Picking microseismic first arrival times by Kalman filter and wavelet transform

Baolin Qiao and John C. Bancroft

ABSTRACT

Due to the high energy content of the ambient noise, microseismic monitoring system records considerable erroneous data. To pick up the first arrival times, special techniques must be applied due to the very low signal to noise ratio data. This paper presents three techniques: wavelet transform is applied to de-noising the noisy data; Kalman filter and modified STA/LTA method are implemented to pick up the first arrival times. The results show that the first arrival times are picked up accurately even in very noisy data by incorporating these techniques.

KALMAN FILTER

In 1960, R.E. Kalman published his famous paper describing a recursive solution to the discrete-data linear filtering problem (Kalman, 1960). The kalman filter (KF) addresses the general problem of trying to estimate the state x of a discrete-time controlled process that is governed by the linear stochastic different equation

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}, \tag{1}$$

with a measurement

$$z_k = Hx_k + v_k. \tag{2}$$

Here x_k is state vector, A is $n\times n$ state transition matrix, B is $n\times r$ optional control matrix, u_k is control vector, z_k is measurement vector, and H is $m\times n$ measurement matrix. The random variables w_k and v_k represent the process and measurement noise respectively, they are assumed to be independent of each other, white, and with normal probability distributions $p(w)\sim N(0,R)$, $p(v)\sim N(0,Q)$. R and Q might change with each time step or measurement, however here we assume they are constant.

The process of the KF falls into two groups: time update equations and measurement update equations.

Time update equations:

Priori state estimate at step k:

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1} \tag{3}$$

Priori estimation error covariance at step k:

$$P_k^- = AP_{k-1}A^T + Q \tag{4}$$

Measurement update equations:

Kalman gain at step k:

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \tag{5}$$

Posterior state estimate at step *k*:

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H \hat{x}_k^-) \tag{6}$$

Posterior estimation error covariance at step *k*:

$$P_k = (I - K_k H) P_k^- \tag{7}$$

The time update equations are responsible for projecting forward the current state and error covariance to obtain the priori estimate for step *k*; the measurement update equations are responsible for the feedback, i.e., for incorporating the measurement into the priori estimate to obtain an improved posterior estimate for step *k*, this posterior estimate is also the priori estimate for the next step *k+1*.

SEISMIC WAVELET MODEL

A seismic model is typically modelled as an exponentially decaying cyclic waveform (Sheriff and Geidart, 1982) as follows

$$A(t) = A_0 e^{-h(t-t_0)} \sin[\omega(t - t_0)], \;\; t \geq t_0, \tag{8}$$

where A_0 is initial amplitude, h is damping factor, and ω is dominant angular frequency. To simplify the mathematics and keep the KF in a linear form, this seismic wavelet is modelled as a periodic process with random walk amplitude (Baziw, 2002),

$$x_1(t) = x_2(t) \sin\,[\omega(t - t_0)], \tag{9}$$

where $x_1(t)$ is an approximation to the seismic wavelet defined by (8), and $x_2(t)$ is the random walk process approximating A_0 in (8), which is defined as its derivative being driven by white noise as follows:

$$\dot{x}_2(t) = w(t), \tag{10}$$

where $E[w(t)w(\tau)] = q(t)\delta(t - \tau)$.

The linear continuous differential equation defining the seismic wavelets is outlined as follows

$$\dot{x}_1(t) = \omega x_2(t) \cos\,(wt). \tag{11}$$

The discrete form of (11) is

$$x_1(k) = x_1(k-1) + \Delta\omega \cos[\Delta\omega(k-1)]\, x_2(k-1), \tag{12}$$

where Δ is the sample rate.

AMBIENT NOISE MODEL

A Gauss-Markov process is a good candidate to model the microseismic environmental noise (Baziw, 2002), its autocorrelation function is defined by

$$\emptyset_{nn}(\tau) = \sigma^2 e^{-\beta|\tau|}, \tag{13}$$

where σ^2 is the variance and β is called the reciprocal of time constant.

