Nonlinear Dyn manuscript No.
(will be inserted by the editor)

Realizing Logic Gates with Time-Delayed Synthetic Genetic Networks

Amit Sharma · Vivek Kohar · Manish Dev Shrimali · Sudeshna Sinha

Received: date / Accepted: date

Abstract We demonstrate the realization of fundamental logic operations, as well as a memory element, with engineered delayed synthetic gene networks. Further, we investigate the effect of time delay in different kinds of processes, on the operational range of this biological logic gate. We show that this delay can either enhance or diminish logic behavior, depending on its functional form. Lastly, we show that the desired response to inputs can be induced, even in the absence of noise, by time delay alone.

Keywords Time delay · Logical stochastic resonance · synthetic gene network

1 Introduction

In recent times, a large body of research has focused on the cooperative interplay of noise and nonlinearity in dynamical systems, leading to phenomena such as stochastic resonance (SR) [1, 2]. In this direction, recently, Murali *et al* [3] investigated the response of a bistable system to an external signal, encoding logic inputs. It was demonstrated that in an optimal band of noise, the output of the system, determined by its state, was a consistent logical combination of the inputs. This phenomenon has been termed *logical stochastic resonance* (LSR), and it suggests a new way of implementing reconfigurable and reliable logic gates in the presence of noise [4, 5, 6]. The main feature of LSR is the capability of the nonlinear device to work optimally in a range of environmental noise; hence LSR is a practical and reasonable answer for computational devices wherein the noise-floor cannot be suppressed.

In another line of research, the important effects of time delay in dynamical systems has been brought to light. Time delay reflects the transmission times related to the transport of matter, energy, and information through the system [7]. Therefore time-delay systems can be regarded as simplified, but very useful, descriptions of systems involving a reaction chain or a transport process. Recently the combined effect of noise and time delay has been the subject of considerable interest in many complex systems such as genetic regulatory networks [8, 9, 10].

In this work we bring together the concept of logical stochastic resonance and time-delayed nonlinear dynamical systems, by investigating the possibility of obtaining robust logic response from a synthetic gene network (SGN). Such system plays an important role in the emerging field of synthetic biology [11], and can potentially lead to new forms of cellular control which could have important applications in functional genomic, nanotechnology, and gene and cell therapy [12]. Very recently, realization of morphable logic gates in an engineered gene network was reported [13, 14], while in Ref. [15] it was shown that repressilator with autoinducer is also able to show the fundamental logical behavior in synthetic gene regulatory network. The logical behavior in the electronic analog of a synthetic genetic network,

Amit Sharma[1,*], Vivek Kohar[2], Sudeshna Sinha[3]
Indian Institute of Science Education and Research (IISER) Mohali, Knowledge City, SAS Nagar, Sector 81, Manauli 140 306, Punjab, India
([*]Present address: FIRST, Collaborative Research Center for Innovative Mathematical Modelling, Institute of Industrial Science, The University of Tokyo, 4-6-1 Komaba Meguro-ku, Tokyo 153-8505, Japan.)
E-mail: [1]sharma.a28@gmail.com, [2] vivek.kohar@gmail.com, [3] sudeshna@iisermohali.ac.in

Manish Dev Shrimali
Department of Physics, Central University of Rajasthan, NH-8, Bandar Sindri, Ajmer 305 801, Rajasthan, India
E-mail: shrimali@curaj.ac.in

composed of two repressive and two constructive promoters is also demonstrated in Ref. [16].

Here we demonstrate the realization of logic operations with engineered *time-delayed* genetic networks, and show that this biological system can reliably emulate and morph into different logic gates in an optimal window of noise. Further, we investigate the effects of the interplay of noise, nonlinearity and time delay on the operational range of this biological logic gate under different scenarios.

2 Single-gene Network with noise and time delay

In this work, we use the quantitative model, describing the regulation of the P_{RM} operator region of λ phage [11]. This network is a DNA plasmid consisting of promoter region P_{RM} (contain three operator sites OR_{1-3}) and cI gene [11]. In basic biochemical reactions, the gene cI expresses repressors, which in turn dimerizes and binds to the DNA as transcription factor. The binding can be with one of the three operator sites sequentially. Positive feedback arises due to the fact that downstream transcription is enhanced by binding at operator sites (OR_2), while binding at OR_3 represses transcription, effectively turning off production and thereby constituting a negative feedback loop. By considering the total concentration of DNA promoter sites to be constant, the chemical reactions of the model under ambient noise, is given by suitable rescaling as [11, 13]

$$\begin{aligned}\dot{x} &= \frac{m(1+x^2+\alpha\sigma_1 x^4)}{1+x^2+\sigma_1 x^4+\sigma_1\sigma_2 x^6} - \gamma x + D\eta(t) \\ &= F(x) + D\eta(t) \qquad (1)\end{aligned}$$

where x is the concentration of the repressor. In Eq. (1) the first term on the right hand side represent the production of the repressor due to transcription. The even polynomials in the x occur due to dimerization and subsequent binding to the promoter region. Parameter m represents the number of plasmids per cell. Parameter γ determines the steady state concentration of repressor and is directly proportional to protein degradation rate and can be a tunable parameter in construction of artificial network. For the operator region of λ phage, we have to fix the other parameters at $\sigma_1 = 2, \sigma_2 = 0.08$ and $\alpha = 11$ [11, 13].

