Design Proposal: Mach Zehnder Interferometer

Abstract:

This report presents the design analysis of a Mach-Zehnder Interferometer (MZI) with different arm lengths (delay lengths) to study their impact on free spectral range (FSR) and wavelength group index. The MZI transfer function was analyzed for different unbalanced interferometers. The results of the simulations are compared against the results from the experiment.

1. Introduction:

The Mach-Zehnder Interferometer (MZI) is an optical device commonly used to precisely measure and control phase differences between two light paths. It operates by splitting a coherent light beam into two separate paths using a beam splitter. Each path takes a different route, and when the beams are recombined, they interfere with each other. The resulting interference pattern depends on the phase difference between the paths, which can be affected by changes in path length, refractive index, or the presence of a material in one of the arms [1]. In this study, three different waveguide widths—200 nm, 500 nm, and 800 nm were tested under TE polarization, and their optical characteristics were analyzed. Mach-Zehnder Interferometers (MZI) were tested with different path lengths, Montecarlo simulations and corner analysis were performed to realise the fabrication variabilities, the simulated was compared against the experimental results.

2. Theory

The optical properties of a silicon photonic waveguide are characterized by its effective index (n_{eff}) and group index (n_g). The effective index describes the phase velocity of light in the waveguide and depends on the waveguide geometry, material, and operating wavelength. For a given geometry (e.g., 220 nm height and 500 nm width), n_{eff} as a function of wavelength (λ) can be obtained via numerical simulation and fitted using a polynomial compact model [2]:

$$n_{eff}(\lambda) = a_0 + a_1(\lambda - \lambda_0) + a_2(\lambda - \lambda_0)^2$$

where a_0, a_1, and a_2 are fitting coefficients, and λ_0 is a reference wavelength (e.g., 1.55 μm).

The group index n_g quantifies the propagation of optical pulses and is given by:

$$n_g(\lambda) = n_{eff}(\lambda) - \lambda\, (d\, n_{eff} / d\lambda)$$

This parameter is crucial for determining the spectral response of interferometric devices.

A Mach-Zehnder Interferometer (MZI) consists of two arms of (possibly) different lengths, connected by input and output splitters. When a coherent optical signal enters the MZI, it is split into two paths, accumulates a relative phase shift, and then recombines, producing interference at the output.

For an unbalanced MZI (with a path length difference $\Delta L = L_2 - L_1$), the normalized output intensity as a function of wavelength is:

$$I_{out}(\lambda) = (1/2)\, I_{in}\, [1 + \cos(2\pi\, n_{eff}\, \Delta L / \lambda)]$$

The free spectral range (FSR), which is the wavelength spacing between adjacent transmission maxima, is given by:

$$FSR(\lambda) = \lambda^2 / (n_g(\lambda)\ \Delta L)$$

This relationship allows extraction of the group index from measured or simulated transmission spectra:

