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MATHEMATICS
3D GEOMETRY

Nishant Vora

B.Tech - IIT Patna

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SEP 25	Saturday	2 PM	JEE Adv PYQs - Probability
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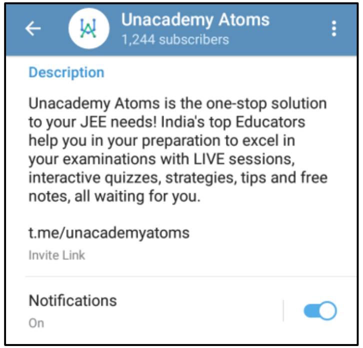

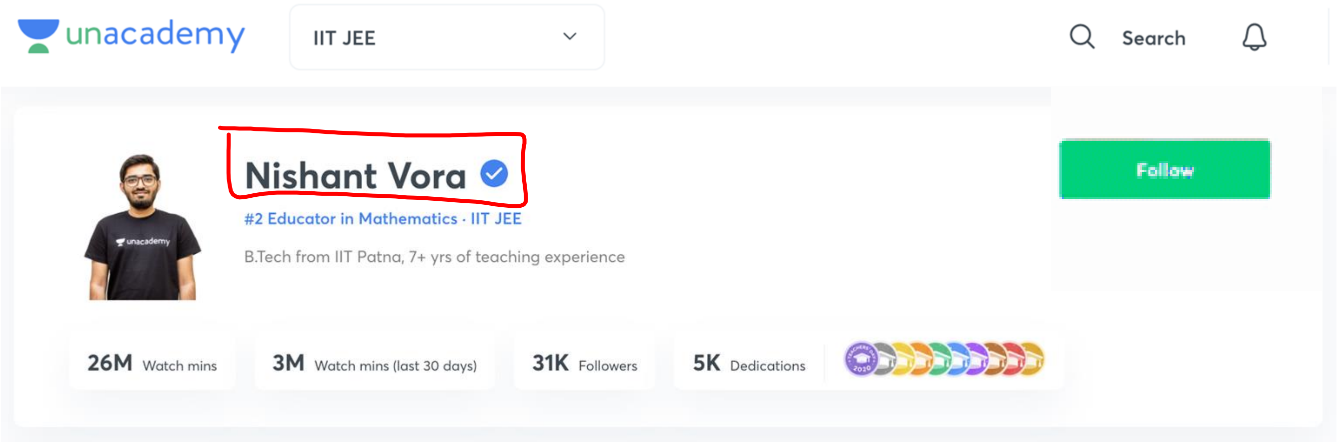

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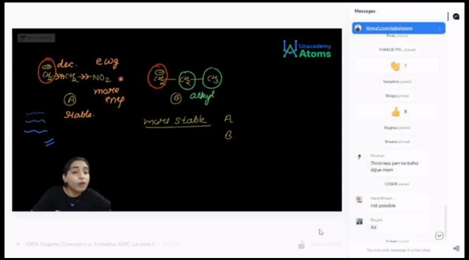

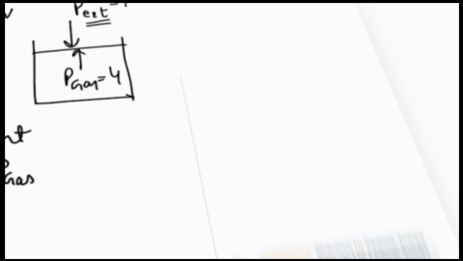

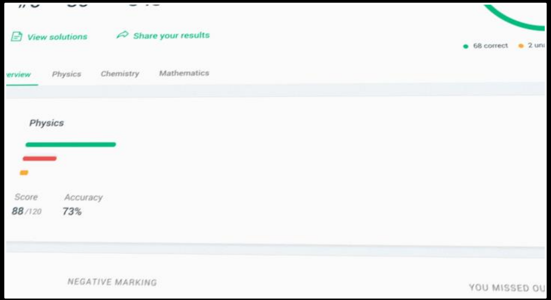

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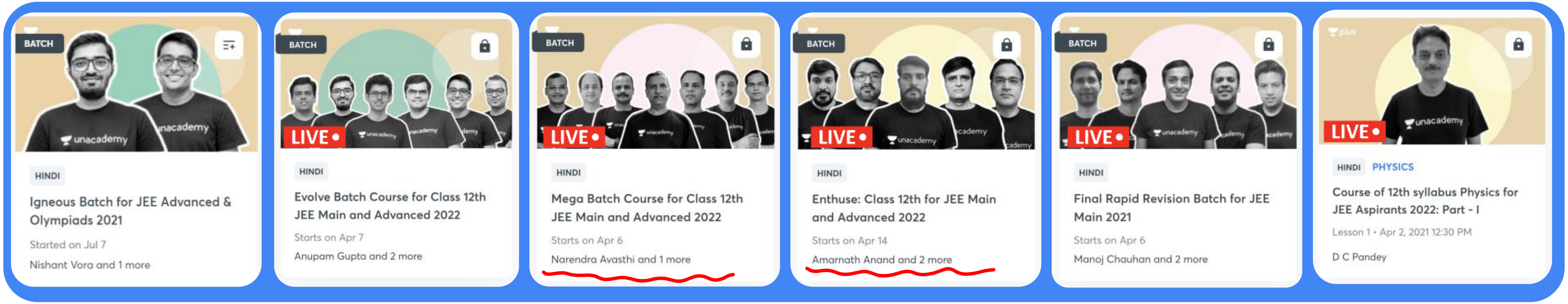

