Microseismic Event Detection Kalman Filter

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Introduction

- Kalman filter approach used to detect the P-wave and S-wave
- Two 3 state Kalman filters can be used to detect P wave and S wave separately
- A single 5 state Kalman filter used to detect P and S wave simultaneously

P and S wave modeling

P and S wavelets modeled by

$$A(t) = A_0 e^{-h(t-t_0)} sin(\omega(t - t_0)), t > t_0 \tag{1}$$

Approximation of the wavelets

The seismic wavelets are approximated by

$$x_1(t) = x_2(t) sin(\omega(t - t_0))$$

- $x_2(t)$ is random walk. Event detection done by the amplitude of x_2
- $\dot{x}_2(t) = w(t)$ where $w(t)$ is the white noise
- $\dot{x}_1(t) = x_2(t)\omega cos(\omega(t))$ if, $t_0 = 0$
- $\dot{x}_3(t) = -\beta x_3(t) + \sqrt{2\sigma^2\beta} w(t)$

Noise in the simulated signal

Gauss Markov Noise Process

$\beta = T_c^{-1}$ and σ estimated from Discrete Gauss Markov model

$$v[k] = a_w v[k-1] + b_w e[k] \text{ where, } a_w = e^{-\beta\Delta} \text{ and, } \\ b_w = \sigma\sqrt{1 - e^{-2\Delta\beta}} \tag{2}$$

Δ is the sampling frequency.

The noise used in the paper is ARMA$(1, 1)$

State Space Modeling for Kalman Filtering

Continuous time

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \omega cos(\omega t) & \omega \\ 0 & 0 & 0 \\ 0 & 0 & -\beta \end{bmatrix}}_{F(t)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ q(t) & 0 \\ 0 & \sqrt{2\sigma^2\beta} \end{bmatrix}}_{G(t)} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$Z(t) = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + v(t)$$

State Space Modeling for Kalman Filtering

Discrete Time

$$
\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \\ x_3[k+1] \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \Delta\omega cos(\omega\Delta(k)) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-\beta\Delta} \end{bmatrix}}_{\Phi_{k-1}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ q[k] & 0 \\ 0 & \sqrt{2\sigma^2\beta} \end{bmatrix}}_{Q_{k-1}} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{3}
$$

$$
z[k] = \underbrace{\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}}_{H_{k-1}} \begin{bmatrix} x_1[k] \\ x_2[k] \\ x_3[k] \end{bmatrix} + v[k]
$$

Formulation in the papers by Eric Baziw

Kalman Filter Equations

$$\begin{aligned} \vec{x}_k &= \Phi_{k-1}\vec{x}_{k-1} + \vec{w}_{k-1} \text{ where, } \vec{w}_k = \mathcal{N}(\vec{0}, Q_k) \\ \vec{z}_k &= H_k\vec{x}_k + \vec{v}_k \text{ where, } \vec{v}_k = \mathcal{N}(\vec{0}, R_k) \end{aligned} \tag{4}$$

State Estimate Extrapolation

$$\vec{\hat{x}}_k(-) = \Phi_{k-1}\vec{\hat{x}}_k(+)$$

Error Covariance Extrapolation

$$P_k(-) = \Phi_{k-1}P_{k-1}(+)\Phi_{k-1}^T + Q_{k-1} \tag{5}$$

Kalman Gain Matrix

$$K_k = P_k(-)H_k^T[H_kP_k(-)H_k^T + R_k]^{-1}$$

State Estimate Update

$$\vec{\hat{x}}_k(+) = \vec{\hat{x}}_k(-) + K_k[\vec{z}_k - H_k\vec{\hat{x}}_k(-)]$$

