

# Application of Kalman Filter in Microseismic Data Denoising Based on Identified Signal Model

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**Abstract:** Kalman Filter has been used for microseismic data filtering based on mechanism signal model. However, the model was built with many assumptions and simplifications, which did not fully represent the characteristics of microseismic signals and affected the effect of Kalman filtering. In order to establish a more accurate mathematical model of microseismic signals and improve the effect of Kalman filtering, this paper builds an ARMA model for a typical microseismic event by identification methods. The simulation of the theoretical model and the synthetic signals shows that the identification method and model are accurate. Through the processing and analysis with the synthetic signals and the practical ground monitoring data of microseismic, the Gaussian white noise added in the microseismic signals is suppressed and the SNR is improved significantly by Kalman filtering based on identification model, which verifies the feasibility of the identification model and the filtering algorithm.

**Key Words:** Microseismic, ARMA, State Space Model, Kalman Filter

## 1. INTRODUCTION

Microseismic event signals [1] usually have characteristics of the weak energy, low SNR. Sometimes valid event signals are nearly submerged by environmental noises. The accuracy of source location is greatly affected by SNR of event signals. Therefore, microseismic data must be preprocessed by various filtering methods to improve SNR. Many filtering methods have been used for microseismic data filtering and denoising, such as particle filtering [2], polarization filtering [3], wavelet analysis [4]. Some filtering methods are model-based, such as Kalman filter [5], Weiner filter.

Baziw [6] presented a real-time Kalman Filter based on mechanism models of microseismic event signal and background noise. The background noise is modeled as a Gauss-Markov process and the microseismic wavelet is modeled as a periodic process with random walk initial amplitude. The signal model is achieved by simplification and linearization under many strict assumptions. Baziw [7] improved Kalman Filter designs of current microseismic event detection to enhance the automation of P-wave first identification.

Mechanism model must be determined according to various signals, which makes it not applicable in practice. In order to overcome this problem, an ARMA (Auto-Regressive and Moving Average Model, ARMA) model is established by identification method, such as Steiglitz-McBride iteration method. A microseismic wavelet is typically modeled as an exponentially decaying cyclic waveform. In order to apply Kalman Filter to microseismic signal filtering, the ARMA model is further converted into a state-space model [8],

which is suitable for Kalman Filter. In the process of model identification, a lot of assumptions and simplifications existed in mechanism modeling are eliminated.

The simulation results of the theoretical model and synthetic signals verify that the identification method and model are accurate. Through the processing and analysis with the synthetic signals and the practical ground monitoring data of microseismic events, satisfactory results are achieved, which verifies the feasibility of the identification model and the filtering algorithm.

## 2. ARMA MODLE

ARMA model [8] is one of the most commonly used models describing discrete stationary random signals in modern time series analysis. ARMA model can be seen as a time series model generated by a linear system under excitation of white noise.

The input-output model of a discrete system is given by the form of linear differential equation as follows.

$$\begin{aligned} y(k) + a_1 y(k-1) + \cdots + a_p y(k-p) \\ = b_1 u(k-1) + \cdots + b_q u(k-q) + e(k) \end{aligned} \quad (1)$$

The ARMA model can be simplified as a more general form as

$$A(z^{-1})y(k) = B(z^{-1})u(k) + e(k) \quad (2)$$

where  $z^{-1}$  is the backward shift operator.

Given that

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots$$

and

$$B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + \cdots$$

where the parameters  $p$  and  $q$  are the orders of the autoregressive process and moving average process,

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$a_i (i=1,2,\dots)$  and  $b_j (j=1,2,\dots)$  are the coefficients of the autoregressive process and moving average process, respectively.

Particularly, when all coefficients of the input items are set to 0, the model can be written as

$$y(k) + a_1 y(k-1) + \dots + a_p y(k-p) = e(k) \quad (3)$$

Eq. (3) is called  $p$ -order autoregressive model, i.e.,  $AR(p)$  model.

When all coefficients of the output items are set to 0, the model can be written as

$$b_1 u(k-1) + \dots + b_q u(k-q) = e(k) \quad (4)$$

Eq. (4) is called  $q$ -order moving average model, i.e.,  $MA(q)$  model.