The discrete model for the Gauss-Markov process can be written as

$$n_{k+1} = a_w n_k + b_w w_k, \tag{14}$$

where $a_w = e^{-\beta\Delta}$, and $b_w = \sigma\sqrt{(1-e^{-2\beta\Delta})}$.

KF GOVERNING EQUATIONS

The discrete form of the KF governing equation is

$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} 1 & \Delta\omega\, cos[\Delta\omega(k-1)] & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-\beta\Delta} \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ x_3(k-1) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ q(t)\Delta & 0 \\ 0 & \sigma\sqrt{(1-e^{-2\beta\Delta})} \end{bmatrix} \begin{bmatrix} w_1(k-1) \\ w_2(k-1) \end{bmatrix} \tag{15}$$

where $w_1(k\text{-}1)$ and $w_2(k\text{-}1)$are zero mean, unity variance, Gaussian white noise processes.

For microseismic recording data, there is only one scalar measurement available, which is a combination of both the seismic wavelet (state x_1) and the ambient noise (state x_3),

$$z(k) = x_1(k) + x_3(k). \tag{16}$$

This results in the following measurement matrix:

$$H_k = [1 \quad 0 \quad 1]. \tag{17}$$

DATA SIMULATION

The wavelet is generated by equation (8) with parameters listed in Table-1:

Table-1: Wavelet parameters.

	Frequency (Hz)	Initial amplitude (mm/s^2)	Damping factor (1/s)	Arrival time (ms)	Sample rate (ms)
p-wave	200	160	80	150	0.05
s-wave	70	200	50	400	0.05

Gauss-Markov ambient noise is simulated by equation (14). There are five Gauss-Markov processes are simulated in order to test our techniques on different levels of noisy data. Their parameters are listed in Table-2.

Table-2: Gauss-Markov process parameters.

	noise1	noise2	noise3	noise4	noise5
β	10000	10	1	0.1	0.05
σ^2	1000	1000	1000	2000	2000

The synthetic data are shown in Fig. 1. The noise free signal and noises are shown on the left side; the signals for test are shown on the right side. We can see that these data are very noisy. It will be difficult to pick up the first arrival times of *p*- and *s*-wave if nothing is done beforehand.