$$n_g(\lambda) = \lambda^2 / (FSR(\lambda)\ \Delta L)$$

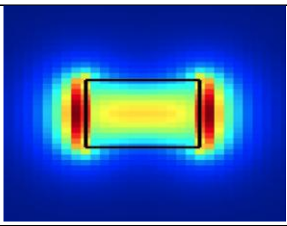
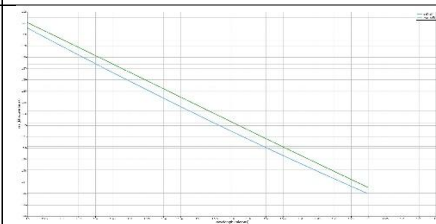
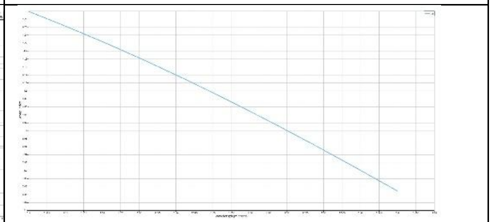
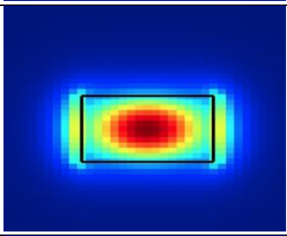
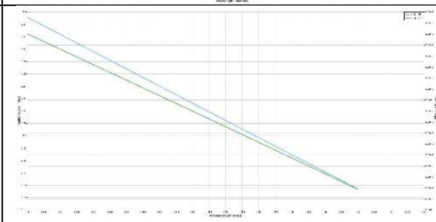
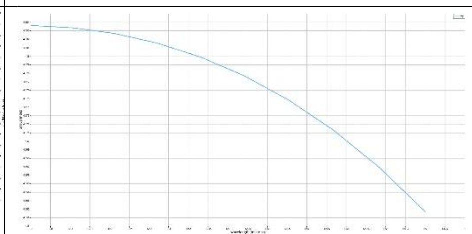
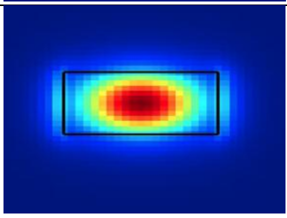
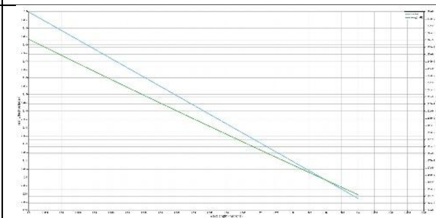
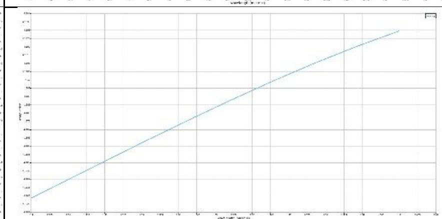
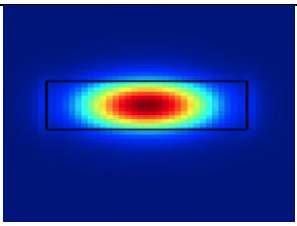
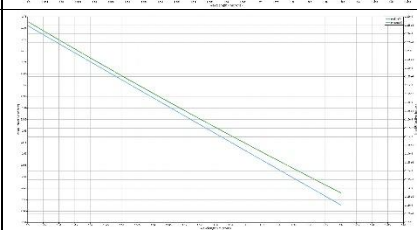
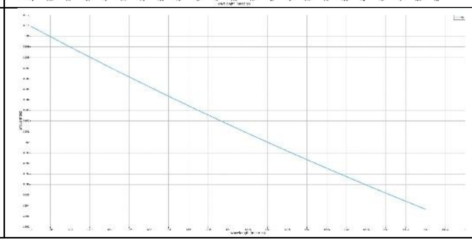
By designing MZIs with different arm length differences (ΔL), one can study how the FSR and group index depend on the waveguide and device parameters.

3. Modelling and Simulation

3.1 Waveguide Geometry and Mode Analysis

The chosen waveguide geometry is a silicon strip waveguide with a height of 220 nm. Waveguides with widths of 200 nm, 300 nm, 400 nm, 500 nm, and 800 nm were studied. The simulated mode profiles for the fundamental transverse electric (TE) mode were obtained using Lumerical MODE Solutions. A wavelength sweep from 1500 nm to 1600 nm was performed to extract the effective index (n_{eff}) and group index (n_g) of the fundamental TE mode. The results are shown in Figure1.

Table 1: Simulated effective index (n_eff) and group index (n_g) versus wavelength for the fundamental TE mode.

Waveguide width (nm)	Te polarization	Effective index	Group index
300			
400			
500			
800			

The effective index data was fitted to a second-order polynomial compact model:

$n_{eff}(\lambda) = a_0 + a_1(\lambda - \lambda_0) + a_2(\lambda - \lambda_0)^2$

where a_0, a_1, and a_2 are fitting coefficients, and λ_0 is the reference wavelength (typically 1.55 μm).