The work of parametric modeling is to determine the orders  $p$  and  $q$  of the ARMA model, and estimate the values of coefficient arrays  $a_i (i=1,2,\dots)$  and  $b_j (j=1,2,\dots)$ .

In many cases, the values of the orders  $p$  and  $q$  are given. But there are some cases which still need to determine the orders of the model to further estimate the values of the coefficients.

After the model is determined, the next step is to estimate the parameters of the model. Similar to signal analysis, the methods of parametric modeling can be divided into two categories: time domain methods and frequency domain methods. This paper focuses on the time-domain modeling methods which assumes that the object system is an AR model or an ARMA model, and then estimate the model parameters according to the response information from the given system, or the correlation of input and output sequences.

### 3. MICROSEISMIC WAVELET MODELING

A microseismic wavelet is typically modeled as an exponentially decaying cyclic waveform as follows [9][9].

$$A(t) = A_0 e^{-h(t-t_0)} \sin[w(t-t_0)], t \geq t_0 \quad (5)$$

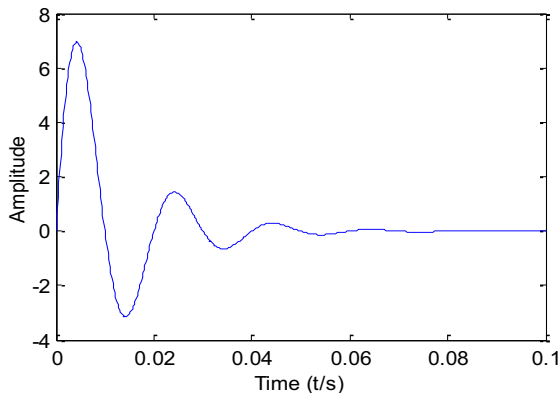
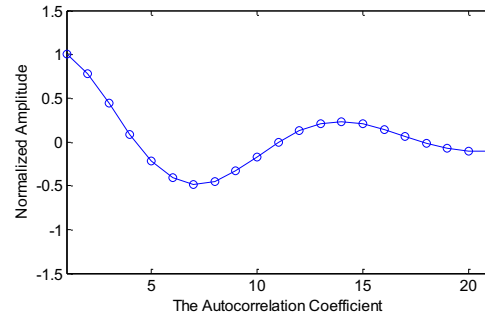


Fig 1. Numerical simulation of the synthetic microseismic wavelet.

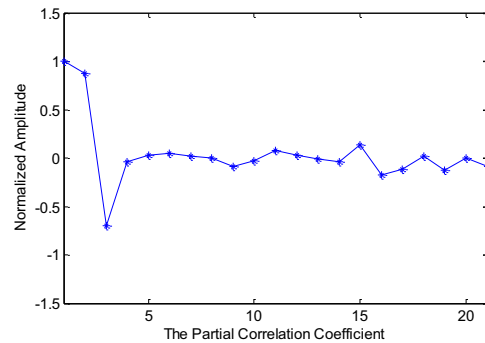
In eq.(5),  $A_0$ ,  $h$  and  $w$  are the initial amplitude, damping factor, dominant angular frequency, respectively. The parameter  $t_0$  determines initial phase. In numerical

simulation, the sampling rate of the signal is assumed to be 1000 Hz, i.e., the sampling interval is 1ms. The wavelet with previous parameters specified at the appropriate value is available (see Figure 1).

The sample data of the synthetic microseismic wavelet is analyzed by correlation analysis method [10], and the results show that the autocorrelation function of the sample data and the partial autocorrelation function of the sample data all have the characteristics of trailing. So we can conclude that the identification model of the microseismic wavelet is an ARMA model.



(a) The curve of autocorrelation coefficient



(b) The curve of partial correlation coefficient

Fig.2 The curves of autocorrelation and partial correlation coefficient.

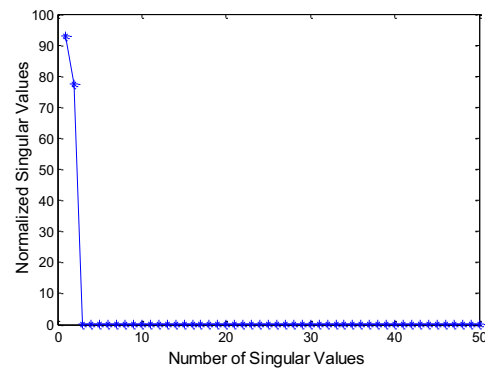


Fig.3 The normalized singular values curve.

