RM 294 – Optimization I
Project 1
Advertising | Group 4

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Objective

The purpose for this report is to deliver the best possible solution for how much the marketing department should be investing in each of the various media channels to provide the firm with the best possible return on investment (ROI) while working within the set of constraints the marketing department has available and an overall budget of $10 million. This report will go through how we formulated the problem and solved for the optimal solution and then discuss a sensitivity analysis for the solution.

Problem Formulation

Objective Function:

Since the objective is to maximize the ROI of marketing investments our objective function can be written as the sum of all of the ROI estimates for a channel multiplied by the amount invested in that medium.

ROI = Σ (Estimated ROI of medium) * (amount allocated to the medium)

Constraints:

In addition to the budget of $10 million the project is bound by a few other constraints imposed by management including:

- The amount invested in print and TV should be no more than the amount spent on Facebook and Email.
- The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords.
- For each platform, the amount invested should be no more than $3M.

Optimal Allocation Analysis

ROI 1:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	3.1%	4.9%	2.4%	3.9%	1.6%	2.4%	4.6%	2.6%	3.3%	4.4%

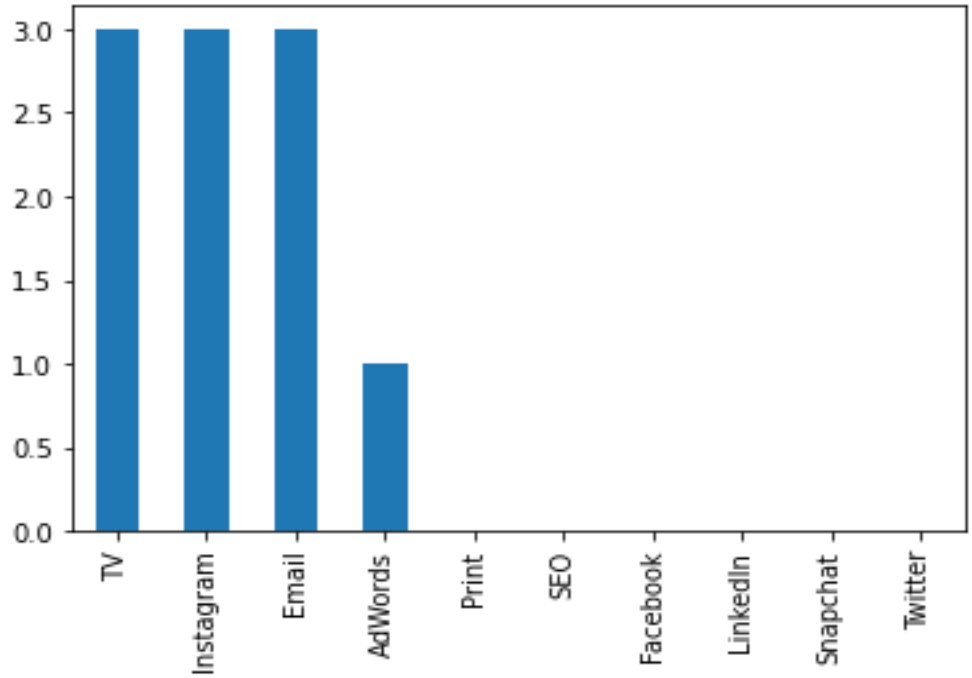
ROI 2:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	4.9%	2.3%	2.4%	3.9%	4.4%	4.6%	2.6%	1.9%	3.7%	2.6%

Allocation based on ROI 1:

```
The optimal revenue we can gain is 0.4560 million

Print          0.0
TV             3.0
SEO            0.0
AdWords        1.0
Facebook       0.0
LinkedIn       0.0
Instagram      3.0
Snapchat       0.0
Twitter        0.0
Email          3.0
```



