ROBOTIX
TECHNOLOGY ROBOTIX SOCIETY

Time: 3 hour | *Term* | Maximum Marks: 60

1 Instructions

1. For a surface $\vec{r} = \vec{r}(u\cos v, u\sin v, f(u))$. Write down the first fundamental form of the surface. Show that the parametric curves are orthogonal. [06]

2. Prove that necessary conditions for the curve $u = u(t), v = v(t)$ on a surface $\vec{(r)} = \vec{(r)}(u, v)$ to be geodesic is that
$$U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} \tag{1}$$
where
$$U = \frac{d}{dt}\Big(\frac{\partial T}{\partial \dot{u}}\Big) - \frac{\partial T}{\partial u} = \frac{1}{2T}\frac{dT}{dt}\frac{\partial T}{\partial \dot{u}}$$
$$V = \frac{d}{dt}\Big(\frac{\partial T}{\partial \dot{v}}\Big) - \frac{\partial T}{\partial v} = \frac{1}{2T}\frac{dT}{dt}\frac{\partial T}{\partial \dot{v}}$$
[10]

3. For the curve
$$x = a(3u - u^3), \qquad y = 3au^2, \qquad z = a(3u + u^3)$$
show that
$$\tau = k = \frac{1}{3a(1 + u^2)^2}$$
[8]

4. A curve is uniquely determined except as the position in space, when its curvature and torsion are given functions of its arc length. [8]

5. Show that there exists an infinite family of involutes for a gives curve. [8]

6. Give short answers of the following questions.

    1. Define Helicoids?
    2. Define spherical indicatrix?
    3. Define the intrinsic equation?
    4. Write the statement of existence theorem for space curve?
    5. The normal curvature k_n is equal to the what?
    6. Prove that $L = -n_1 \cdot r_1$ and $N = -n_2 \cdot r_2$?
    7. Define the geodesic?
    8. Write down the equation of tangent plane?
    9. If equation of the circle is $x^2 + y^2 = a^2$ then the parametric equations of circles are_________?