The identification model's orders of the microseismic wavelet are determined by SVD [11] (Singular Value Decomposition, SVD). SVD is a widely used completely orthogonal decomposition method, which is implemented by constructing special matrices and obtaining the decomposed singular values. The singular values are

normalized processing and sorted in the order from large value to small value. By observing the normalized singular values curve (See Figure 3), the order of the model is determined, i.e. ARMA(2,1).

The ARMA model of the exponentially decaying cyclic signal is established by model identification method, Steiglitz-McBride iteration method, which is realized by Matlab function *armax*. The ARMA model is as follows.

$$A(q)y(t) = B(q)u(t) + e(t)$$

where

$$A(q) = 1 - 1.9648q^{-1} + 0.9689q^{-2}$$

$$B(q) = 0.6181q^{-1}$$

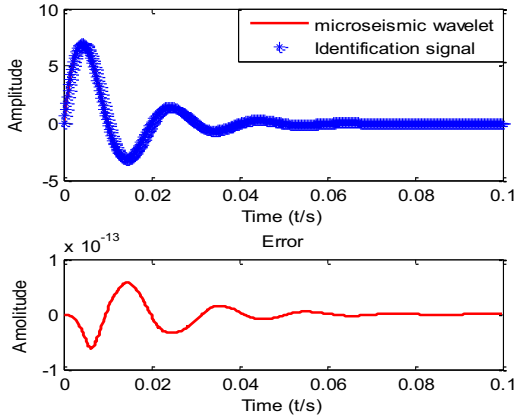


Fig 4. Identification curve of the microseismic wavelet.

#### 4. STATE-SPACE MODEL of MICROSEISMIC WAVELET

In order to apply Kalman Filter to microseismic signal filtering and verify the accuracy of the identification model, the ARMA model is converted into a state-space model [8], which is suitable for Kalman Filter. The state-space model is as follows.

$$\begin{cases} \begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{pmatrix} 1.9648 & 1 \\ -0.9689 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \begin{pmatrix} 0.6181 \\ 0 \end{pmatrix} u(t) \\ y(k) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} \end{cases} \quad (6)$$

Based on the state-space model, the microseismic wavelet added with certain Gaussian white noise is synthesized shown as Figure 5.

#### 5. KALMAN FILTER

The Kalman Filter [12] is a method for estimating a state vector  $x$  from measurement  $z$ , which is applicable for systems that can be defined by a first-order differential equation in  $x$  and a linear equation in  $z$ . The state vector may be corrupted by a noised process vector  $w$  and the measurement vector is corrupted by a noised observation vector  $z$ .

The state-space model of a discrete system is given by

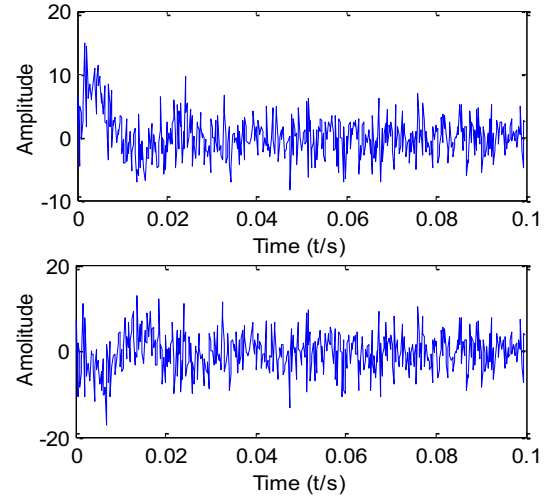
$$x_k = Ax_{k-1} + Bu_k + w_k \quad (7)$$