The nonlinearity of Eq. (1) leads to a bistable regime in steady state concentration of repressor with parameter γ and its potential U_{eff} shown in Fig. 1(a). The bistability in the system arises due to competition between production of x along with dimerization and its degradation. The final concentration of x is determined by the initial concentration of repressor.

In above equation, $\eta(t)$ is an additive zero mean Gaussian noise with unit variance and D denote the noise intensity. Such an additive noise source alters the background repressor production and represents the inherent stochasticity of biochemical processes such as transcription and translation, and the fluctuations in the concentration of a regulatory protein can cascade through a genetic network [11, 17].

The chemical reactions describing the network are naturally divided into two categories: fast (dimerization, protein DNA binding/unbinding) and slow (transcription, translation and degradation). It is well known that the transcription and translation processes involve compound multistage reactions and the sequential assembly of long molecules [11]. Therefore time delay is very relevant in this system, and should be explicitly considered in this model. This time delay could appear at any level of the gene regulatory network, like in the production of repressors due to transcription in synthesis or in the degradation process of chemicals.

Case I: Time delay in degradation process

First we consider a local time delay in the protein degradation process. Such a time delay τ in the degradation process can affect repressor concentration x. Therefore Eq. (1) becomes

$$\dot{x} = \frac{m(1+x^2+\alpha\sigma_1 x^4)}{1+x^2+\sigma_1 x^4+\sigma_1\sigma_2 x^6} - \gamma x_\tau + D\eta(t) \qquad (2)$$

where the x_τ (time delay) is previous to the time when dx/dt is computed. Since γx_τ is dependent linearly on the repressors concentration, for simplicity, we call this form as linear time delay. The potential function of the system under time delay in Eq. (2) is obtained numerically (using a small delay approximation of probability density in [18]) by integrating the right hand side function of equation for different value of γ. It is evident that the presence of small delay decreases the depth of the potential well (see Fig. 1(b)). Time delay in degradation process strongly affects the characteristic of the system and after a critical time delay it destroy the bistability in the system.

Case II: Time delay in the synthesis process

Next we consider a time delay in the synthesis process, where τ reflects the time taken for the production of repressor due to transcription. Therefore Eq.(1) becomes

$$\dot{x} = \frac{m(1+x_\tau^2+\alpha\sigma_1 x_\tau^4)}{1+x_\tau^2+\sigma_1 x_\tau^4+\sigma_1\sigma_2 x_\tau^6} - \gamma x(t) + D\eta(t) \qquad (3)$$

Now x_τ appears in a nonlinear term (first term on the right hand side), for simplicity, we regard this case as a

Table 1 Relationship between the two inputs and the output of the fundamental OR and AND logic, and the complementary NOR and NAND logic operations. All possible logical circuits can be constructed by combining the NOR (or NAND) gates.

Input set (I_1, I_2)	OR	AND	NOR	NAND
(0,0)	0	0	1	1
(0,1)/(1,0)	1	0	0	1
(1,1)	1	1	0	0

nonlinear time delay case.

Case III: Global delay
When there is a time delay in both processes (degradation and synthesis) in the gene regulatory network, then the governing equation can be considered as:

$$\dot{x} = \frac{m(1 + x_\tau^2 + \alpha\sigma_1 x_\tau^4)}{1 + x_\tau^2 + \sigma_1 x_\tau^4 + \sigma_1\sigma_2 x_\tau^6} - \gamma x_\tau + D\eta(t) \quad (4)$$

This case combines the effects of the above two cases.

3 Logical Stochastic Resonance (LSR)

We now explicitly demonstrate LSR in the noisy time delayed synthetic gene network. Namely, we consider the system:

$$\dot{x} = F(x_\tau) + I + D\eta(t) \quad (5)$$

where $F(x)$ is a synthetic gene network function given in Eq. (1), which has two distinct stable potential wells [11]. I is the low amplitude input signal. A logical input-output can be obtained by associating the input $I = I_1 + I_2$, where I_1 and I_2 are two (aperiodic) trains of square pulses encoding the two logic inputs. Without loss of generality, we consider the input strength value 0.5 when the logic input is 1, and value 0 when the logic input is 0. The logic inputs being 0 or 1, produce four sets of binary inputs (I_1, I_2): (0,0), (0,1), (1,0), and (1,1). So the four distinct input conditions give rise to three distinct values of I. Hence, the input signal $I = I_1 + I_2$, is a three-level aperiodic wave form. We solve Eq.(5) by employing the Euler–Maruyama numerical method for integrating stochastic delay differential equations with $\Delta t = 0.005$ for 1×10^5 time steps. We held the value of one input set constant over 1000 time steps, and simulated the system over a sequence of 100 such sets. The logical output of the system is determined by the value of x. We prescribe a threshold value x^* for output determination: a logical 1 is obtained if $x(t) \geq x^*$ and a logical 0 is obtained if $x(t) < x^*$.