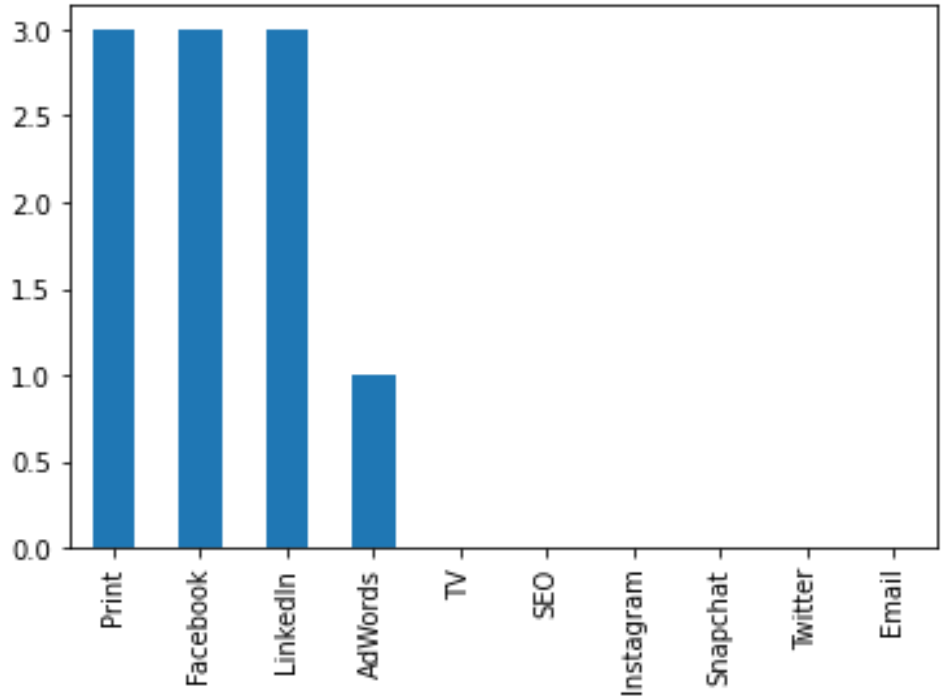
The optimal revenue we gain from the expected ROI is 0.456 million USD. For the optimal revenue, we need to invest 3M in TV, 1M in Adwords, 3M in Instagram and 3M in Email.

To get the optimal revenue, we put in the expected ROI as objective function. For the constraint of channel allocation limit, we constructed an identity matrix to put in the constraint equations. Also, we constructed the constraint for the total amount and social medias.

Allocation based on ROI 2:

```
The optimal revenue we can gain is 0.4560 million

Print        3.0
TV           0.0
SEO          0.0
AdWords      1.0
Facebook     3.0
LinkedIn     3.0
Instagram    0.0
Snapchat     0.0
Twitter      0.0
Email        0.0
```



By running the model again with different ROI, we get the same optimal revenue but with different allocation this time.

Allocation comparison

No, they are not the same. Both have the same investments into Facebook, AdWords, and SEO but differ on the last allocation. The first allocation places the investment in email, while the second places it in LinkedIn advertising.

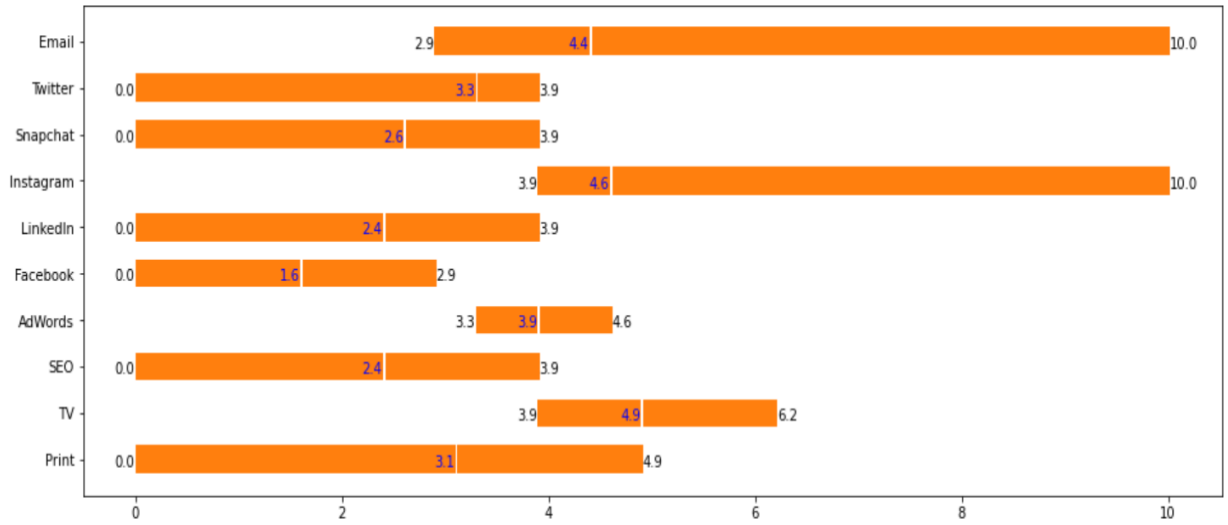
Assuming the first ROI data is correct, and we use the second allocations - The return is $0.204 million lower than what it would be using the optimal solution.