3.2 Mach-Zehnder Interferometer Simulation using K-layout

Several Mach-Zehnder Interferometer (MZI) circuits with varying path length differences (ΔL) were simulated to study the effect on the free spectral range (FSR). The MZI transfer function as a function of wavelength is given by:

$$I_{out}(\lambda) = (1/2) * I_{in} * [1 + \cos(2\pi * n_{eff} * \Delta L / \lambda)]$$

The FSR for each design is calculated using:

$$FSR(\lambda) = \lambda^2 / (n_g(\lambda) * \Delta L)$$

Conversely, the group index can be extracted from the FSR using:

$$n_g(\lambda) = \lambda^2 / (FSR(\lambda) * \Delta L)$$

3.3 Parameter Sweep and Results

A set of MZI designs with different path length imbalances (ΔL) was simulated. Table 2 summarizes the parameter variations and the corresponding simulated FSR values and group index values at 1550nm.

ΔL(μm)	FSR	n_g
500	1.143E-9	4.150
300	1.903E-9	4.208
200	2.885E-9	4.164
100	5.674E-9	4.234
50	1.171E-8	4.103

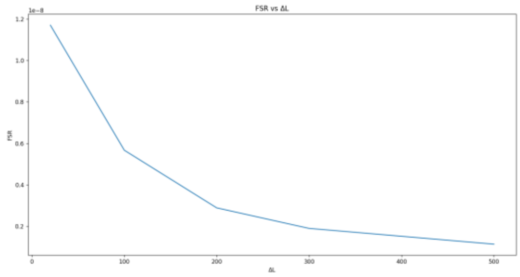


Figure 1: Free spectral range vs path length difference

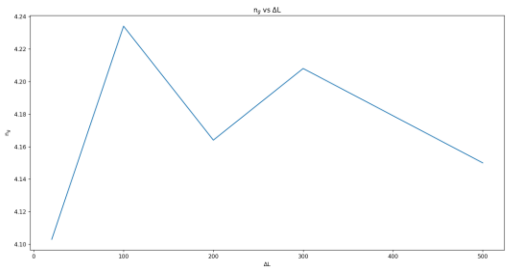


Figure 2: Group index vs path length difference

The design of the MZIs was implemented using KLayout, leveraging the EBeam Process Design Kit (PDK) to ensure compatibility with silicon photonics fabrication processes. SiEPIC – Ebeam library was used to access essential photonic components. Design verification in KLayout is performed using the "Functional Layout Check" tool from the SiEPIC plugin, which identifies layout errors and ensures compliance with photonic design rules.

