

Module-3 Individual Task-3

Bayes' Theorem in Real Life (Medical Testing Example)

In real life, medical tests are used to detect diseases. Many people believe that if a test result is positive, it means they definitely have the disease. But in reality, this is not always true.

Bayes' Theorem helps us calculate the actual probability of having a disease after getting a positive test result.

Step 1: Real-World Scenario

Let us imagine a disease that is rare in the population.

Only 1% of people have this disease.

So, probability of having disease = 0.01

Now assume there is a medical test for this disease:

- If a person has the disease, the test gives positive result 99% of the time.
(This is called sensitivity)
- If a person does NOT have the disease, the test gives negative result 95% of the time.

That means 5% false positive results.

Now suppose a person takes the test and the result is positive.

Question: What is the probability that the person actually has the disease?

This is where Bayes' theorem is used.

Step 2: Bayes' Theorem Formula

Bayes' theorem formula is:

$$P(D|T) = P(T|D) \cdot P(D) / P(T)$$

Where:

$P(D|T)$ = Probability of disease after positive test

$P(T|D)$ = Probability test is positive if disease exists

$P(D)$ = Probability of disease in population

$P(T)$ = Total probability of getting a positive test

Step 3: Calculate Total Positive Test Probability

We know

- $P(D) = 0.01$
- $P(T|D) = 0.99$
- $P(T|\neg D) = 0.05$ (false positive rate)
- $P(\neg D) = 0.99$

Now calculate probability of positive test:

$$P(T) = P(T|D) P(D) + P(T|\neg D) P(\neg D)$$

$$P(T) = (0.99 \times 0.01) + (0.05 \times 0.99)$$

$$P(T) = 0.0099 + 0.0495$$

$$P(T) = 0.0594$$

Step 4: Apply Bayes' Theorem

$$P(D|T) = 0.99 \times 0.01 / 0.0594$$

$$P(D|T) = 0.0099 / 0.0594$$

$$P(D|T) \approx 0.1666$$

$$P(D|T) \approx 16.6\%$$

Step 5: Interpretation of Result

Even after testing positive, the probability that the person actually has the disease is only 16.6%.

This looks surprising because the test is 99% accurate.

But the disease is very rare, so many positive results are false positives.

This is why doctors usually ask for second confirmation tests before final diagnosis.

Conclusion

Bayes' Theorem helps us make better decisions using probability.

In medical testing, it shows that:

- A positive result does NOT always mean the person has the disease.
- We must consider how common the disease is in the population.
- It helps doctors avoid panic and make smarter medical decisions.

This is a simple real-life application of Bayes' Theorem.