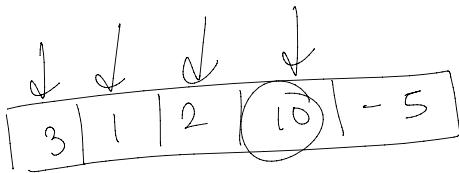


## Binary Search



$$R = 10$$

## Linear Search

R, A

```

for(int i=0; i<A.size(); i++) {
    if(A[i] == k) {
        return true;
    }
}
```

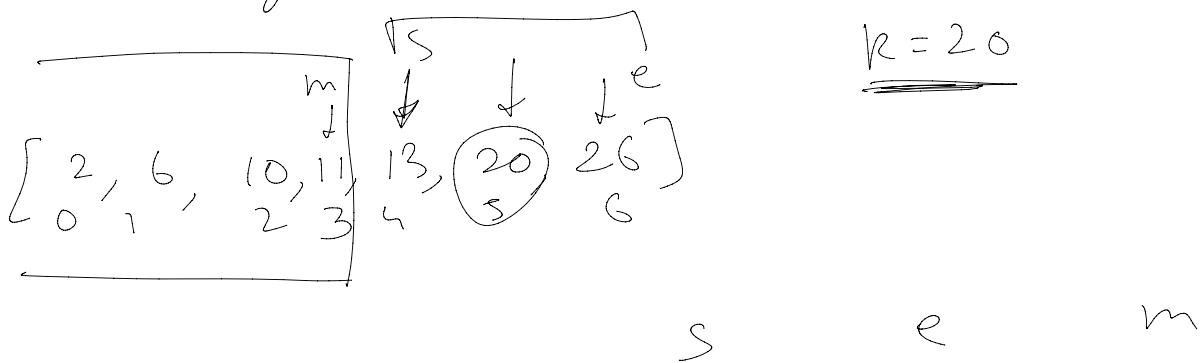
$T: O(n)$   
 $S: O(1)$

## Binary

→ Sorted array

→ Unsorted array

→ No array



$$\underline{k = 20}$$

s e m

$$\begin{aligned}
 s &= 0 \\
 e &= A.size() - 1 \\
 m &= s + \frac{e-s}{2}
 \end{aligned}$$

s	e	m
0	6	3
4	6	5

if ( $A[m] \leq k$ ) {  
     s = m + 1;

}

s = 0, e = A.size() - 1, m;

while (s <= e) {

$$\rightarrow m = s + \frac{e-s}{2};$$

if ( $A[m] == k$ ) {  
     return true;

}

else if ( $A[m] > k$ ) {

$$e = m - 1;$$

else {

$$s = m + 1;$$

}

3

\* At any given point we should be able to decide whether we have to make start = mid + 1 or end = mid - 1.

$$T: O(\log_2 n)$$

$$(2^u) \left(\frac{1}{2}\right) \left(\frac{1}{2}\right) \left(\frac{1}{2}\right) \left(\frac{1}{2}\right) = 1$$

$$(\log_2 n) \quad \log_2(2^u) \Rightarrow \textcircled{u}$$

Square root of integer (Binary search on a search space)

## Square root of integer (Binary search)

(25)

(1-25)

$$n \\ s=1 \\ e=25$$

$$\text{int ans} = 0$$

while ( $s \leq e$ ) {

$$m = s + \frac{e-s}{2};$$

if ( $m * m == n$ ) {

return m;

3

else if ( $m * m < n$ ) {

ans = m;

$$s = m+1;$$

3  
else {

$$e = m-1;$$

3

return ans;

3

$$(m=17) \quad \sqrt{20}$$

(34)

$$\begin{array}{cccc} s & e & m & \text{ans} \\ 1 & 34 & 17 & \\ \end{array}$$

$$\begin{array}{ccc} 1 & 16 & 8 \\ \end{array}$$

$$\begin{array}{ccc} 1 & 7 & 4 \\ \end{array}$$

$$\begin{array}{ccc} 5 & 7 & 6 \\ \end{array}$$

$$\begin{array}{ccc} 5 & 5 & 5 \\ \end{array}$$

(5)