In the simulations below we take the output determination threshold x^* to be 0.75 to obtain AND/OR/SR-Latch logic, i.e. we interpret the state $x < 0.75$ as logic output 0 and state $x \geq 0.75$ as logic output 1. Alternately, interpreting the state $x \geq 0.75$ as logical 0 and the state $x < 0.75$ as logical output 1 yields the complementary logic output, namely NAND/NOR. In a completely analogous way, we can obtain OR/NOR gate by varying the parameter γ in stochastic SGN system.

4 Results

To understand the consequences of time delay on LSR, we study the response of time-delayed synthetic gene network (SGN) system under varying noise intensities, in the three different scenarios given by Eqs. (2-4). Note that when there is no time delay in the SGN ($\tau = 0$), it has been demonstrated that there is an optimal window for reliably obtaining logical outputs NAND/AND and NOR/OR [13], namely one observes clearly defined LSR in such systems. *So here we will determine how robust the phenomenon of LSR is under time delay, and determine the change in the optimal range of logic operations under increasing delay.*

Specifically, we study the qualitative behavior of the time-delayed SGN under different noise levels by examining the time series (as displayed in Fig. 2). Further, we quantify the consistency (or reliability) of obtaining a given logic output by calculating the probability of successfully obtaining the desired output for different input sets, averaged over a 100 different runs consisting of permuted input sets. If the logic output, as obtained from $x(t)$, matches the logic output in the truth table (after leaving transience) for *all* the input sets in the run, it is considered a success; otherwise it is considered a failure. The ratio of successful runs to the total number of runs is denoted as $P(logic)$. When $P(logic)$ is ~ 1 the logic operation is obtained completely reliably, namely the system always yields the correct output. This measure of the success of the logic operation is very stringent, and it checks if any random combination of logic inputs, streaming in any random sequence, yields the correct result.

First consider the time delay in the degradation term as represented by Eq. (2) with noise intensity $D = 0.12$ and $\gamma = 6.1$. We observe that the response of the system is similar to the output of the SGN without time delay τ, while it fails to do so for very large time-delay. This is evident in the results displayed in Fig. 2.

Fig. 3 shows the probability $P(logic)$ of obtaining the different logical output (NAND and NOR) with different time delays τ. It is clear from Fig. 3 (top row) that time delay in the degradation term shrinks the range for obtaining correct logic gates. Further, it shifts the optimal noise window needed for successful logic operations towards the lower end. In this scenario, while the time delay reduces the operational range of the logic gate, it improves the performance of the system at low noise levels vis-a-vis the SGN without delay. For sufficiently larger delay, the probability of obtaining a reliable NAND/NOR gate vanishes. So the effect of large delay is analogous to very high noise in the system, and it leads to random hopping between wells giving inconsistent output.

Next we consider the time-delayed SGN given by Eq. (3) which represents a delay in the production of the repressor due to transcription. We fixed the noise intensity at $D = 0.3$ and $\gamma = 6.1$, and studied the response of the system with respect to varying time delays τ. The results are displayed in Fig. 4, and interestingly it shows a trend that is very different from arising in Eq. (2). Now, at small delay, the system is unable to yield the correct logical output, while higher delays enhanced the performance of the system, as evident from Fig. 4. So in this scenario the *time delay aids the system to obtain the desired logical output*.

We also show $P(logic)$ calculated for the time-delayed SGN in Eq. (3) for different delays shown in Fig 3 (middle row). It is clear again from this quantitative measure, that time delay enhanced the probability of obtaining the correct output and helped the system perform better at higher noise levels in this case.

Furthermore, we study the effect of time delay in both terms of the dynamical equation of the SGN, as given in Eq. (4). The results are displayed in Fig. 3 (bottom row), and it is evident that the presence of time delay shrinks the optimal range (as in the case of Eq. (2)) and also extends the range towards higher noise levels (as in the case of Eq. (3)). However, since the effect of delay in the first term opposes the effect of delay in the second, the shrinking of the optimal LSR regime is less pronounced in the case of delay in both terms vis-a-vis the case of delay in the second term alone. This is clear from the comparison of Fig. 3 bottom and top row.

Implementation of a Memory element:
Further, we observed that within some parameter range, time-delayed SGN can also operate as a Latch, which is a fundamental building block of a computing machine [19]. A latch system has two stable states and is used to store information. The system can be made to change state by signals applied to one or more control inputs. Specifically, either the system maintains its previous state or it can be set to either 1 or 0. The truth table of SGN latch implemented in our simulations is given in Table 2. If both the inputs are 0, the output of the system goes to lower state and if both the inputs are

Table 2 Relationship between the two inputs and the output of SGN latch and Set-Reset latch.

