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Application of Kalman Filtering and Smoothing for Airborne Gravimetry

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Abstract. Due to the interference of noise, filtering technology is applied to achieve gravity anomaly for airborne gravimetry. Kalman filtering and smoothing are discussed and implemented for data processing of airborne gravimetry in this paper. Firstly, the algorithms of Kalman filtering and smoothing are introduced. Then, the system model for solving the gravity anomaly is established which is based on the dynamic equation and the hardware design equations. Finally, the result of Kalman filtering and smoothing would be compared with digital FIR low pass filter, and it is proved that Kalman filter and smoother could obtain more accurate result than FIR low pass filter as that the solving error of Kalman filter and smoother is improved within *1* mGal compared with the theory standard obtained by GT-1A software.

Introduction

Airborne gravimetry is a new kind of technique for surveying the gravity acceleration over the earth's surface, which is based on aircraft, including gravimeter, GPS, altimeter and attitude sensor etc.[1]. The main problem is how to obtain the gravity anomaly from measurement results of gravimeter which is disturbed by perturbing acceleration due to the dynamic measurement. Although the several parts of that could be calculated by correctional formulas, the gravity anomaly regarded as siganl is hardly to obtain according to the deletion of accurate formula[2],[3]. However, noise is in high frequency compared with low frequency gravity signal. Therefore, the noise of vertical acceleration could be eliminated by low pass filter[4]. However, transition zone is not clearly between gravity signal and noise in the spectrum of airborne gravimetry data. This adds heavy burden on filter design. Therefore, Kalman filtering, which is an effective filtering and estimation technology, is applied to process the airborne gravimetry data for higher resolution and accuracy. Kalman filtering is an optimal recursive data processing algorithm which is based on system model. Kalman filtering could contribute to band limitation which equals to low pass filtering[5][6]. In this paper, Kalman filtering and fix interval smoothing algorithms are presented for airborne gravimetry data to obtain the gravity anomaly based on the system model. Then, the results of Kalman filtering and smoothing are compared with digital FIR low pass filter which is based on window function. As a result, it could be seen that Kalman filtering and smoothing supply more accurate results than FIR low pass filter.

1. Algorithms of Kalman filter and smoother

Discrete Kalman filter. The state equation and observation equation of a linear discrete system are assumed as follows[7]:

$$X_k = \phi_{k,k-1} X_{k-1} + \Gamma_{k-1} W_{k-1} \tag{1}$$

$$Z_k = H_k X_k + V_k \tag{2}$$

In the equations (1) and (2), X_k is the state vector at time k; $\Phi_{k,k-1}$is the one-step transition matrix from time $k-1$ to time k, and Γ_k is the driving transaction matrix of system noise. Besides, Z_k is the measurement vector at time k while H_k is the measurement matrix. Moreover, V_k is the measurement noise and W_k is the system noise sequence. Assume that both of W_k and V_k are unrelated white noise or Gaussian white noise, whose means are 0. Those are [5]:

$$E\left[W_k\right]=0,\ \mathrm{Cov}\left[W_k,W_j\right]=E\left[W_k,W_j^T\right]=Q_k\delta_{kj} \tag{3}$$

$$E\left[V_k\right]=0,\ \mathrm{Cov}\left[V_k,V_j\right]=E\left[V_k,V_j^T\right]=R_k\delta_{kj} \tag{4}$$

$$\mathrm{Cov}[W_k,V_j]=E[W_k,V_j^T]=0 \tag{5}$$

At different time k, the measurement vector Z_k is applied to estimate optimal state vector X_k. In other words, optimal estimation is implemented via the corresponding state equation and statistical characteristic of noise. The algorithm steps are illustrated as follows:

$$\hat{X}_{k,k-1}=\phi_{k,k-1}\hat{X}_{k-1} \tag{6}$$

$$\hat{X}_k=\hat{X}_{k,k-1}+K_k\left[Z_k-H_k\hat{X}_{k,k-1}\right] \tag{7}$$

$$K_k=P_{k,k-1}H_k^T\left[H_kP_{k,k-1}H_k^T+R_k\right]^{-1} \tag{8}$$

$$P_{k,k-1}=\phi_{k,k-1}P_{k-1}\phi^T{}_{k,k-1}+\Gamma_{k,k-1}Q_{k-1}\Gamma^T{}_{k,k-1} \tag{9}$$