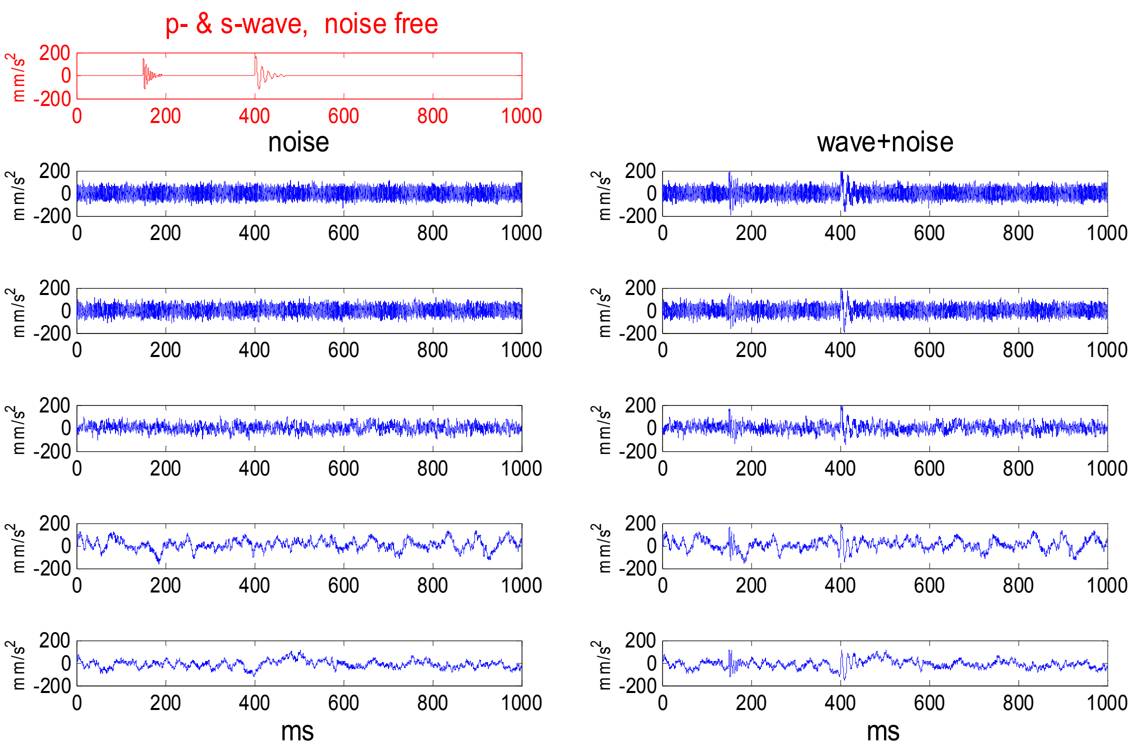

Fig. 1: Wavelet and noises.

The frequency contents of wavelet and noises are plotted in Fig. 2. We can see that the wavelet has two dominant frequency, 70 Hz and 200 Hz; noise1 is almost white noise; the frequency contents of noise2 cover 0~6000 Hz with a dominant frequency at 700 Hz; noise3 has frequency contents of 0~500 Hz with a dominant frequency at 80 Hz; noise4 and noise5 are mainly low frequency (0~60 Hz). Note that the frequency contents of noise3~5 are overlapped with that of the wavelet.

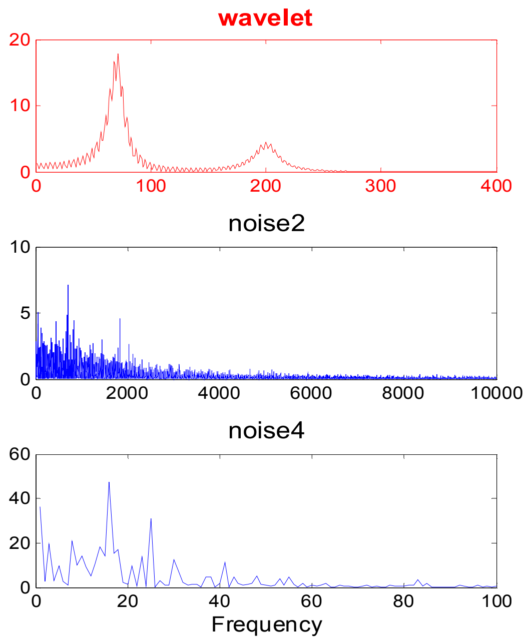

Fig. 2: Frequency contents of wavelet and noises.

KALMAN FILTERING RESULTS

The time update and measurement update equations were programmed to implement the Kalman filtering process. It is found that the best event detection state to track is the seismic wavelet amplitude (Baziw, 2002), i.e., the state x_2, see Fig. 3. We can see that we have obtained a dramatic improvement in the SNR when comparing these results to the initial seismic time series in Fig. 1.