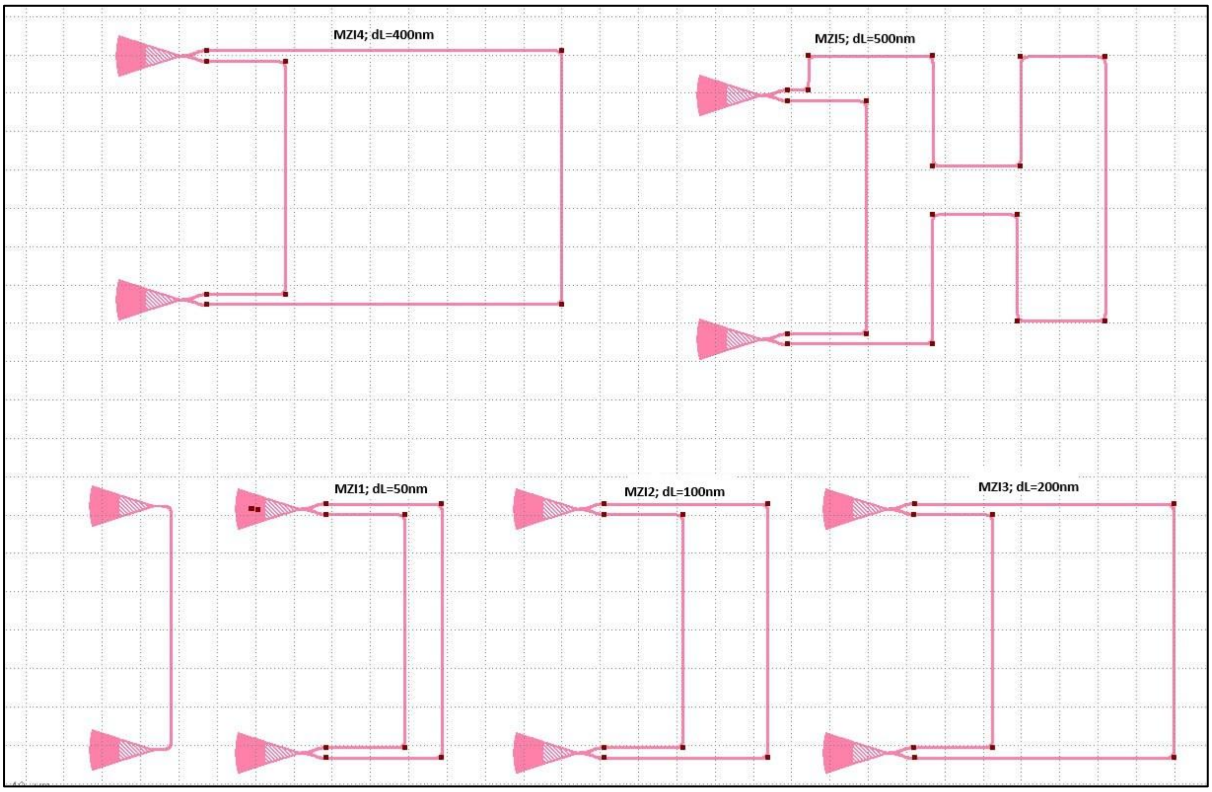


Figure 3: MZI with different path length differences

4. Fabrication and variability

Electron Beam Lithography (EBL) is a high-resolution nanofabrication technique that utilizes a focused beam of electrons to define extremely fine patterns on a substrate coated with an electron-sensitive resist. The process begins with the generation of a highly focused electron beam by an electron gun. This beam is shaped and directed using electron-optical components such as lenses and apertures. The scanning of the electron beam across the resist-coated substrate is controlled with high precision by digital-to-analog converters (DACs) and deflection systems, following patterns specified by computer-aided design (CAD) software.

Hydrogen silsesquioxane (HSQ) is used as the resist. HSQ is a negative-tone resist, meaning that the regions exposed to the electron beam become insoluble in the developer, while the unexposed regions are removed during development. After exposure, the substrate is immersed in a developer solution to selectively remove the unexposed resist. The resulting resist pattern serves as a mask for subsequent pattern transfer, typically achieved through an etching process. Finally, the remaining resist is stripped away, leaving the desired nanostructure on the substrate.

This approach enables the realization of complex and high-resolution features, making EBL with HSQ particularly suitable for advanced applications such as silicon photonics and other nanoscale device fabrication [3].

4.1 Corner Analysis and Monte Carlo simulations

Corner analysis was performed on the waveguide strip to evaluate variations in circuit performance resulting from fabrication-induced parameter deviations. Corner analysis is essential because small variations in waveguide dimensions, like width and thickness, can significantly affect effective index and group index, which in turn influence device performance. The figures below illustrate the variation of the effective index (η_{eff}) and the group index (η_{g}) across a wavelength range of 1500 nm to 1600 nm for different combinations of waveguide heights and widths. The simulations indicate that the effective index varies between 2.355 and 2.506, while the group index varies between 4.125 and 4.245 at the central wavelength of 1550 nm.

