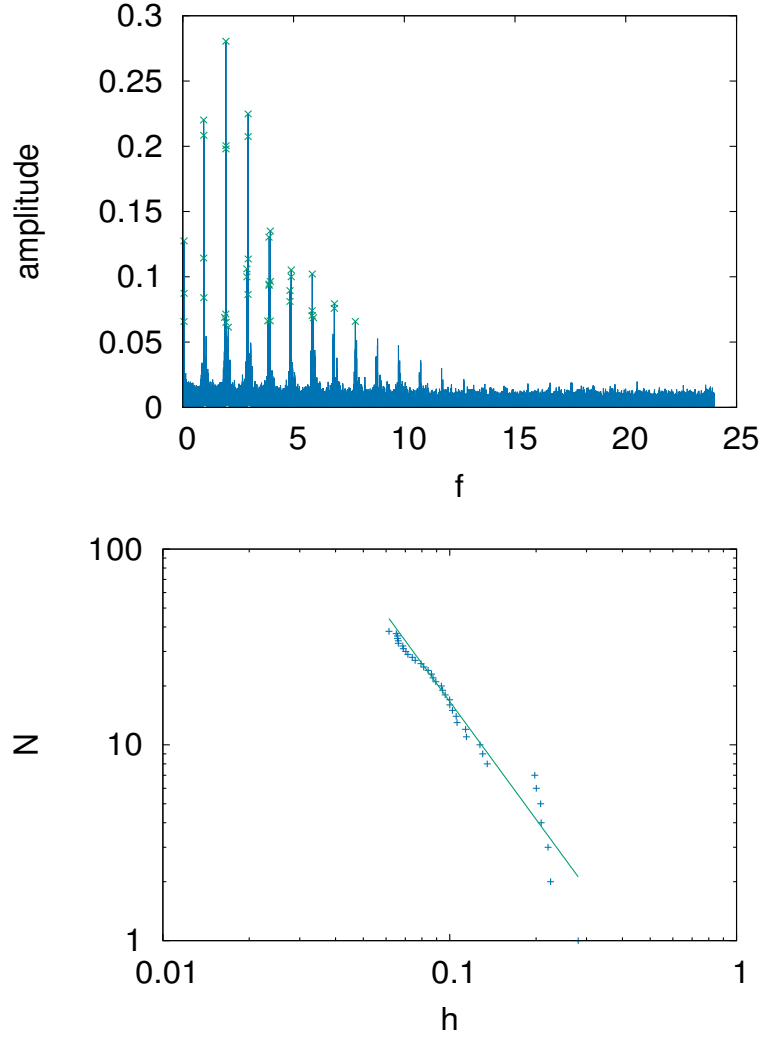
observe that RRab and RRc stars are clearly separated in terms of scaling exponent. Thus the phenomenon of roughness of periodogram of RRc stars is valid in general. Secondly the stars form a very prominent cluster which suggests that this type of scaling is a distinguishing feature of these stars. We observed similar scaling law for RRc stars using Kepler data. Both long cadence and short cadence data from Kepler gave an exponent of  $-1.5$ . As the number of data points in Kepler data was much higher, it indicates that for sufficiently large number of data points we can get a good estimate of the scaling exponent which saturates near  $-1.5$ .

Netzel et. al [29] reported that about 27 percent of high cadence stars in OGLE IV data of galactic bulge show a mode near period ratio of  $(0.60 - 0.64)$ . We then selected this particular sample of stars and in Fig. 4 we plot the scaling exponent for this subset of stars. We observe that the stars with additional modes form a very close cluster with a scaling exponent of  $-1.5$  consolidating our hypothesis that additional modes are reflective of singular spectra.

Next we studied the impact on scaling exponent when the number of data points is low. When the number of data points decreases, the SNR of peaks decreases and we get very few peaks which are above our  $SNR \geq 10$  threshold. For some reasonable scaling we need the number of peaks to be more than 10 and so we lowered our threshold to  $SNR \geq 4$  and the corresponding scaling exponents are shown in Fig. 5. The periodogram of OGLE data has strong 1 day aliases and these corrupt our analysis for lower SNR values. If we decrease the SNR threshold, there is higher likelihood that peaks due to noise and aliases will also be counted in our analysis. We observe that when we lower the SNR threshold then the noise and aliasing terms become dominant and both RRab and RRc have similar scaling exponent though