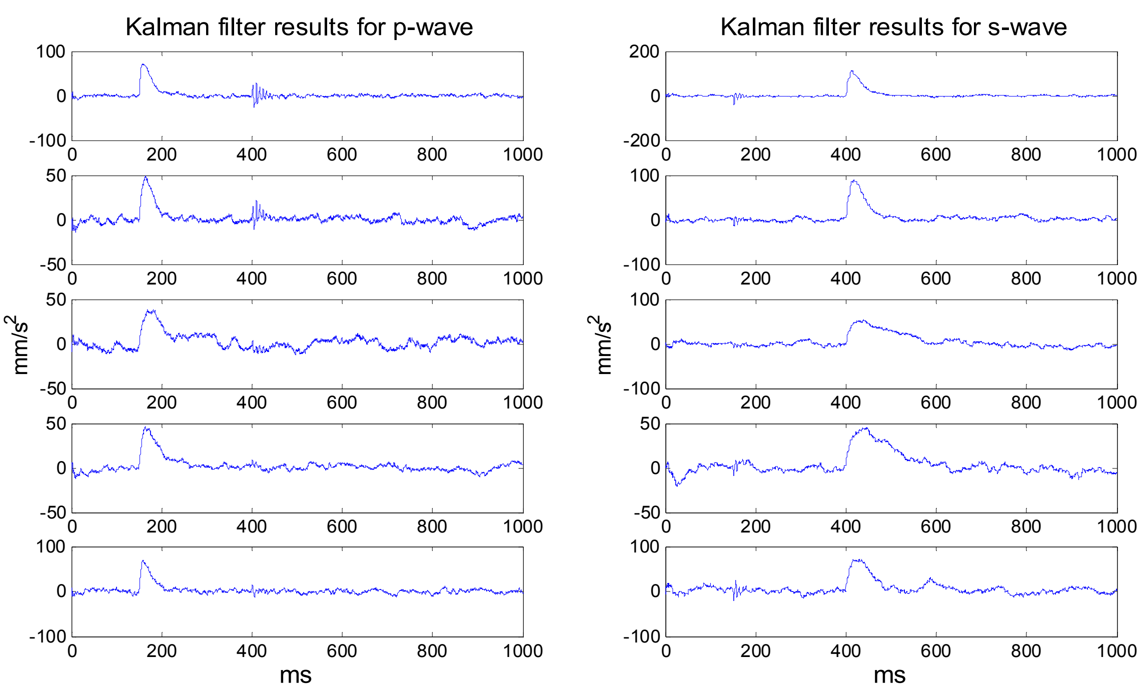

Fig. 3: Kalman filtering results (state x_2).

Although Fig. 3 shows the first arrival times are around 150 ms and 400 ms, we still can't get accurate values if we look closely in Fig. 3. Picking up arrival times will be depended on the experience and decisions of operators, which is also time consuming if huge data is given.

To obtain the accurate *p*- and *s*-wave arrival time, STA/LTA or MER methods was often used to get the arrival times (Han et al., 2009; Chen and Stewart, 2006). In this paper we use a modified STA/LTA method,

$$eratio(i) = |grm(i)|^m \left[\frac{l_1}{l_2}\frac{\sum_i^{i+l_2} grm(i)^2}{\sum_{i-l_1}^{i} grm(i)^2}\right]^n, \quad (18)$$

where *grm(i)* is the seismogram value at point *i*. A diagram of computing the modified STA/LTA is illustrated in Fig. 4.

