A brief introduction to quantum computing

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Abstract

Quantum computing is computing using quantum-mechanical phenomena, such as quantum superposition and entanglement. Quantum computing is a chance a solving problems that are considered unsolvable classical computers. Quantum computing is a relatively new and complicated field in this paper I provide a brief overview of quantum computing systems.

Keywords: Quantum Computing

1. Introduction

The pace at which computer systems evolve is overwhelming. From cell phones to distributed systems computers the evolution of computers happens to be unnoticed. As time goes on classical computers will reach an end in potential, according to Moore's law the number of transistors per square inch on integrated circuits doubled every year since their invention. But what if we can not go any smaller in transistors. Quantum computers perform operations on elections which have a diameter of about 10^{-18}m which is much smaller than the transistors of a quantum computer. Due to the many advancements in quantum computing technology these computers are finally within reach.

The world we live in is a quantum one. Classical computers are abstract and do not align with the world. Quantum computers will help us achieve a better understanding of the world.

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2. What is a qubit?

A qubit is a unit of quantum information, essentially the quantum version of a classical bit. What makes a qubit differ from a classical bit is quantum superposition and quantum entanglement. It is esential to understand both of of these concepts before we continue with further study of qubits.

2.1. Quantum superposition

Quantum superposition is a fundamental principle in which a system can be in all possible states until measured. After it has been measured it selects one of the basis states from the superposition. This can be explained by the double silt experiment. Lets begin with a two walls one with two spaces in it and another string wall. On the outside of the first wall two tennis balls will be thrown though the cracks in the wall the blue dots on the second wall illustrate where the balls hit.

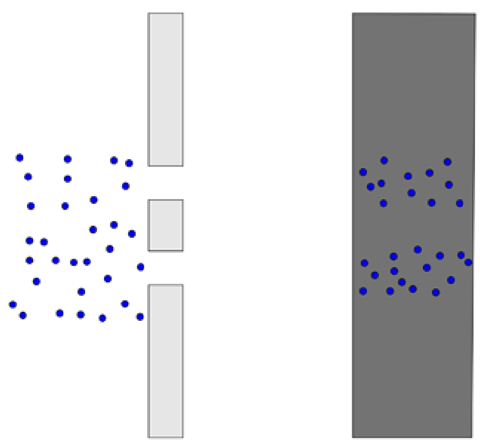

Figure 1: The Double-slit experiment: expected result

The first part of an example yields an expected result. Now instead of tennis balls being thrown through the wall imagine a light of a single wavelength shining through. This is displayed in *Figure 2*. As the waves pass through the spaces in the wall it then splits into two more waves. Where the waves overlap a brighter light can be seen on the second wall but the waves also interfere with each other and result in a lesser light.

Now lets move on the quantum part of the problem. Imagine firing electrons between the two spaces in *Figure 3*, but lets now block of one of the

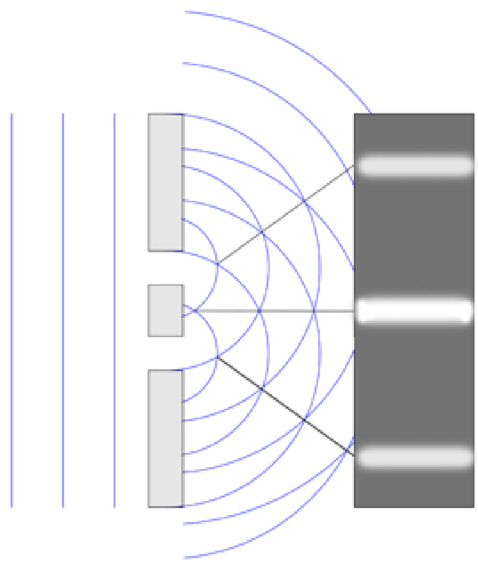

Figure 2: The Double-slit experiment: shinning light

spaces for a moment. You will notice that the electrons go through the spaces in the wall just like the tennis balls did but what is different in this example is that electrons appear on the second wall just like the wave interference pattern from *Figure 2*.