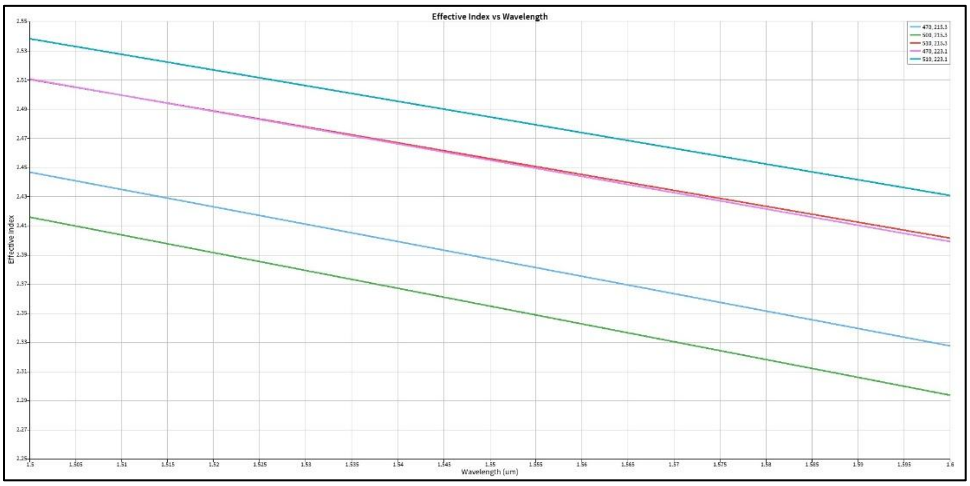


Figure 4: Effective index - Corner analysis for a strip waveguide

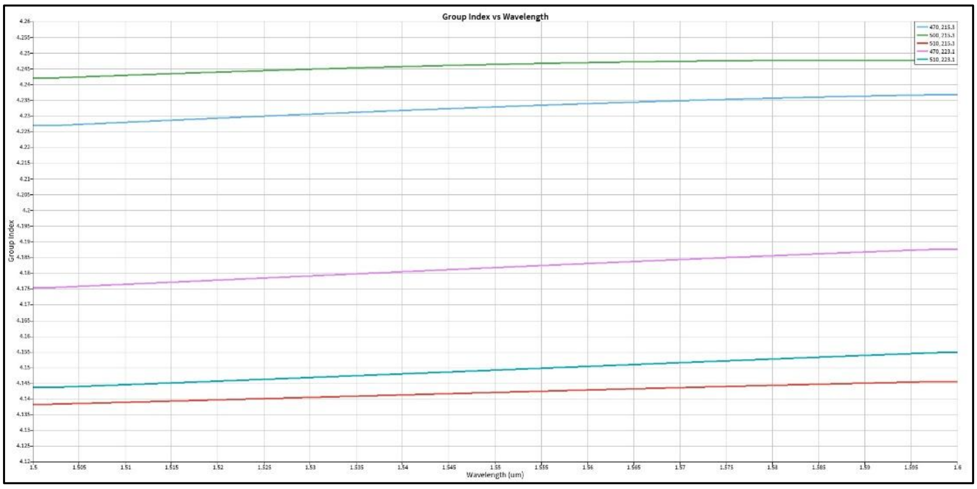


Figure 5: Group index - Corner analysis for a strip waveguide

4.2 Monte-Carlo simulation

A Monte Carlo simulation was performed on a Mach-Zehnder Interferometer (MZI) to assess the impact of fabrication-induced variations on its spectral response. The simulation incorporated random variations in key device parameters, and the resulting ensemble of spectra demonstrates the expected spread in the transmission characteristics. Analysis of the central wavelength histogram reveals a distribution of resonance positions, indicating the degree of wavelength shift due to process fluctuations. The free spectral range (FSR) histogram shows the statistical variation in FSR values, while the gain histogram quantifies the spread in insertion loss across the simulated devices. Monto-carlo simulation for a MZI with a path length difference of 50 nm.