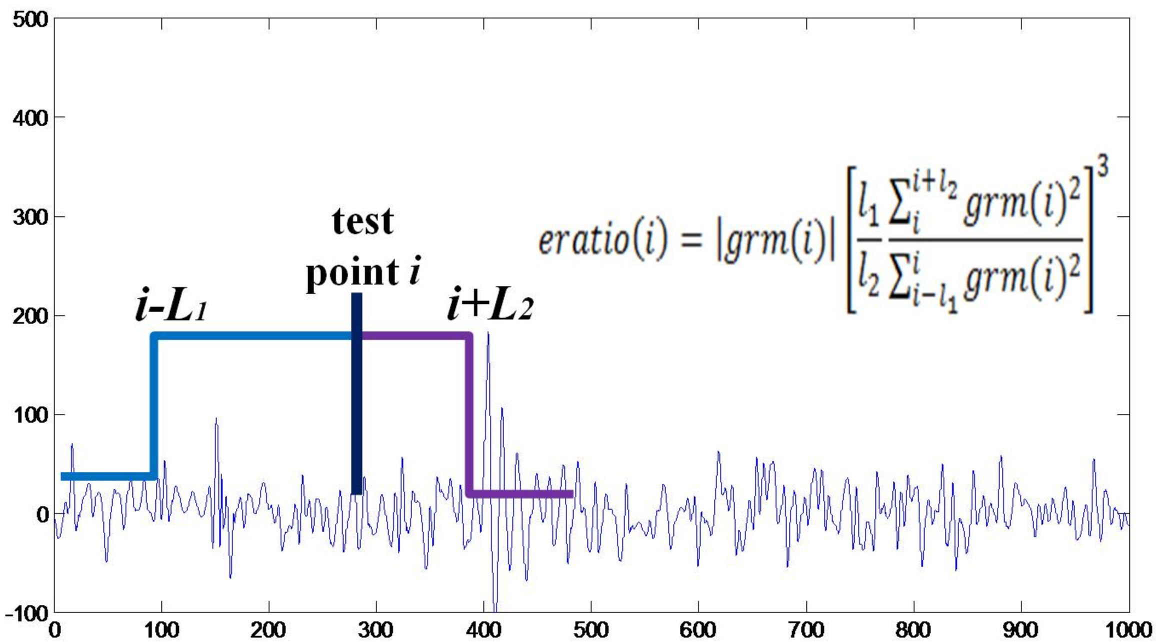

Fig. 4: Diagram of the modified STA/LTA.

From our test, we found that when m=1, n=3, and L_1 and L_2 were set to 40 and 10 sample points respectively, we can get the least estimation error. These results are shown in Fig. 5 and Table-3.

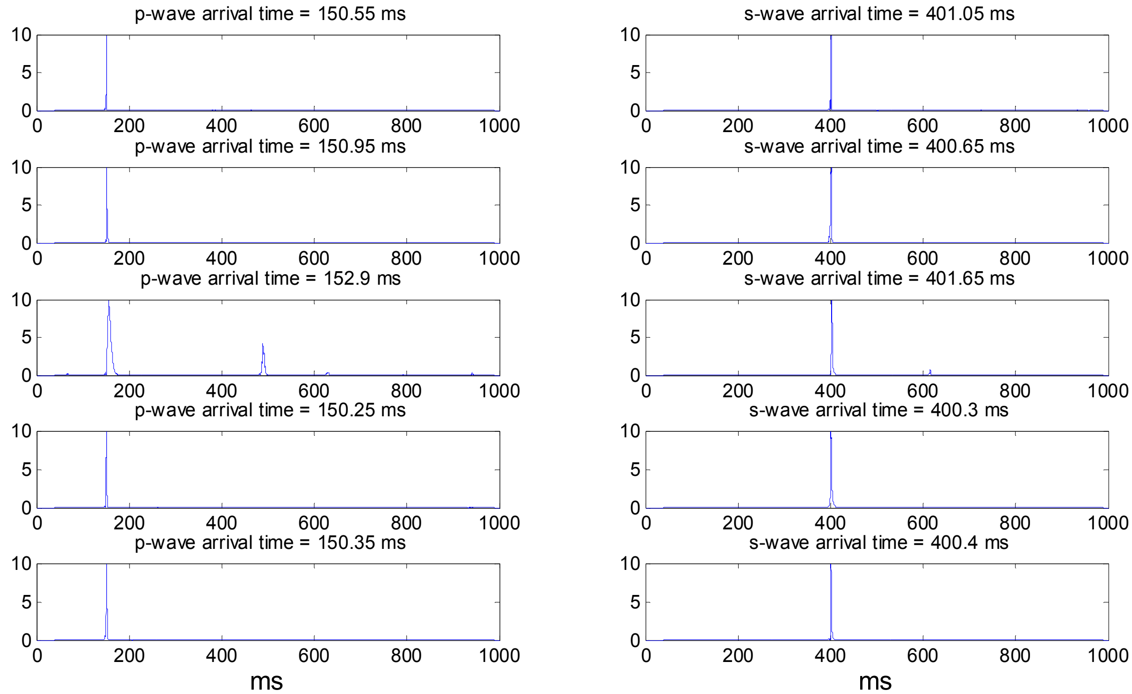

Fig. 5: Estimation of first arrival times using modified STA/LTA method.