$$z_k = Hx_k + v_k \quad (8)$$

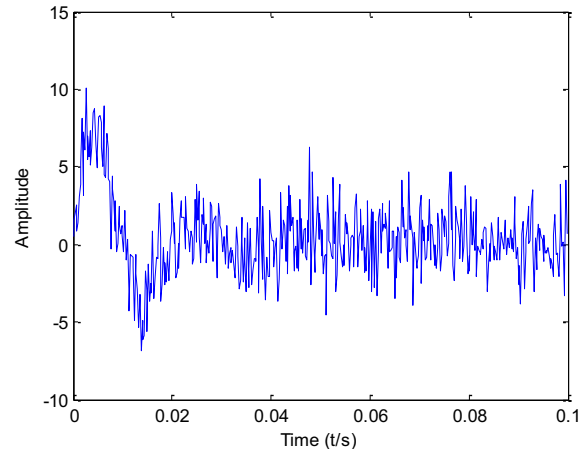
where  $x$  is an  $n$ -vector,  $w$  is a  $p$ -vector.  $z$  and  $v$  are  $m$ -vectors. The matrixes  $A$ ,  $B$  and  $H$  are coefficients of the model. The subscript  $k$  denotes time. The random processes  $w$  and  $v$  are independent white noise processes and are assumed to be zero mean, so that

$$w_k \sim \quad , \quad \sim$$

where  $Q$  and  $R$  are the covariance matrices of the process noise  $w$  and the observation noise  $v$ , respectively.



(a) State components added with Gaussian white noise



(b) Observations added with Gaussian white noise

Fig 5. The microseismic wavelet added with certain Gaussian white noise.

Kalman filtering is a method of state estimation which can ensure a minimum mean square error. The filtering algorithm is briefly summarized as follows [12].

- 1) Estimate the system state of the next time step

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1} \quad (9)$$

- 2) Predict the next estimation error covariance

$$P_k^- = AP_{k-1}A^T + Q \quad (10)$$

- 3) Compute the Kalman gain matrix of the filter

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \quad (11)$$

- 4) Update the state estimation with an current observation

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H \hat{x}_k^-) \quad (12)$$

- 5) Update the estimation error covariance

$$P_k = (I - K_k H) P_k^- \quad (13)$$

where  $I$  is the identity matrix in eq. (13).

With initial conditions:

$$\hat{x}_0 = E[x_0] \quad (14)$$

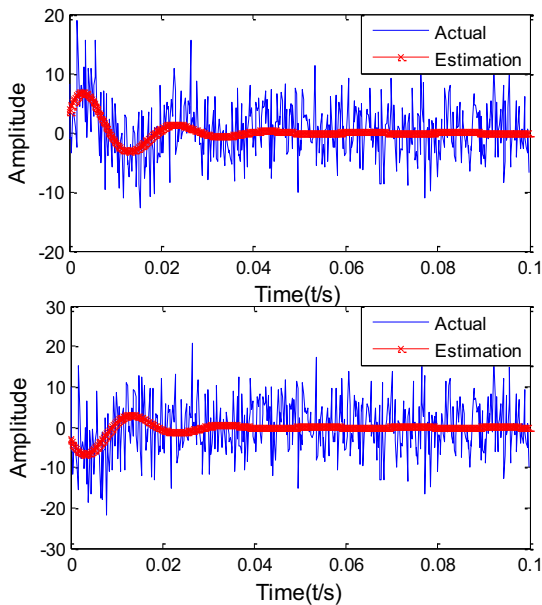
$$P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \quad (15)$$

The steps of the Kalman Filter are presented as follows.

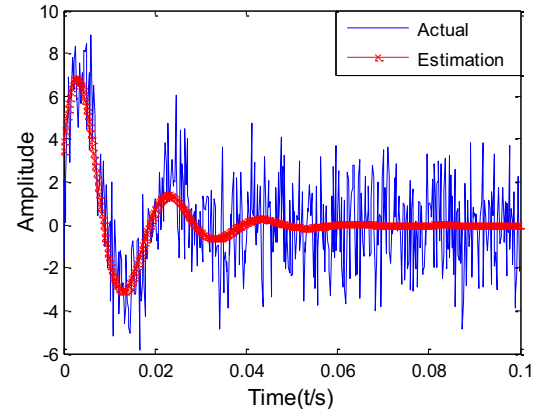
1. Specify initial conditions and initialize the system state and error covariance matrix.
2. The time update process: calculating a priori state estimation  $\hat{x}_k^-$  and a priori error covariance  $P_k^-$ .
3. The state update process: computing the Kalman gain matrix  $K_k$ , and updating system state  $\hat{x}_k$  with the current observation and adjusting error covariance  $P_k$ .
4. If the algorithm does not end, then go back to the second step until the filtering algorithms is end, that is, the last observation is reached.