(6 5 X)

## First and last occurrence of $x$

5

[ $\underset{1}{\overset{\uparrow}{1}}, \underset{2}{\overset{\uparrow}{3}}, \underset{3}{\overset{\uparrow}{5}}, \underset{4}{\overset{\uparrow}{5}}, \underset{5}{\overset{\leftarrow}{5}}, \underset{6}{\overset{\uparrow}{6}}, \underset{7}{\overset{\uparrow}{7}}, \underset{8}{\overset{\uparrow}{8}}, 123, 125$ ]

$\{1, 3, 5, 5, 5, 6, 7, 8\}$   
 ↓ ↑ ↑ ↑ ↑

$e=4 \not\in S$

## I (Linear Search)

$s = -1, e = -1; k$   
 $\text{for } i \text{ from } 0 \text{ to } A.size() \text{ do } \{$   
 $\quad \text{if } (A[i] == k) \{$   
 $\quad \quad s = i;$   
 $\quad \quad \text{for } j \text{ from } i \text{ to } A.size() \text{ do } \{$   
 $\quad \quad \quad \text{if } (A[j] == k) \{$   
 $\quad \quad \quad \quad e = j;$   
 $\quad \quad \quad \}$   
 $\quad \quad \}$   
 $\quad \quad \text{else } \{$   
 $\quad \quad \quad \text{break};$   
 $\quad \quad \}$   
 $\quad \}$   
 $\}$

?

## Find first occurrence

$s = 0, e = A.size() - 1, k, m;$   
 $s = -1, e = -1;$   
 $\text{while } (s <= e) \{$   
 $\quad m = s + \frac{e-s}{2};$

$\quad \text{if } (A[m] == k) \{$   
 $\quad \quad s_i = m;$   
 $\quad \quad e = m - 1;$

$s \quad e$   
 $\downarrow \quad \downarrow$   
 $1, 3, 5, 5, 5, 6, 7, 12, 3, 12, 5$

$s_i$	$s$	$e$	$m$
4	0	8	4
0	0	3	1
-	-	2	?

$e = -1;$   
 $c = m - 1;$   
 3  
 else if ( $A[m] < k$ ) {  
 $s = m + 1;$

$3$   
 else {  
 $e = m - 1;$   
 $3$

$3$   
 Last occurrence

$e = -1$   
 $s = 0, e = A.size() - 1, m$   
 while ( $s \leq e$ ) {  
 $m = s + \frac{e-s}{2};$

$\text{if } (A[m] == k) {$   
 $e = m;$   
 $s = m + 1;$

$3$   
 else if ( $A[m] < k$ ) {  
 $s = m + 1;$

$3$   
 else {  
 $e = m - 1;$   
 $3$

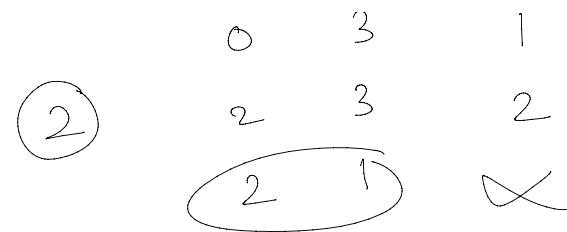
$T: O(\log n)$   
 $S: O(1)$

find a peak element (Binary Search on sorted array)

$[1, 2, 3, \underline{4})$

$[1, \underline{2}, 3, 12, \underline{16})$

$[1, \underline{20}, 2, \underline{30}, 4, \underline{50}, 6]$



if ( $A.size() == 1$ ) {  
    return  $A[0]$ ;

}

if ( $A[0] >= A[1]$ ) {  
    return  $A[0]$ ;

if ( $\frac{A[A.size() - 1]}{A[A.size() - 2]} >= A[A.size() - 2]$ ) {  
    return  $A[A.size() - 1]$ ;