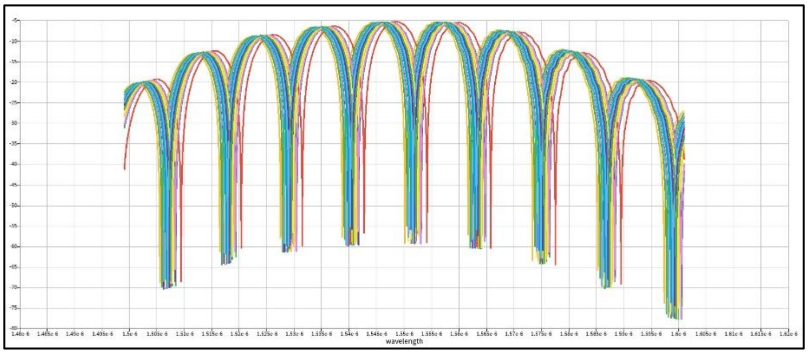


Figure 6: Montecarlo simulation for a MZI with path length difference of 50nm

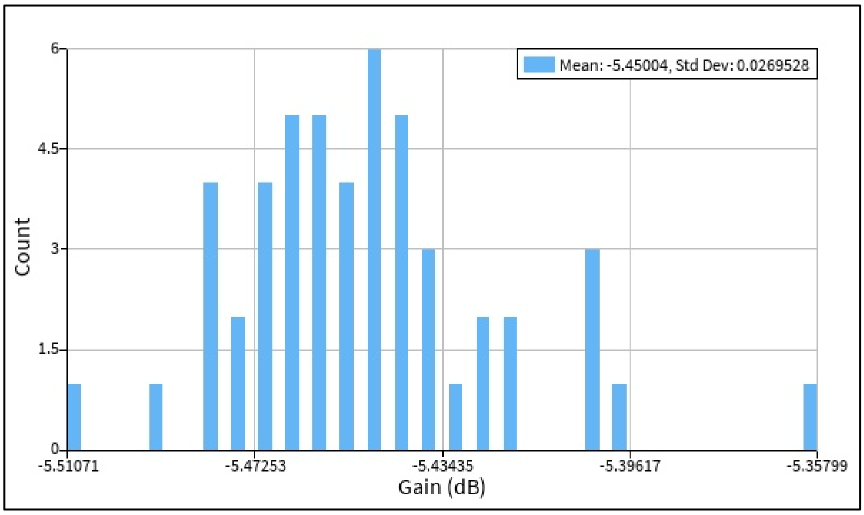


Figure 7: Histogram- Gain (in dB) for a MZI with path length difference of 50nm

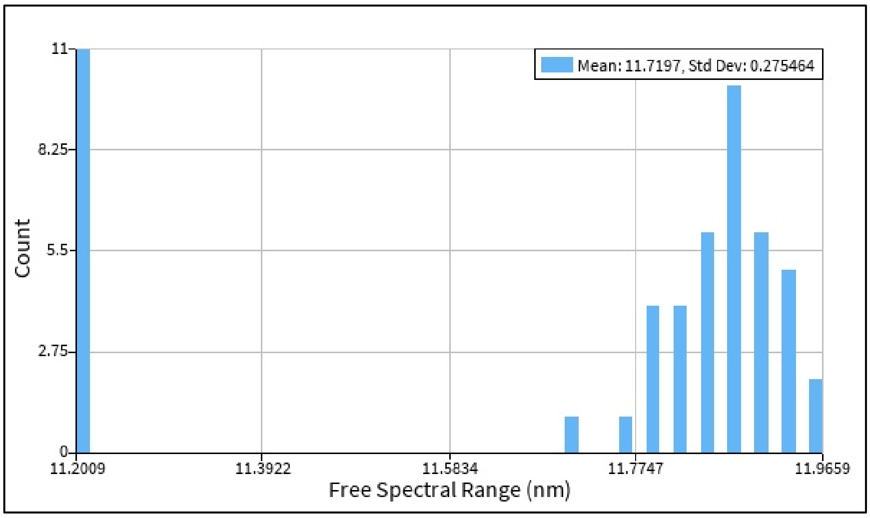


Figure 8: Histogram- FSR for a MZI with a path length difference of 50nm

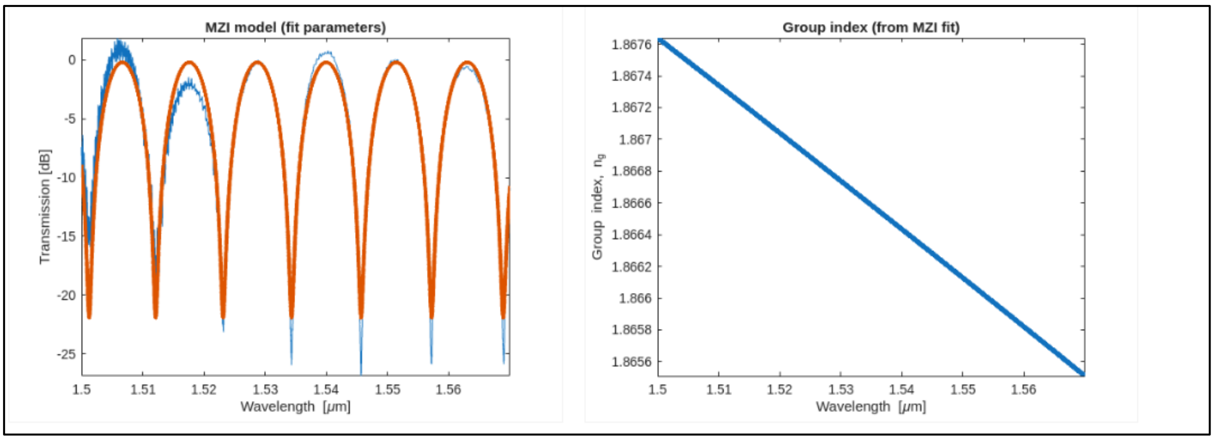


Figure 9: Response fit and group index of measured data

Table 2: Comparison of simulated and experimental data.

Path length difference	ΔL = 50nm	ΔL = 100nm	ΔL = 200nm	ΔL = 300nm	ΔL = 500nm
FSR simulated	11.719E-9	5.793E-9	2.909E-9	1.988E-9	1.127E-9
FSR experimental	11.325E-9	5.670E-9	2.855E-9	1.905E-9	1.130E-9
n_g simulated	4.100	4.220	4.129	4.028	4.263
n_g experimental	4.1611	4.1556	4.1265	4.1229	4.1703

5. Conclusion

This report has presented a comprehensive analysis of Mach-Zehnder Interferometers (MZIs) and silicon strip waveguides, focusing on the impact of geometric and fabrication-induced variations on device and circuit performance. Through both corner analysis and Monte Carlo simulations, the study demonstrated that even small deviations in waveguide width and thickness can lead to measurable changes in effective index, group index, and spectral response. The simulations revealed that the effective index varies between 2.355 and 2.506, and the group index between 4.125 and 4.245 at 1550 nm, highlighting the sensitivity of photonic devices to process variations. Monte Carlo analysis further illustrated the statistical spread in resonance wavelength, FSR, and insertion loss, emphasizing the importance of considering variability for robust photonic circuit design.

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Reference:

[1] Lukas Chrostowski, Michael Hochberg, chapter 12 in "Silicon Photonics Design: From Devices to Systems", Cambridge University Press, 2015.

[2] B. E Little, S. T Chu, H. A Haus, et al. Microring resonator channel dropping filters, IEEE Xplore, 1997.

[3] R. J. Bojko, J. Li, L. He, T. Baehr-Jones, M. Hochberg, and Y. Aida, "Electron beam lithography writing strategies for low loss, high confinement silicon optical waveguides," J. Vacuum Sci. Technol. 2011