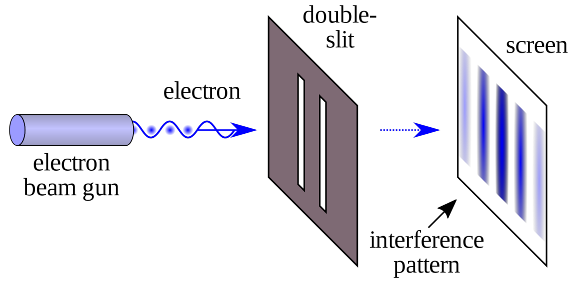

Figure 3: The Double-slit experiment: electron beam gun

The two spaces in the barrier allow for the passage of electrons. The result of this experiment is an interference pattern not predicted by classical mechanics. How did this result happen? Perhaps, shooting the elections together somehow interfere with each other resulting in this patter but even if the electrons are fired one by one the experiment yields the same result.

Before we go any further notice qubits are represented by a vector of complex numbers. For example, a qubit can be described by:

$$\alpha \left|0\right\rangle + \beta \left|1\right\rangle$$

in dirac's notation. This paper will not go over dirac's notation futher study can be found in [3].

Lets look at another example of superposition. In this example we will have a qubit with two possible configurations up and down.

$$c_1 \left|\uparrow\right\rangle + c_2 \left|\downarrow\right\rangle$$

The coefficients c_1 and c_2 are the probabilities of being in either state up or down. The probability of a configuration can be given by the absolute value of the coefficient squared. So just as in basic probability we know that the probabilities should add up to 1. Therefore, the electrons must be in one of these states

$$p_{\text{down}} = \mid c_2 \mid^2 \tag{1}$$

$$p_{\text{up}} = \mid c_1 \mid^2 \tag{2}$$

$$p_{\text{up or down}} = p_{\text{up}} + p_{\text{down}} = 1 \tag{3}$$

A particle can have many different positions this which is denoted by

$$|\alpha\rangle$$

where α is the value of the position.

Superposition guarantees that there are states which are arbitrary superpositions with complex coefficients [4].

$$\sum_x \psi(x)|x\rangle$$

The sum is defined only if x is dicrete. If it is not the $\sum$ is replaced with an $\int$. $\psi(x)$ is the wavefunction of the particle which is the mathematical description of a quantum state that is interpreted. $\psi(x)$ is a given way to get the probability of finding the qubit at a given point in space. $\psi(x)$ can be obtained from Schrdinger equation below

$$i\hbar\frac{\partial}{\partial t}|\Psi(\mathbf{r},t)\rangle = \hat{H}|\Psi(\mathbf{r},t)\rangle$$

Once $\psi(x)$ has been found it can be used to determine the probability of finding the qubit at a given point in space as follows

$$|\ \psi(x)\ |^2 = Probability$$

since the Schrdinger equation is linear, any linear combination of solutions will also be a solution. Although there may be many interpretations $\psi(x)$, this does not stop people from applying quantum mechanics to different scenarios.

2.2. Quantum entanglement

Quantum entanglement is a powerful correlation that exists between qubits. Two or more qubits can be linked together even if they are separated by great distances. When placed at opposite ends of the universe they are still linked. Quantum entanglement is essential to superdense coding and quantum teleportation which will be discussed in a later section [2].

3. Quantum circuits

Quantum logic gates are basic quantum circuits operating on a small number of qubits. Quantum gates can be applied to qubits by multiplying their matrix representations by the vector representations of qubits. Below is a vector representation of a single qubit

$$v_0|0\rangle + v_1|1\rangle \rightarrow \begin{bmatrix} v_0 \\ v_1 \end{bmatrix}$$

In this section we will go over widely used quantum logic gates.

3.1. Pauli-X gate

The Pauli-X gate of a quantum computer is practically the same as a NOT gate of a classical computer. It maps $|0\rangle$ to $|1\rangle$ and $|1\rangle$ to $|0\rangle$. It can be represented by the following matrix.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

3.2. Swap gate

The swap gate swaps two qubits and can be given by the following matrix.

$$\text{SWAP} = S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.3. Hadamard gate

The Hadamard gate acts on a single qubit, making the measure of having 1 or 0 have equal probabilities. It maps $|0\rangle$ to $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and $|1\rangle$ to $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ therefore, creating superposition.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

4. Design Challenges

In this section I will go over some challenges that quantum computers face and how the methods we use in classical computation will not work in this case.

4.1. No free lunch

As we already know quantum computers differ from classical computers. So the same way we tackle fault tolerance in classical computer wont always work for a quantum computer. For example, when classical computer experience a failure they can revert back their previous state. But a quantum computer cannot do such a thing due to the no-cloning theorem which prevents us from cloning a quantum state of the system.

To tackle the problem of cloning quantum states we could make use of quantum teleportation which allows us to convert a quantum state into a sequence of classical bits, copy those bits to some new location, and recreate a copy of the original quantum state in the new location with the help of a EPR channel which is just quantum entanglement.