I_1	I_2	SGN Latch	SR Latch
0	0	0	No change
0	1	No change	0
1	0	No change	1
1	1	1	Restricted Set

1, the output corresponds to high state. When the two inputs are different, i.e. one is 0 and other 1, the system maintains its previous state or in other words, it acts like a memory element. Fig. 3 marks (light blue) the points in the $\gamma - D$ parameter space where the system mirrors a latch output (in accordance with Table. 2 with complete consistency, namely the probability of producing a latch operation is ~ 1 (after transience)). *Significantly, unlike traditional latches built by concatenating two logic elements, here we need only one SGN to implement the latch truth table.* Moreover, by applying a NOT operation to the second input we can obtain the conventional Set-Reset latch functionality (see table 2) from SGN. Note that if the restricted input set (1,1) is applied in this latch, the output will maintain its previous value (no change). The NOT operation can be done by encoding the input in reverse, i.e. we encode the input as 0.5 when the logic input is 0 and 0 when the logic input is 1. So one can say that a single SGN can function as a $S\overline{R}$ latch.

Noise-Free LSR:
From the above results, it is also evident that noise is not a necessary condition to obtain a consistent logic response here [20]. *In the presence of time-delay in the dynamics, it is possible to have phenomena completely analogous to LSR, without any noise.* So the role of noise, which induces the desired hopping in response to inputs in LSR, is played by time delay in the system. That is, the system needs only sufficient delay τ, in order to change its state to the desired well.

We show the response of the system under the time delay τ, without noise in Figs. 5. We also quantify the consistency of obtaining a given logic output (P(NAND/ AND), P(NOR/OR), P(Latch)) for input strength values 0.5 and 0.75 in $\gamma - \tau$ parameter space (see Fig.6). When the input signal value is 0.5, we only observe NAND/ NOR gate and SR-Latch (Fig.6(a)), while with input signal strength 0.75, all three logic operation are observed and the operational range is also extended. (Fig.6(b)).

5 Conclusion

We have demonstrated the implementation of logic operations with engineered time-delayed genetic networks, and shown that this biological system can reliably emulate and morph into different logic gates and latch in an optimal window of noise. Further we investigated the effects of the interplay of noise, nonlinearity and time delay on the operational range of this biological logic gate. We have reported the variation in the range of robust logic operations, including the Set-Reset latch, under different kinds of delay, and shown how delay can both enhance or diminish logic behavior, depending on its functional form. Lastly, we have demonstrated that the desired "logical" response to the inputs can be induced, even in the absence of noise, by time delay alone.

In summary, our observations provide an understanding of the information processing capacity of time-delayed synthetic genetic networks. In particular, we have indicated what kind of time delay aids logic patterns and yields a larger range of robust operations. So time delay suggests a new direction in enhancing the computational ability in such systems. Thus it provides new inputs to the design of biologically inspired computing modules and circuits. So this work has potential relevance to various cellular engineering applications, like biomolecular computing, controlling of biological systems, and multi-step biosynthetic pathways [21].

6 Acknowledgement

AS acknowledges support by the Aihara Project, the FIRST program from JSPS, initiated by CSTP. VK acknowledges research fellowship from CSIR, India.

Appendix

Time delay in the degradation process is given by following equation

$$\frac{dx(t)}{dt} = \frac{m(1+x^2+\alpha\sigma_1 x^4)}{1+x^2+\sigma_1 x^4+\sigma_1\sigma_2 x^6} - \gamma x(t-\tau) + D\eta(t) \qquad (6)$$

The presence of small delay changes the position and depth of the wells in the bistable potential.

The dynamics in Eq.(6) is a non-Markovian process. Using the probability density approach, the non-Markov process can be reduced to a Markov process, and the approximate time-delay Fokker-Plank equation is

$$\frac{\partial P(x,t)}{dt} = -\frac{\partial[h_{eff}P(x,t)]}{\partial x} + D\frac{\partial^2 P(x,t)}{\partial^2 x} \qquad (7)$$

Here the conditional average h_{eff} is

$$h_{eff} = \int_{-\infty}^{\infty} h(x,x_\tau)P(x_\tau, t-\tau|x,t) \qquad (8)$$

where $x_\tau = x(t-\tau)$
$h(x,x_\tau) = \frac{m(1+x^2+\alpha\sigma_1 x^4)}{1+x^2+\sigma_1 x^4+\sigma_1\sigma_2 x^6} - \gamma x_\tau$
$h(x) = \frac{m(1+x^2+\alpha\sigma_1 x^4)}{1+x^2+\sigma_1 x^4+\sigma_1\sigma_2 x^6} - \gamma x$