## 6. PROCESSING RESULTS

Based on the research of identification modeling and Kalman filtering algorithm, the microseismic wavelet added with certain Gaussian white noise was processed by Kalman Filter, and satisfactory results are achieved. In Figure 6, the Gaussian white noise added in microseismic wavelet has been significantly removed and the effect of Kalman filtering is obvious, which verifies the feasibility of the identification model and the Kalman filtering algorithm.



(a) The filtering results of state components



(b) The filtering results of observations

Fig 6. The filtering results of the microseismic wavelet added with certain Gaussian white noise.

In order to further verify the practical application effect of the identification model and Kalman filtering algorithm, the microseismic events monitoring data of a gas well in Sichuan (see Figure 8) has been processed. Seen from the processing results of single channel microseismic monitoring data (see Figure 7) and multi-channel microseismic monitoring data (see Figure 9), the noise added in the microseismic signals is suppressed and the SNR is improved significantly by Kalman filtering.

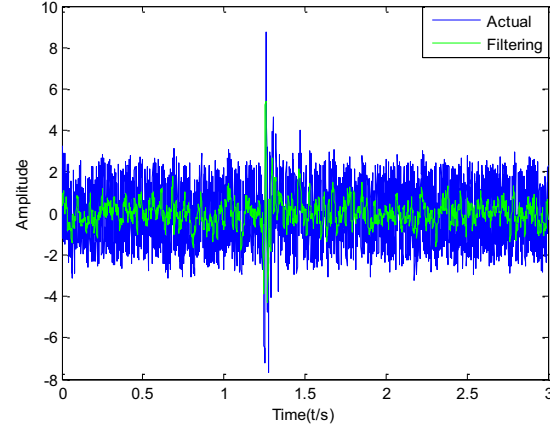


Fig 7. The filtering result of single channel microseismic monitoring data.

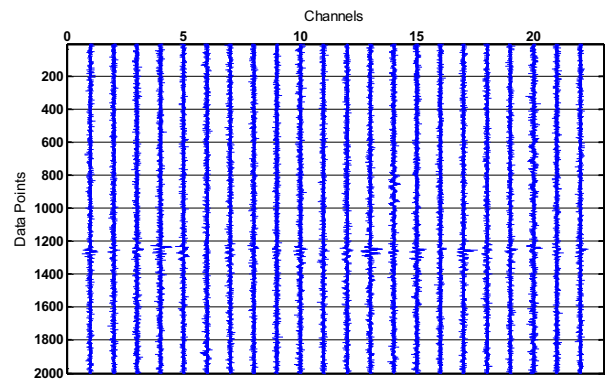


Fig 8. Multi-channel microseismic signals of a gas well in Sichuan.

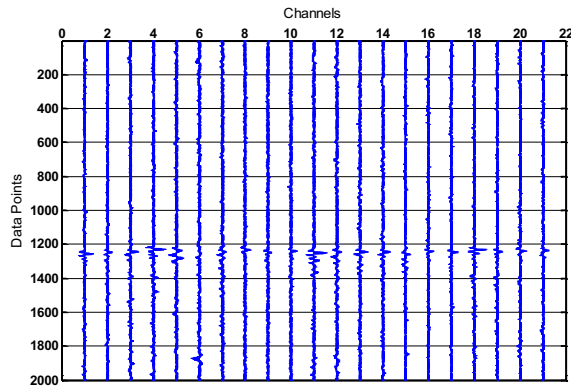


Fig 9. The filtering result of multi-channel microseismic signals of a gas well in Sichuan.

## 7. CONCLUSIONS

In this paper, a microseismic wavelet is typically modeled as an exponentially decaying cyclic waveform and a more accurate ARMA model, which eliminates a lot of assumptions and simplifications existed in mechanism modeling, is established by using Steiglitz-McBride iteration method. In order to apply Kalman Filter, the ARMA model is further converted into a state-space model which is suitable for Kalman Filter. The simulation results of the theoretical model and synthetic signals show that the identification method and model are accurate. Through dealing with the synthetic signals and the microseismic events monitoring data, the Gaussian white noise added in the microseismic wavelets has been suppressed and the SNR is improved significantly by Kalman Filter based on identification model, which verifies the feasibility of the identification model and the filtering algorithm.

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