As illustrated, the error of picking arrival times is under 3 ms. It is interesting that with noise4 and noise5, we get better results comparing to noise1~3, noise3 has the biggest errors. The reason of this is not clear at this moment.

Table-3: Estimation error of the first arrival times using modified STA/LTA.

	noise1	noise2	noise3	noise4	noise5
p-wave error (ms)	0.55	0.95	2.9	0.35	0.35
s-wave error (ms)	1.05	0.65	1.65	0.3	0.4

DENOISING BY WAVELET TRANSFORM

The wavelet transform (WT) properties such as localization, which is essential for the analysis of transient signals, provide a filter to extract characteristics of interest such as energy and predominant timescales. This information is subsequently exploited for microseismic events detection. WT can also be used to de-noising seismic data (Fu,2005; Chen and Chao, 2004; Zhang and Ulrych, 2003).

The de-noising procedure proceeds in three steps: first of all, choose a wavelet to compute the wavelet decomposition of the signal s at level N; then for each level from 1 to N, select a threshold and apply soft thresholding to the detail coefficients; in the final, compute wavelet reconstruction based on the original approximation coefficients of level N and the modified detail coefficients of levels from1 to N.

In this paper we use wavelet *db4* and decompose signals up to level 6. The results of WT are shown in Fig. 6.

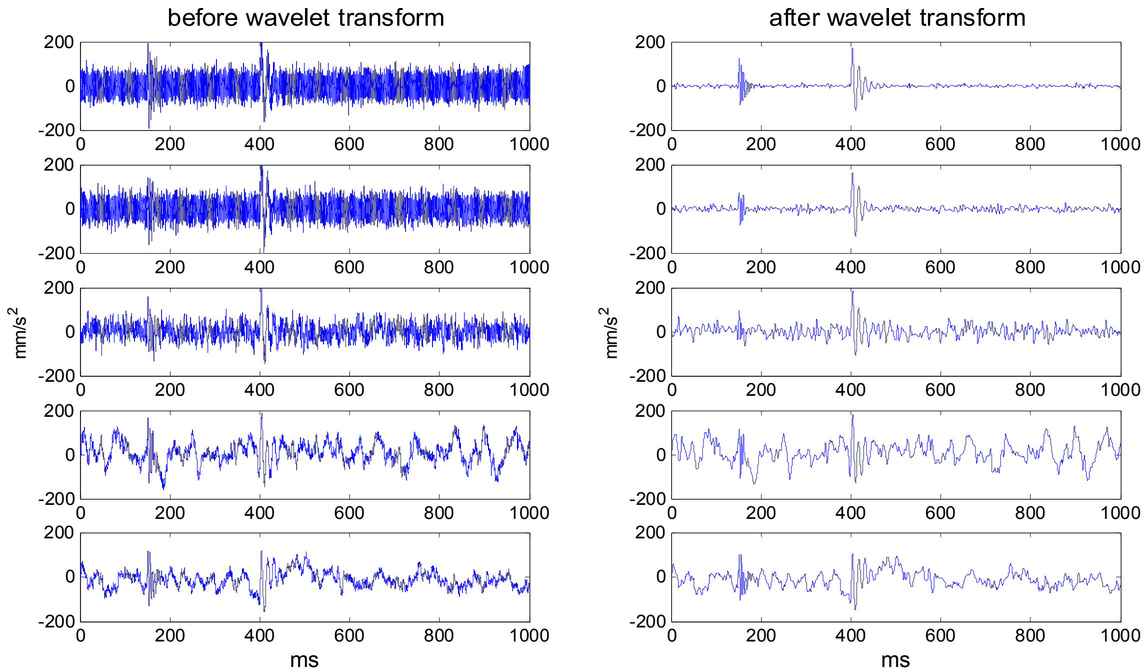

Fig. 6: The results of wavelet transform.