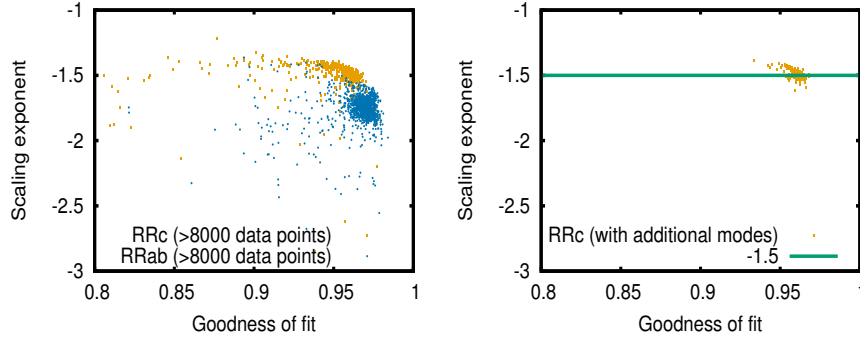


**Fig. 2** The LS periodogram of OGLE-BLG-RRLYR-05301. Peaks with  $SNR \geq 10$  have been marked. Day and night cycle introduce additional aliases. The scaling of the number of peaks above a certain threshold versus the threshold is shown on the figure below. A line of slope  $-1.5$  is shown as a guide to eye.

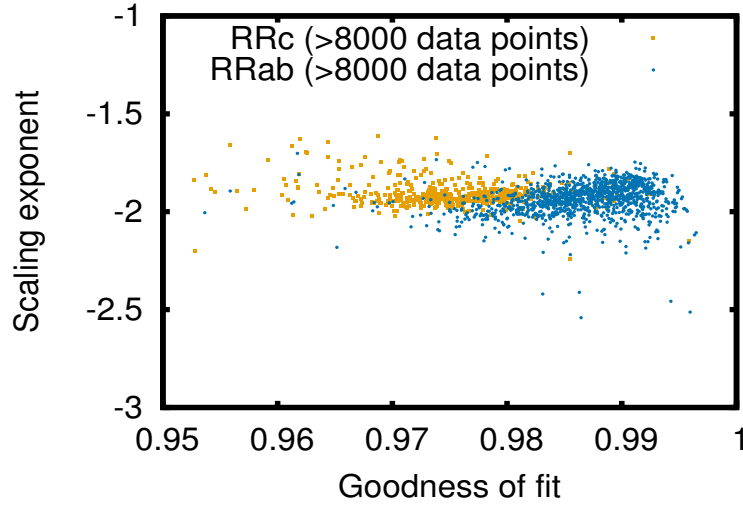


**Fig. 3** LS periodogram and scaling exponent of a RRab star OGLE-BLG-RRLYR-03599. Peaks with  $SNR \geq 10$  have been marked. A line of slope  $-2$  is fitted to the data.

the goodness of fit is better for RRab stars and the two clusters of stars are still distinguishable. Note that the Kepler database had more than 60000 data points in long cadence and even more for short cadence and the scaling exponents were still similar for peaks with  $SNR \geq 10$ . This indicates that the scaling exponents saturate for sufficiently large number of data points so that peaks due to noise and aliases do not contribute in our analysis.