$P(x_\tau, t-\tau|x,t)$ is the zeroth order approximate Markovian transition probability density

$$P(x_\tau, t-\tau|x,t) = \frac{1}{\sqrt{4\pi D\tau}}\exp\left(-\frac{(x_\tau - x - h(x)\tau)^2}{4D\tau}\right) \qquad (9)$$

Substituting Eq.(9) into Eq.(8), we obtain

$$h_{eff} = (1-\gamma\tau)\frac{m(1+x^2+\alpha\sigma_1 x^4)}{1+x^2+\sigma_1 x^4+\sigma_1\sigma_2 x^6} - (1-\gamma\tau)\gamma x_\tau \qquad (10)$$

So, the effective Langevin equation for Eq.(7) becomes

$$\frac{dx(t)}{dt} = (1-\gamma\tau)\frac{m(1+x^2+\alpha\sigma_1 x^4)}{1+x^2+\sigma_1 x^4+\sigma_1\sigma_2 x^6} \qquad (11)$$
$$-(1-\gamma\tau)\gamma x_\tau + D\eta(t) \qquad (12)$$

The effective time-delay potential function of Eq.(6) is

$$U_{eff} = -(1-\gamma\tau)\int \frac{m(1+x^2+\alpha\sigma_1 x^4)}{1+x^2+\sigma_1 x^4+\sigma_1\sigma_2 x^6}dx \qquad (13)$$
$$+(1-\gamma\tau)\int \gamma x dx \qquad (14)$$

The effect of time delay on the effective potential given by Eq.(14) is shown in Fig. 1.

References

1. Gammaitoni L., Hänggi P., Jung P., Marchesoni F.: Stochastic resonance, Rev. Mod. Phys. **70** 223287 (1998).
2. Douglass J. K., Wilkens L., Pantazelou E., Moss F.: Noise enhancement of information transfer in crayfish mechanoreceptors by stochastic resonance, Nature **365**, 337-340 (1993).
3. Murali K., Sinha S., Ditto W. L., Bulsara A. R.: Reliable Logic Circuit Elements that Exploit Nonlinearity in the Presence of a Noise Floor, Phys. Rev. Lett. **102**, 104101 (2009).
4. Sinha S., Cruz J. M., Buhse T., Parmananda P., Europhys. Letts. **86**, 60003 (2009).
5. Guerra D. N., Bulsara A. R., Ditto W. L., Sinha S., Murali K., Mohanty P.: Noise-Assisted Reprogrammable Nanomechanical Logic Gate, Nano Letters **10**, 1168-1171 (2010).
6. Singh K. P., Sinha S.: Enhancement of Logical responses by noise in a Bistable Optical System, Phys. Rev. E **83**, 046219 (2011).
7. Wang C., Yi M., Yang K.: Time-delay accelerated transition of gene switch and enhanced stochastic resonance in bistable gene regulatory model, IEEE International Conference on Systems Biology (ISB), Zhuhai, China, September 24, 101 (2011).
8. Craig E. M., Long B. R., Parrondo J. M. R., Linke H.: Effect of time delay on feedback control of a flashing ratchet, Eur. Phys. Lett. **81**, 10002 (2008).
9. Nie L. R., Mei D. C.: Noise and time delay: Suppressed population explosion of the mutualism system , Eur. Phys. Lett. **79**, 20005 (2007); Effects of time delay on symmetric two species competition subject to noise, Phys. Rev. E **77**, 031107 (2008).

10. Zhang D., Song H., Yu L., Wang Q. G., Ong C.,: Set-values filtering for discrete time-delay genetic regulatory networks with time-varying parameters, Nonlinear Dyn **69**, 693 (2012).
11. Hasty J., Isaacs F., Dolnik M., McMillen D., Collins J. J.: Designer gene network: toward fundamental cellular control, Chaos **11**, 207-220 (2001).
12. Gardner T. S., Cantor C. R., Collins J. C.: Construction of a genetic toggle switch in Escherich coli, Nature **403**, 339-342 (2000).
13. Ando H., Sinha S., Storni R., Aihara K.: Synthetic gene networks as potential flexible parallel logic gate, Eur. Phys. Lett. **81**, 50001 (2011).
14. Dari A., Kia B., Bulsara A. R., Ditto W.: Creating morphable logic gates using logical stochastic resonance in an engineered gene network, Eur. Phys. Lett. **93**, 18001 (2011).
15. Sharma P. R., Somani P., Shrimali M. D.: Bio-inspired computation using synthetic genetic network, Phys. Lett. A **377**, 367-369 (2013).
16. Hellen E. H., Dana S. K., Kurths J., Kehler E., Sinha S.: Noise-aided Logic in an Electronic Analog of Synthetic Genetic Networks, (to appear in PLoS One).
17. Hasty J., Pradines J., Dolnik M., Collins J. J.: Noise-based switches and amplifiers for gene expression, Proc. Natl. Acad. Sci. U.S.A. **97**, 2075-2080 (2000).
18. Frank D.: Delay Fokker-Plank equation, perturbation theory, and data analysis for nonlinear stochastic systems with time delay, Phys. Rev. E **71**, 031106 (2005).
19. Kohar V., Sinha S.: Noise-Assisted Morphing of Memory and Logic function, Phys. Letts. A **376**, 957-962 (2012).
20. Gupta A., Sohane A., Kohar V., Murali K., Sinha S.: Noise-free logical stochastic resonance, Phys. Rev. E **84**, 055201(R) (2011).
21. Y. Benenson, *Science* **340**, 554 (2013).