Error Covariance Update

$$P_k(+) = [I - K_kH_k]P_k(-)$$

Formulation in the papers by Eric Baziw

Problems

- Q_k is the noise covariance matrix of the inputs which are considered to be the white noise sequences w_1,w_2
- The $\vec{w}$ used differently in the matrix form and the Kalman Filter equations
- Assuming Q_k is from eq(3) it's 2×3,eq(5) is incorrect
- Abuse of the Q_k matrix. Being used as the covariance matrix as well as the control matrix
- Further to solve real-life example, the noise profile is assumed to be ARMA$(1, 1)$

Correct formulation

Tweaks to the solution

- Q_k is assumed to be the control matrix
- Covariance matrix (Q_{k-1})of the states is hardcoded, a 3×3 matrix, to make the eq(5) correct
- R_k in eq(4), which is measurement noise is also hardcoded

Ideal way to follow the KF approach

- Solve the state space identification
- Constraining the state transition matrix and the observation model matrix
- Use known P wave and S wave models to constrain the matrices

Results by tweaking the solution

3 state Kalman Filter to detect P and S waves separately

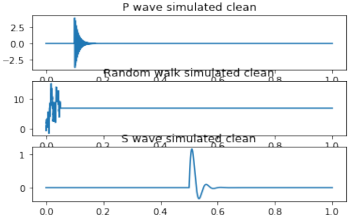

Figure 1: Simulated Signals

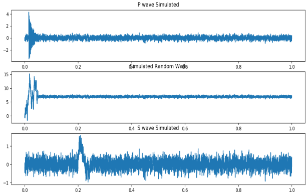

Figure 2: Signals with noise

- SNR for the simulated P wave is 1
- SNR for random walk approximated signal is 15
- SNR for the simulated S wave is 0.32

Results by tweaking the solution

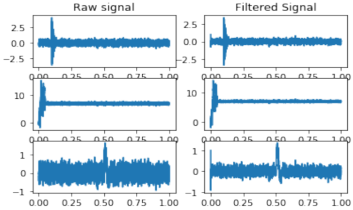

Figure 3: Filtered Signal

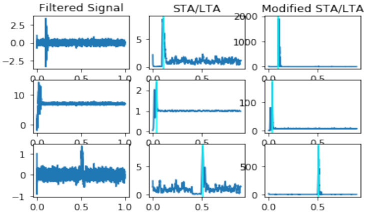

Figure 4: Signal Detection

- P wave, Random Walk and S wave time $0.1s, 0.025s, 0.5s$
- P wave, Random Walk and S wave time estimates $0.1004s, 0.0361s, 0.5113s$

Results by tweaking the solution

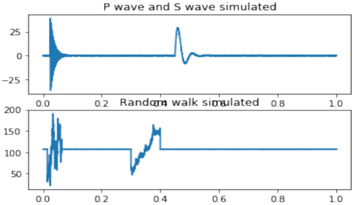

Figure 5: P and S wave signal

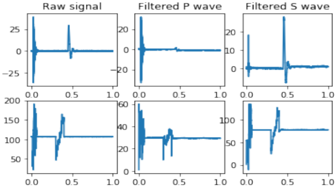

Figure 6: P and S wave filtered

- SNR for the simulated signal is 300
- SNR for random walk approximated signal is 3240

Results by tweaking the solution

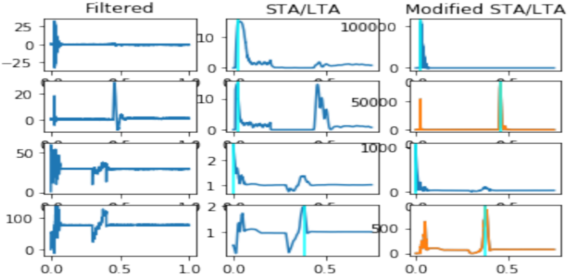

Figure 7: P and S wave signal detection

- P wave, S wave, P-random walk and S-random walk times $0.025s, 0.45s, 0.04s, 0.35s$ and estimated are
- P wave, S wave, P-random walk and S-random walk estimated times $0.0277s, 0.4584s, 0.00035s, 0.377s$

References

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