We can see that after applying WT, SNR of signals are improved especially for noise1 and noise2. Using the results of WT, we apply Kalman filtering process again and the results (state x_2) are shown in Fig. 7.

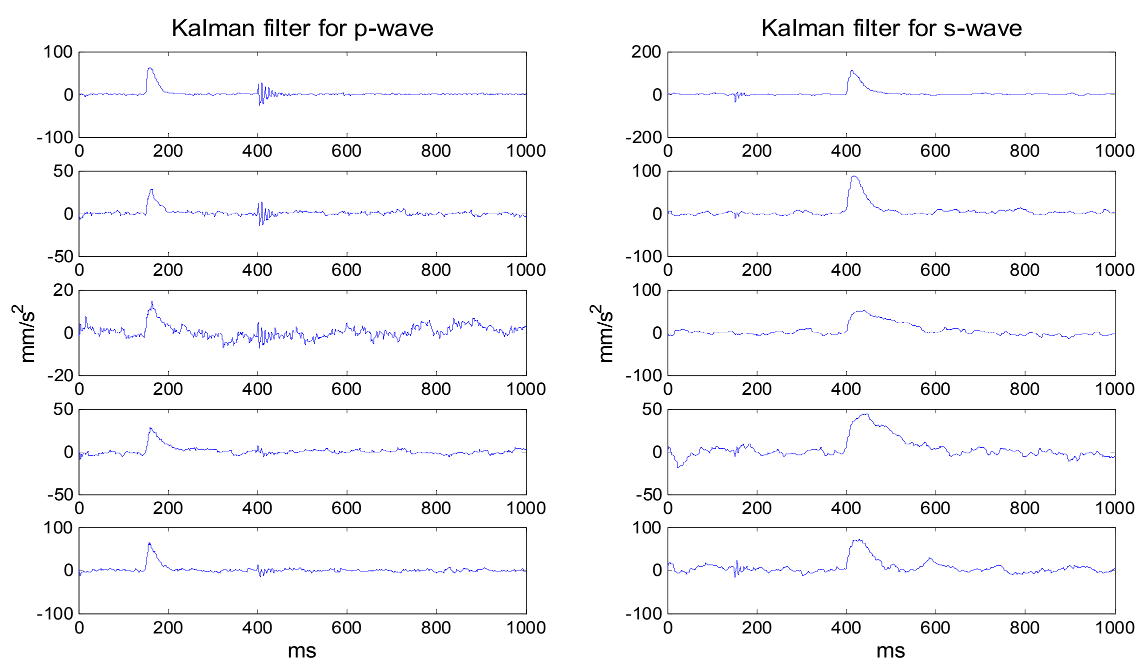

Fig. 7: Kalman filtering results (using WT data).

Again, we got very good results. Comparing Fig. 7 with Fig. 3, we can see that state x_2 is smoother than that without WT.

Using the new Kalman filtering results, we apply the modified STA/LTA method again to estimate the arrival times. The results are plotted in Fig. 8 and Table-4.