}

for (int i = 1; i < A.size() - 1; i++) {  
    if ( $A[i] >= A[i - 1] \& \& A[i] >= A[i + 1]$ ) {  
        return  $A[i]$ ;

3

$O(n)$      $O(1)$

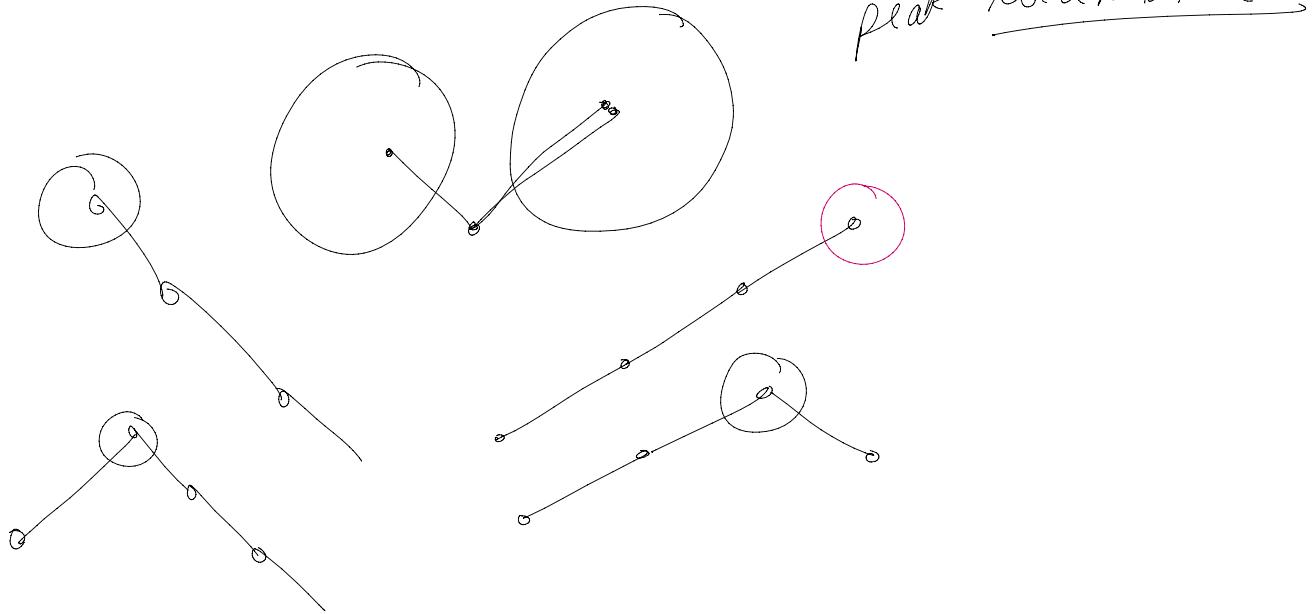
3

1) Can we sort the given array?

$\left[ \begin{matrix} 2 & 6 & 3 & 7 & 9 \end{matrix} \right]$



peak = local maxima



- 1) start Binary search:  $s=0, e=A.size()-1$ .
- 2) update  $m = s + \frac{e-s}{2}$ .
- 3) If  $A[m]$  is peak, return it.
- 4) else if  $A[m] < A[m+1]$ ,  $s=m+1$ .
- 5) else  $e=m-1$ .

Search in a row column sorted matrix

13	36	38
36	53	60
40	51	69

(62)

$$r = 3 \times 2 \quad (3)$$

$$c = 2 \quad 1$$

1) Binary search on each row.

	0	1	2
0	16	20	30
1	12	28	36
2	45	50	80

50

$$n = \emptyset \\ \underline{n++} \\ c = 2 \\ \underline{c--}$$

	0	1	2
0	10	20	30
1	15	35	60
2	25	40	80

25

$$n = 6 \\ c = 21$$