$$P_k=\left[I-K_kH_k\right]P_{k,k-1} \tag{10}$$

In the above equations, $\hat{X}_k$ is the state estimation at time k while $\hat{X}_{k,k-1}$ means the state prediction from time $k-1$ to time k. The P_k is the estimation error covariance at time k, and $P_{k,k-1}$ is the prediction error covariance from time $k-1$ to time k. Moreover, K_k is the filter gain at time k and I is the unit matrix.Once the initial state vector $\hat{X}_0$ and estimation error variance P_0 are given, based on the measurement Z_k at time k, the optimal estimation $\hat{X}_k$ is to be estimated by recursive calculation[8].

Fix interval Kalman smoother. Kalman filtering provides optimal estimation which is based on observation data in every measurement time. Therefore, the estimation is propitious to real-time processing. Generally, smoothing offer better result than filtering since smoothing is the process which utilizes the past, current and future observations to achieve optimal estimation.

Fix interval smoothing utilizes measurement values of fix interval for state estimation. Specifically, in a fix interval from time 0 to time M, a forward Kalman filter is utilized to estimate the state vectors, then a backward Kalman filter is applied from time M to 0 for a group of new estimations $(\hat{X}_{M-1/M},\hat{X}_{M-2/M},\dots\dots,\hat{X}_{0/M})$ which is based on the forward result. The algorithm is explained below:

Smoothing equation $$\hat{X}_{j-1/M}=\hat{X}_{j-1}+K_{sm}(\hat{X}_{j/M}-\hat{X}_{j/j-1}) \tag{11}$$

Smoothing gain $$K_{sm}=P_{j-1}\phi^T{}_{j,j-1}P^{-1}{}_{j-1} \tag{12}$$

Smoothing covariance $$P_{j-1/M}=P_{j-1}+K_{sm}(P_{j/M}-P_{j/j-1})K^T{}_{sm} \tag{13}$$

2. Model of gravity anomaly estimation

The dynamic equation ofairborne gravimetry is equation (14):

$$\ddot{h}=f_3+\Delta G_E-G_0(\varphi,h)-\Delta g \tag{14}$$

In equation (14), f_3 is the vertical component of accelerator readings; ΔG_E and $G_0(\phi,h)$ are Eotvos correction, normal gravity field and the free air correction respectively, which could be provided by accurate formulas. Meanwhile, $\ddot{h}$ is the acceleration of SM movement in vertical direction, namely second derivation of altitude h; the gravity anomaly is Δg.

To establish the model, there are other two equations taken into considered, altitude measurement equation (15) and equations (16) of hardware design [8]:

$$h'_{GPS} = h_{GPS} + \delta h \tag{15}$$

$$TF_3\dot{f}_3' + f_3' + KF_3 f_3' = f_{z3} + \delta f_T + \delta f, f_1' = f_{z1} + \delta f_1, f_2' = f_{z2} + \delta f_2 \tag{16}$$