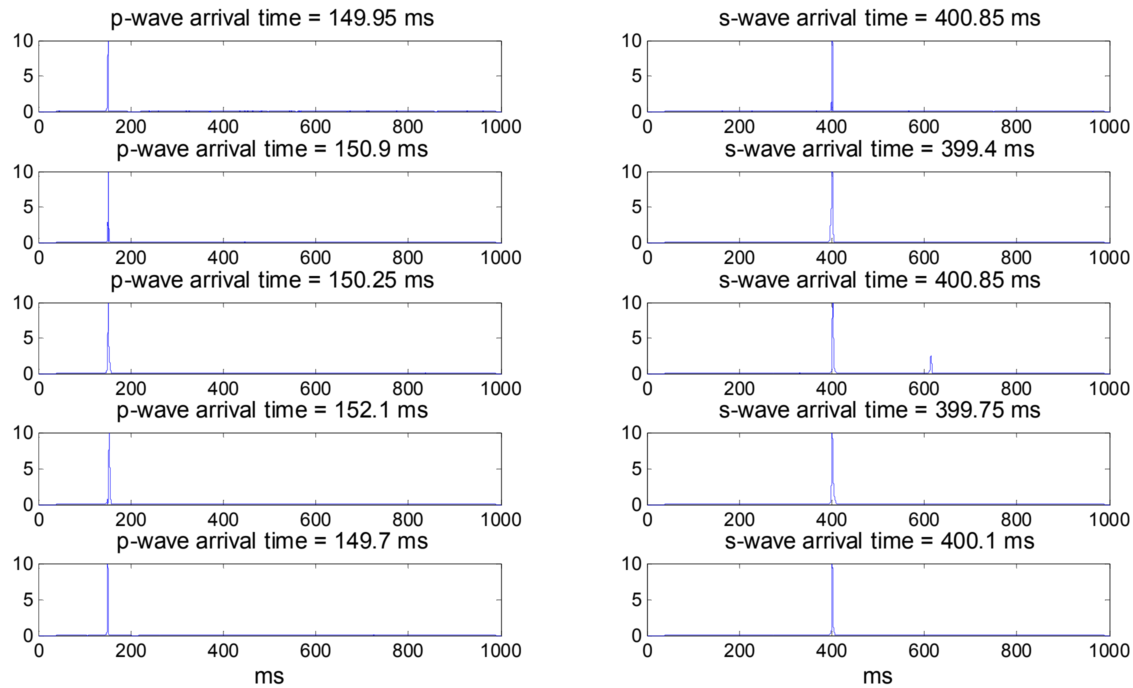

Fig. 8: Estimation of first arrival times (using WT data).

Comparing Table-4 with Table-3, the error is reduced except *p*-wave error of noise4. The reason need to be studied in the future.

Table-4: Estimation error using modified STA/LTA method (using WT data).

	noise1	noise2	noise3	noise4	noise5
p-wave error (ms)	-0.05	0.9	0.25	2.1	-0.3
s-wave error (ms)	0.85	-0.6	0.85	-0.25	0.1

CONCLUSION

By testing our techniques on synthetic data, it shows that wavelet transform can attenuate considerable microseismic ambient noise given appropriate wavelet and decomposing level. The test also shows that the first arrival times can be picked up accurately by combining Kalman filter and modified STA/LTA method. There are some questions still need to be studied further in the future, for example, the parameters of STA/LTA method, the KF governing equations and its parameters, and the effect of choosing different wavelets.

ACKNOWLEDGMENTS

The authors would like to thank all CREWES' sponsors, staff and students for their supports.

REFERENCE

Baziw, E., 2002, Application of Kalman filtering techniques for microseismic event detection: Pure and Applied Geophysics, **159**, 449-471.

Chen, Z. and Stewart, R.R., 2006, A multi-window algorithm for real-time automatic detection and picking of P-phases of microseismic events: CREWES Research Report, **18**.

Chen, X. and Chao, S., 2004, 第二代小波变换及其在地震信号去噪中的应用: Geophysical Prospecting for Petroleum, **43** (**6**), 547-550.

Fu, Y., 2005, Noise eliminated method for seismic signal based on second wavelet transform: Oil Geophysical Prospecting, **40** (**2**), 154-157.

Han,L. et al., 2009, Time picking and random noise reduction on microseismic data: CREWES Research Report, **21**.

Kalman, R.E., 1960, A new approach to linear filtering and prediction problems: Transaction of the ASME-Journal of Basic Engineering, 33-35.

Sheriff, R.E. and Geldart, L.P., 1982, Exploration seismology: Cambridge University Press, **1**, 55.

Zhang, R. and Ulrych, T.J., 2003, Physical wavelet frame denoising: Geophysics, **68** (**1**), 225-231.