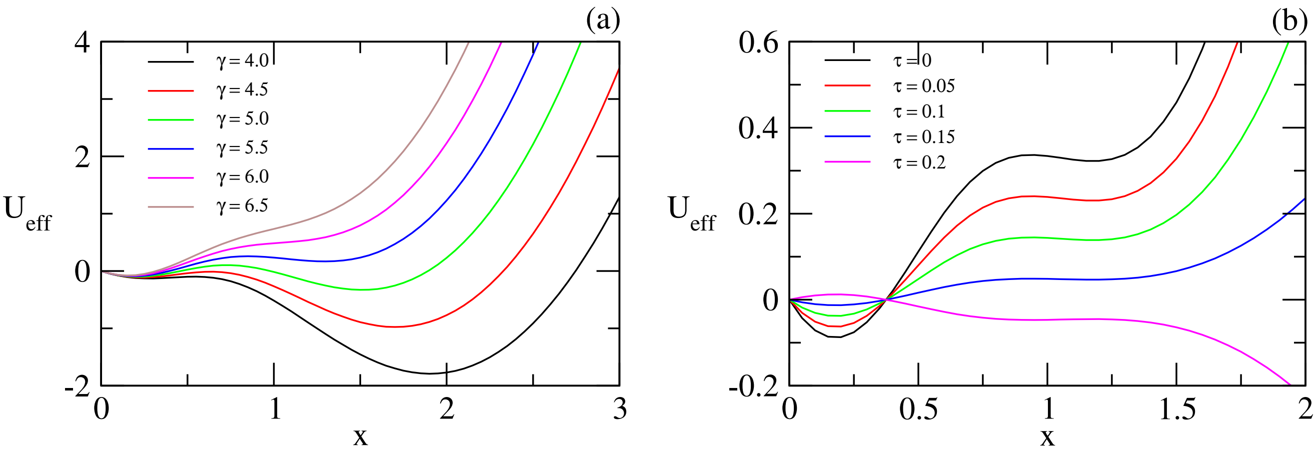

Fig. 1 (left panel) Potential U$_{eff}$ for Eq.(1) at different γ with $\tau = 0$. (right panel) Potential U_{eff} for different delay times with $\gamma = 5.7$ for Eq.(2). Both plots are in the absence of external noise $D = 0$.

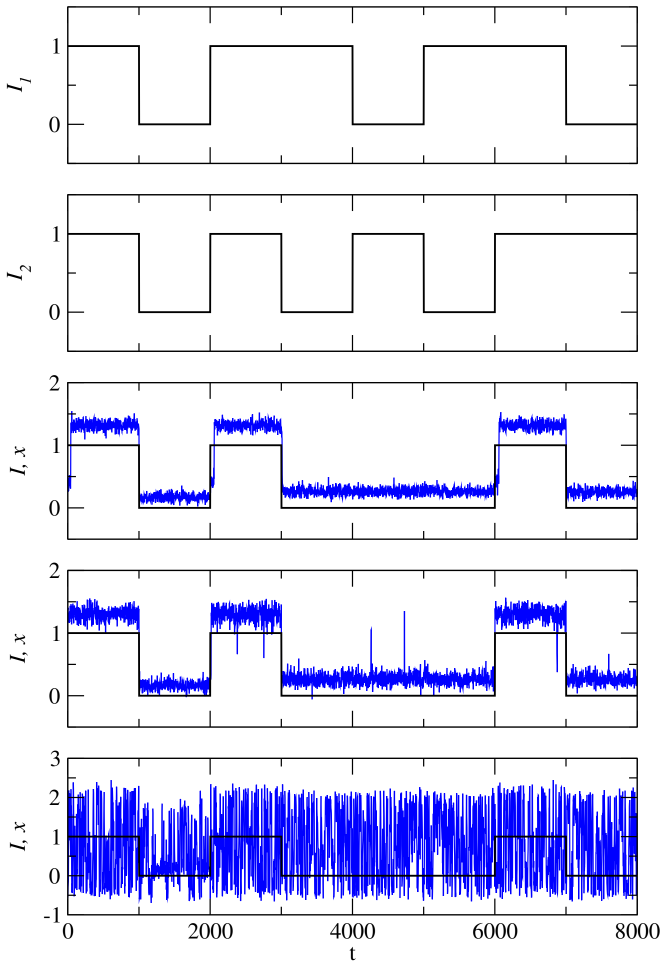

Fig. 2 From top to bottom: panels 1 and 2 show streams of inputs I_1 and I_2. The input strength is 0.5. For I_1, I_2, we have 0 when logic input is 0 and 0.5 when logic input is 1. Panels 3, 4, 5 show the waveforms of $x(t)$ (blue) obtained from the system given in Eq. (2) with $D = 0.12$ and $\gamma = 6.1$, under the time delay (panel 3-5): (i) $\tau = 0$, (ii) $\tau = 0.1$, and (iii) $\tau = 0.2$. The input signal $I = I_1 + I_2$ is indicated by the black line. For zero and small delay (panel 3-4) one obtains the desired AND logic output (with $x^* = 0.75$), while for high delay (panel 5) one does not obtain a consistent response.