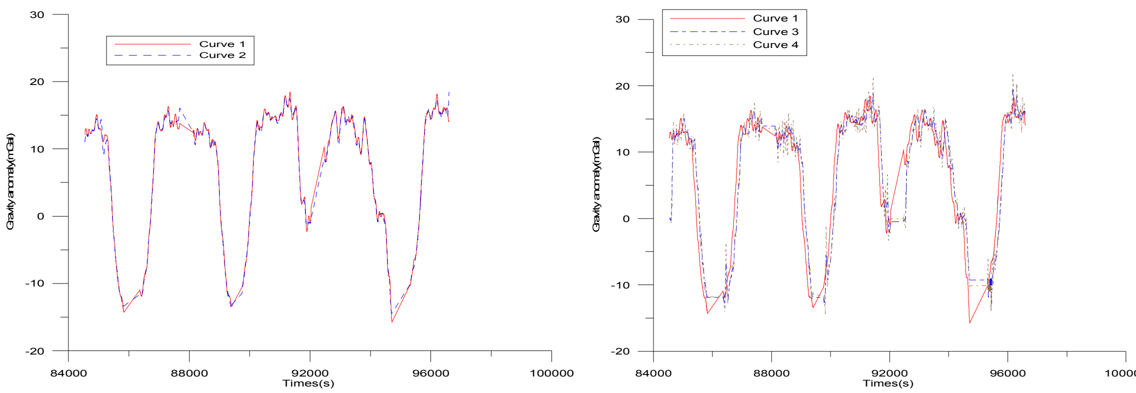
h'_{GPS} means the actual altitude of airplane; h_{GPS} is the measurement value while δh is the measurement noise; f_1' and f_2' are the horizontal measurement accelerations while f_{z1} and f_{z2} are the actual value; δf_1 and δf_2 are the measured noise of horizontal accelerator. Identically, f_3' is the vertical measurement acceleration and f_3 is the actual value. Besides, δf_T and δf are the vertical accelerator noise and system noise; TF_3 and KF_3 are the calibration parameters of the system. The state equation for Kalman filtering is developed from equation (14) and equation (16) while the measurement equation is equation (16). As a result, the model is established as follows[8]:

$$\begin{aligned} X_f &= A_f X_f + B_f q_f + C_f u_f \\ h'_f &= H_f X_f + \delta h \end{aligned} \tag{17}$$

In equations (17), X_f is the stated vector of the filter; A_f, B_f, C_f and H_f are the constant matrixes while q_f and δh are white noises.

3. Experiment results

The program of Kalman filtering and smoothing was developed in this experiment. Then, the result of the program would be compared with GT-1A software. Meanwhile, the resolved gravity anomaly of FIR low pass filter would be shown and compared as well. The design of FIR low pass filter refers to Guo's research [4].



(a) (b)

Figure 1 Curves of resolved gravity anomaly

(a) Kalman filter and smoother (curve 2) v GT-1A software (curve 1),

(b) FIR low pass filters (curve 3, 4)v GT-1A software (curve 1)

The gravity anomaly resolved by Kalman filter and smoother is exhibited in figure 1(a), which is compared with GT-1A software. It should be mentioned that there were *6* survey lines in the measurement area. In the figure, curve 1 is the gravity anomaly of GT-1A software which is regarded as theory standard. Meanwhile, curve 2 is the results of Kalman filter and smoother, and curves 3, 4 in figure 1(b) are those of low pass FIR filters. Specifically, the cutoff frequencies of FIR filters are *0.01Hz* and *0.0167Hz* (the resolution are *6* km and *3.6* km respectively), and the sampling frequencies both are *2* Hz while the orders are *400* [4]. That can be seen from the figure, compared with the FIR low pass filter, the result of Kalman filter and smoother is closer to the standard. That means Kalman filter and smoother could achieve a better result than FIR low pass filter. Besides, the error values compared with the standard is shown in table 1. As can be seen from table 1, the Kalman filter and smoother acquire satisfactory results since that the mean square error and mean error are *0.72* mGal and *0.58* mGal which both of them are less than *1* mGal. Therefore, the result of the research is comparable of GT-1A software.

Table 1 Differences between vertical acceleration of Kalman filter and smoother v theory, FIR low pass filters v theory (Unit: mGal)

Filtering methods	MSE	Mean error	Maximum error	Minimum error
Kalman filter& smoother	0.72	0.58	4.51	-2.15
FIR LP filter(f_c=0.01Hz)	3.59	2.60	10.05	-12.22
FIR LP filter (f_c=0.0167Hz)	3.78	2.81	10.36	-12.96

4. Conclusions

The theory and algorithm of Kalman filtering and smoothing were presented in this paper firstly. Then, the model of Kalman filter and smoother was modeled in this research. At last, the experimental result demonstrated that it is comparable of GT-1A software. Meanwhile, the accuracy of resolving result was enhanced substantially which is compared with traditional digital FIR low pass filter. Hence, Kalman filter and smoother is more suitable to resolve gravity anomaly for airborne gravimetry.

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