Assuming the second ROI data is correct, and we use the first allocations - The return is $0.192 million lower than what it would be using the optimal solution.

I think the third constraint can be useful in situations where you have never tested the medium of advertising because it can possibly minimize loss if the new medium turns out to be a flop. However, it could potentially cause much more loss of return by imposing a rule on a medium that has a predictable, reliable, and high ROI.

Sensitivity Analysis

Sensitivity analysis shows that how much ROI values can vary keeping the other values constant.



The ROI column represents the return on investment given. The low column represents the lowest value possible for any given ROI that keeps the other investment values constant (according to our optimization). The high column represents the highest value possible for any given ROI that keeps the other investment values constant (according to our optimization). The values of 0 and 10 represent that there is no value that the ROI can be decreased and increased, respectively, that will produce a change in how investment is spent.

Optimal allocation for each month

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Total
January	0.0	3.0	0.0	1.000000	0.0	0.0	3.0	0.0	0.000000	3.0	10.000000
February	0.0	3.0	0.0	1.180000	0.0	0.0	3.0	0.0	0.000000	3.0	10.180000
March	0.0	3.0	0.0	1.353920	0.0	0.0	3.0	0.0	0.000000	3.0	10.353920
April	0.0	3.0	0.0	1.505381	0.0	0.0	3.0	0.0	0.010763	3.0	10.516144
May	0.0	3.0	0.0	1.564986	0.0	0.0	3.0	0.0	0.129971	3.0	10.694957
June	0.0	3.0	0.0	1.629829	0.0	0.0	3.0	0.0	0.259657	3.0	10.889486
July	0.0	3.0	0.0	1.699709	0.0	0.0	3.0	0.0	0.399418	3.0	11.099126
August	0.0	3.0	0.0	1.773336	0.0	0.0	3.0	0.0	0.546672	3.0	11.320008
September	0.0	3.0	0.0	1.845758	0.0	0.0	3.0	0.0	0.691517	3.0	11.537275
October	0.0	3.0	0.0	1.908289	0.0	0.0	3.0	0.0	0.816578	3.0	11.724866
November	0.0	3.0	0.0	1.973321	0.0	0.0	3.0	0.0	0.946643	3.0	11.919964
December	0.0	3.0	0.0	2.049697	0.0	0.0	3.0	0.0	1.099393	3.0	12.149090

We are getting a stable allocation.

We have calculated allocation using ROI 1. Based on the allocation received the profit is calculated using the ROI matrix given in the problem. Below is the code snippet:

```
profit = Adv_x4.x @ np.array(roi_mat.loc[i][0:]) / 100
budget += profit*0.5
```

Adv_x4.x: allocation calculated using ROI 1
roi_mat: matrix containing each month actual ROI

The formula iterates over each month actual ROI value and takes a dot product with the allocation.

Conclusion

If it is not a stable allocation from month to month, we can create a new matrix with optimal results from last month. For each iterative run of the optimal result from the second month, we can add a constraint for the allocation.
For example, if the optimal result for the previous month is
[opt_result_Jan1, opt_result_Jan2, opt_result_Jan3, opt_result_Jan4, ...].
Then for the constraint for the current month, we would just write the left-hand matrix as an identity matrix.
The senses would be '<'.

The right-hand side would be
[opt_result_Jan1+1, opt_result_Jan2+1, opt_result_Jan3+1, opt_result_Jan4+1, ...]
This is very similar approach to the constraint that no month can have greater than $3 million in allocation except this set of constraints is based on the previous month's allocation.