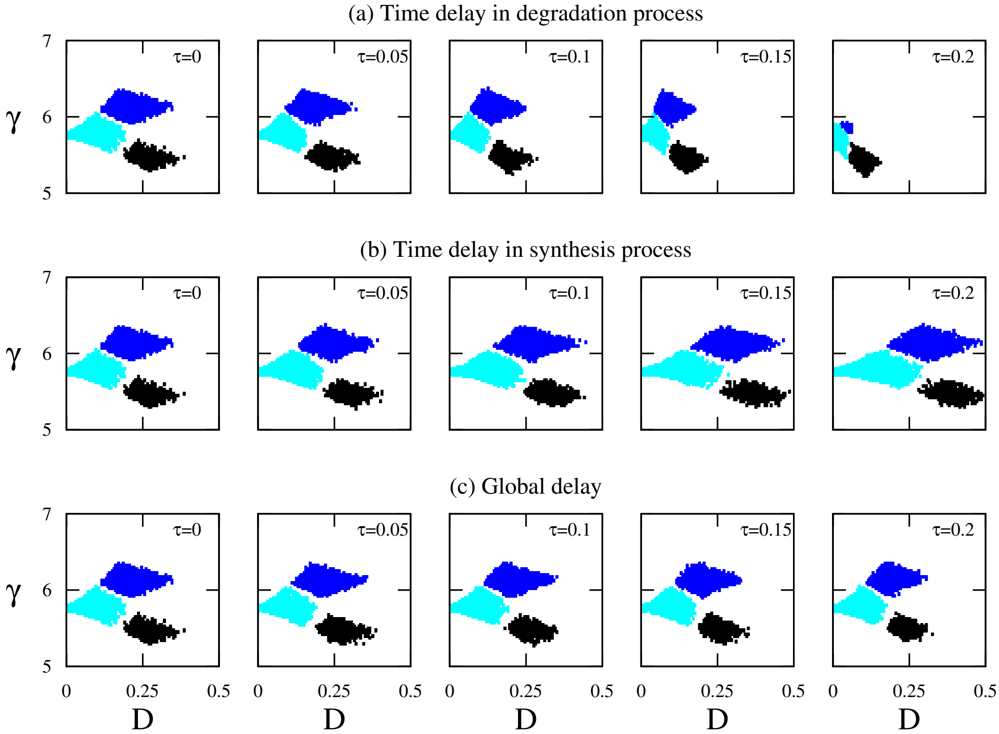

Fig. 3 Points marking where the latch operation (light blue), OR/NOR logic (black) and AND/NAND logic (blue) are obtained with probability ~ 1, with input signal strength 0.5 in the parameter space of $\gamma - D$ for different time delays. The rows show the impact of time delay in the degradation process (upper), the synthesis process (middle) and both (bottom). The columns show different time delays τ: 0, 0.05, 0.1, 0.15 and 0.2 (from left to right).