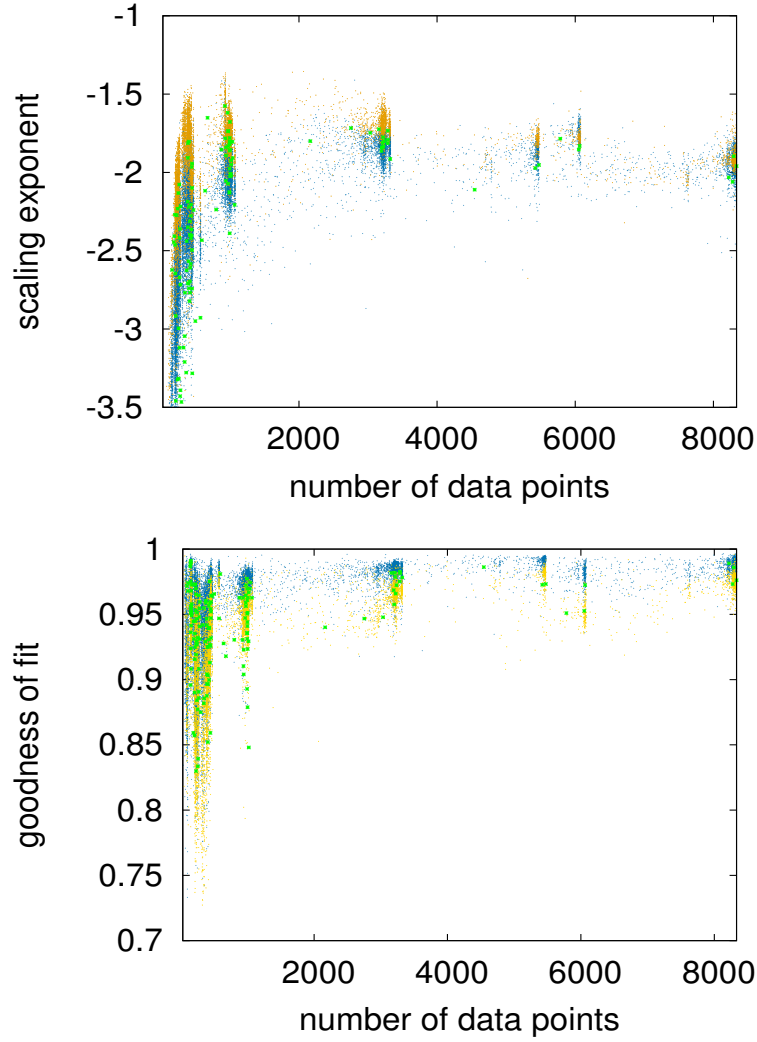


**Fig. 4** The scaling exponent of RRab and RRc stars plotted against goodness of fit for peaks with  $SNR \geq 10$  (left). The scaling exponent of RRc golden stars identified in [29] for peaks with  $SNR \geq 10$  (right).



**Fig. 5** The scaling exponent of RRab and RRc stars plotted against goodness of fit for peaks with  $SNR \geq 4$ .

Next we studied the variation of scaling exponent with number of data points for a fixed SNR. In Fig. 7 we plot the scaling exponent for all the RR Lyrae stars in OGLE-IV Galactic bulge database. We find that low SNR successfully distinguishes RRab and RRc stars for less number of data points but as the number of data points increases, the aliasing peaks become dominant and we need to select higher SNR values to obtain the scaling relation. The number of RRd stars is still small in the database and it is difficult to arrive at concrete generalization but the limited data suggest that they are distributed between the RRab and RRc stars.

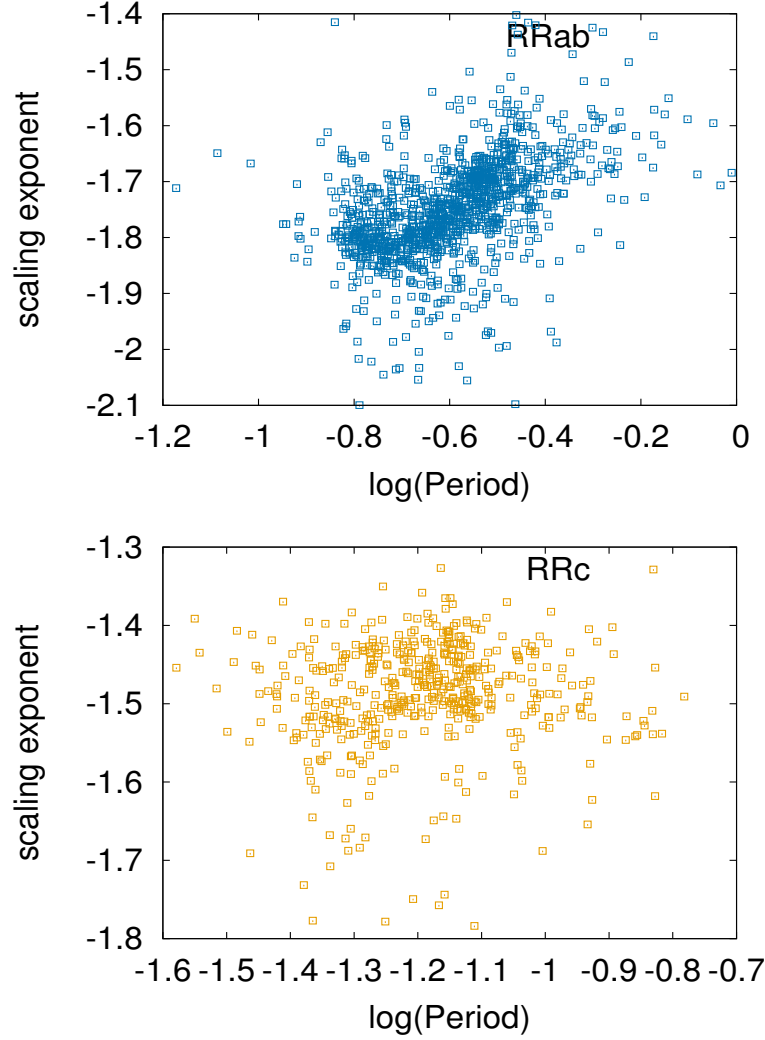


**Fig. 6** The scaling exponent and goodness of fit of RRab(blue), RRc (gold) and RRd (green) stars with number of data points for peaks with  $SNR \geq 4$ .