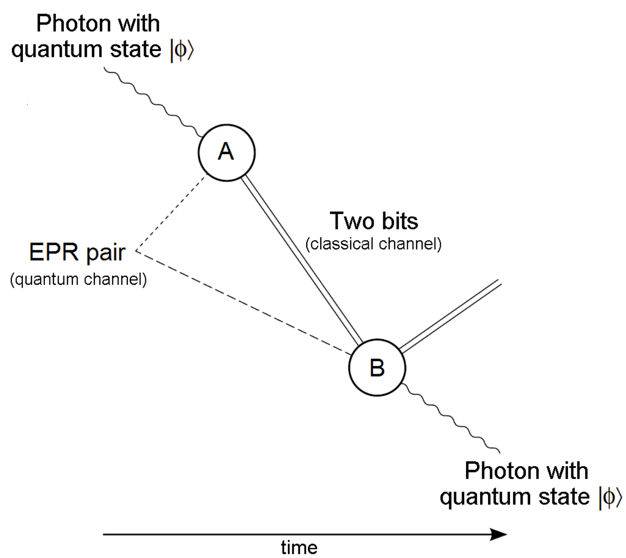

Figure 4: Quantum teleportation

5. Algorithms

Quantum algorithms run on quantum circuits. Although classical algorithms can run a quantum computer, quantum algorithms utilize quantum superposition and quantum entanglement. In this section we will go over basic quantum algorithms and their applications

5.1. Deutsch's algorithm

In this problem we are given a turning machine with a black box called an oracle that implements $f : \{0, 1\}^n \rightarrow \{0, 1\}$ [5]. It takes n digits binary as input and produces either a 0 or a 1. We are told that the function produces constant values of 1 or 0 or half 0s and half 1s [1]. The purpose of the algorithm is to find whether it produces constant values or balanced values (half 1s and half 0s).

We begin with the qubit sate $|0\rangle|1\rangle$ and apply a Hadamard transform to each of our qubits this results in the following

$$\frac{1}{2}(|0\rangle + |1\rangle)(|0\rangle - |1\rangle).$$

Next we are given a quantum function that maps $|x\rangle|y\rangle$ to $|x\rangle|f(x) \oplus y\rangle$ where $\oplus$ is a quantum XOR gate. The following is obtained from applying the quantum function to our two qubits.

$$\begin{aligned}
\frac{1}{2}(|0\rangle(|f(0)\oplus 0\rangle - |f(0)\oplus 1\rangle) + |1\rangle(|f(1)\oplus 0\rangle - |f(1)\oplus 1\rangle))\\
= \frac{1}{2}((-1)^{f(0)}|0\rangle(|0\rangle - |1\rangle) + (-1)^{f(1)}|1\rangle(|0\rangle - |1\rangle))\\
= (-1)^{f(0)}\frac{1}{2}\left(|0\rangle + (-1)^{f(0)\oplus f(1)}|1\rangle\right)(|0\rangle - |1\rangle).
\end{aligned}$$

We ignore the last bit and get the following result

$$\frac{1}{\sqrt{2}}(|0\rangle + (-1)^{f(0)\oplus f(1)}|1\rangle).$$

Now we apply Hadamard transformation to the state to get

$$\frac{1}{2}(|0\rangle + |1\rangle + (-1)^{f(0)\oplus f(1)}|0\rangle - (-1)^{f(0)\oplus f(1)}|1\rangle)$$

$$= \frac{1}{2}((1 + (-1)^{f(0)\oplus f(1)})|0\rangle + (1 - (-1)^{f(0)\oplus f(1)})|1\rangle).$$

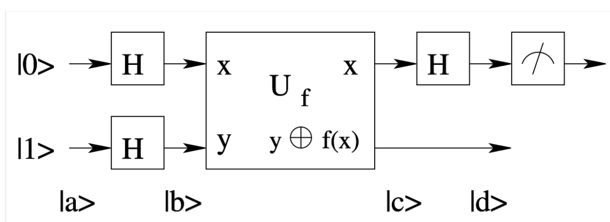

Figure 5: Quantum circuit of Deutsch's algorithm

6. Conclusion

Quantum computing is a very weird and interesting field that exploits quantum superposition and quantum entanglement. Quantum computers give us a better understanding of the world that classical computers cannot give us. Quantum computing may give us revolutionary breakthroughs in optimization problems.

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