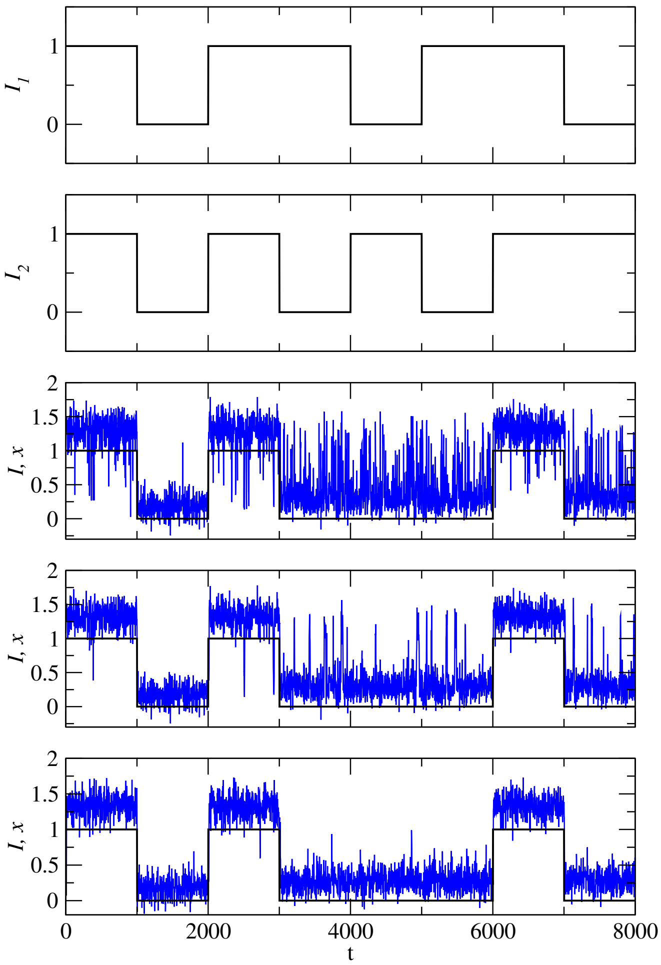

Fig. 4 From top to bottom: panels 1 and 2 show streams of inputs I_1 and I_2. The input strength is 0.5. For I_1, I_2, we have 0 when logic input is 0 and 0.5 when logic input is 1. Panels 3, 4, 5 show the waveforms of $x(t)$ (blue) obtained from the system given by Eq. (3) (with $\gamma = 6.1$ and $D = 0.3$) under the time delay (panel 3-5): (i) $\tau = 0$, (ii) $\tau = 0.1$, and (iii) $\tau = 0.2$. The input signal $I = I_1 + I_2$ is indicated by the black line. Here for larger delay (bottom-most panel) one obtains the desired AND logic output, while small delays do not yield the desired response.

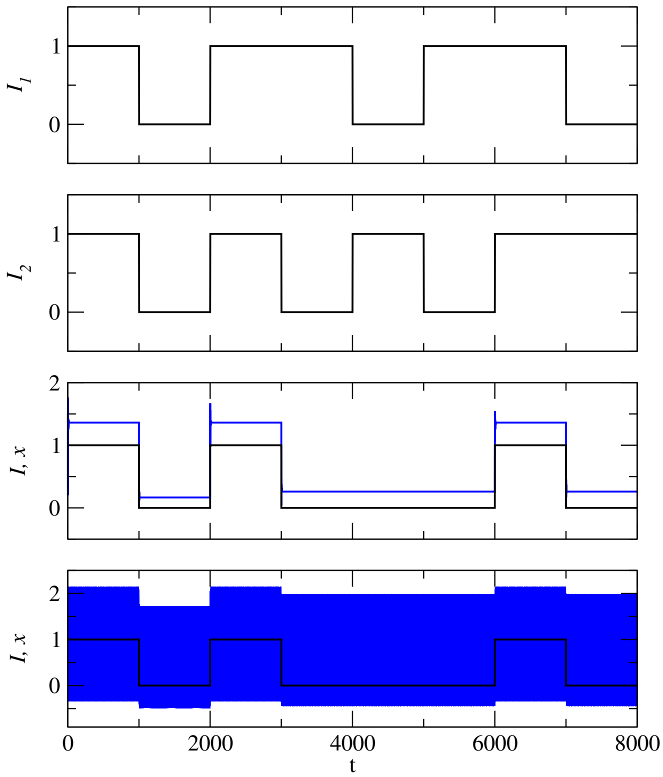

Fig. 5 From top to bottom: panels 1 and 2 show streams of inputs I_1 and I_2. The input strength is 0.75. For I_1, I_2, we have 0 when logic input is 0 and 0.75 when logic input is 1. Time series of $x(t)$ (blue), for (a) AND at $\gamma = 6.7, \tau = 0.1$ (upper panel), (b) SR–Latch at $\gamma = 6.2, \tau = 0.15$ (middle panel) and (c) OR at $\gamma = 5.8, \tau = 0.19$ (lower panel), with noise intensity $D = 0$. The input signal $I = I_1 + I_2$ is indicated by the black line.

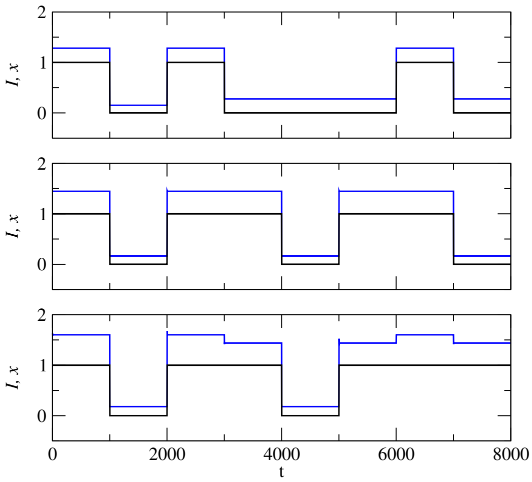

Fig. 6 Points marking where the Set–Reset latch operation (light blue), OR/NOR logic (black) and AND/NAND logic (blue) are obtained with probability 1 in parameter space of $\gamma - \tau$ for Eq.(2) at $D = 0$ for input signal strength 0.5 (left panel) and 0.75 (right panel).