We also studied relation of scaling exponent with dominant period as shown in Fig. 7. We can infer that the periodogram of RRab stars becomes rougher as period increases. The scaling exponent for RRc stars doesn't exhibit any consistent pattern.

We further studied if the Fourier parameters are related with scaling exponent. The results are shown in Fig. 8

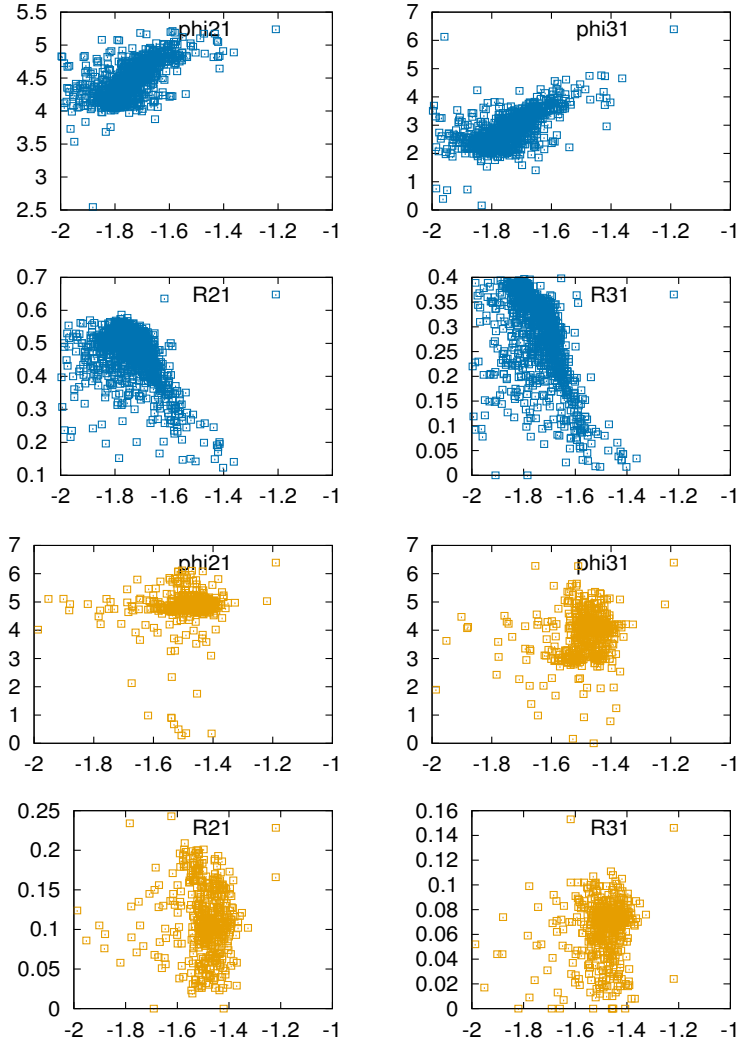




**Fig. 7** The scaling exponent plotted against the dominant period for RRab and RRc stars with more than 8000 data points and peaks with  $SNR \geq 10$

#### 4 Conclusion

We have studied the scaling of periodogram for RR Lyrae stars in OGLE-IV Galactic bulge database. We found that RRc and RRab stars scale differently indicating that the periodogram of RRc stars is rougher compared to RRab stars. This analysis helps us to reclassify the stars in various sub classes based on their dynamical characteristics. As evident from the analysis presented here, the dynamical charac-



**Fig. 8** The Fourier parameters (y axis) plotted against the scaling exponent (x axis) for RRab (blue) and RRC (gold) stars with more than 8000 data points and peaks with  $SNR \geq 10$

teristics closely overlap with the existing classification which is primarily based on the shape of folded light curves. The spectral scaling does depend on factors like the characteristic non sinusoidal oscillations of RRab and RRC, number of data points and the specific SNR threshold elected for cut off peaks but for sufficiently large number of data points the results are robust and similar to results obtained from Kepler database.

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