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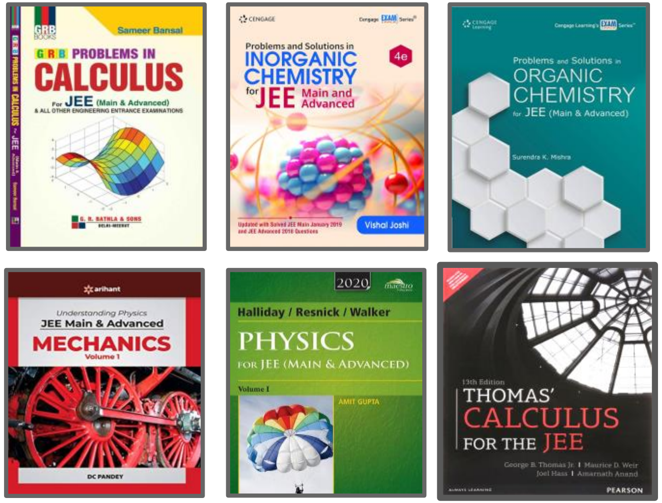


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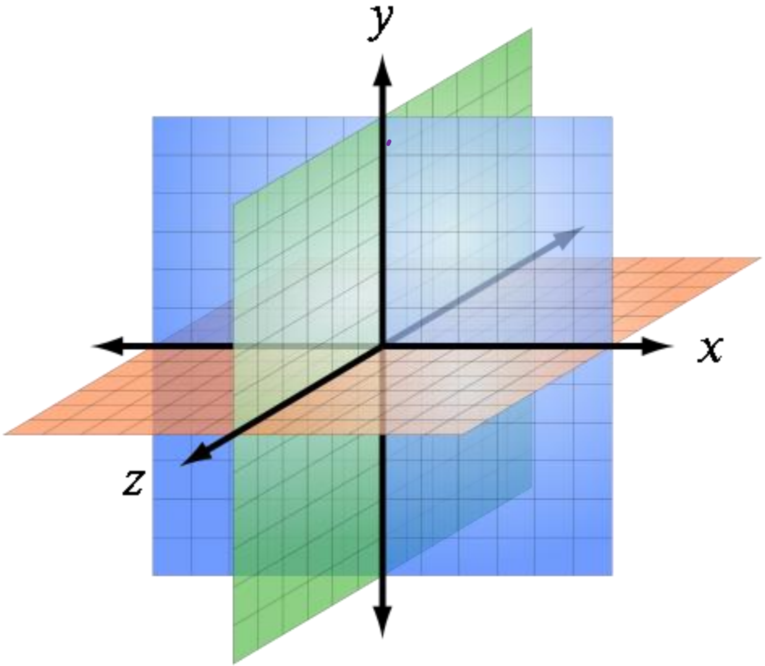
Cartesian Coordinate System

Cartesian Coordinate System in 3D

- 3 Coordinate Axes ✓
- 3 Coordinate Planes (xy , yz , zx)
- 8 Octants

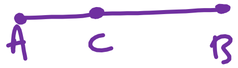


Distance formulae

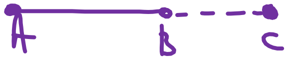
Distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Section formulae

1 Coordinates a point P which divides line joining A (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio m : n internally is given by $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$



2 Coordinates a point P which divides line joining A (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio m : n externally is given by $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$

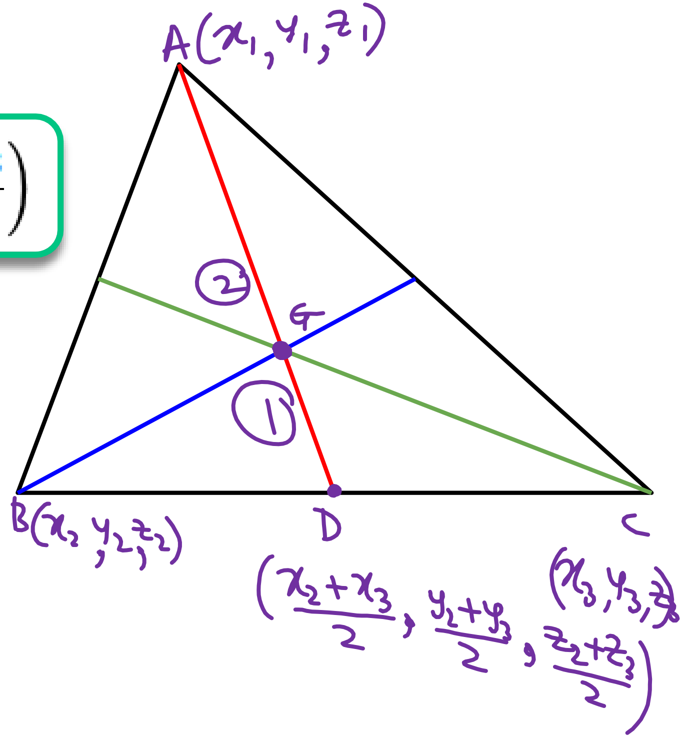


3 Coordinates of midpoint of line joining A (x_1, y_1, z_1) and B (x_2, y_2, z_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Coordinates of Centroid

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

$$G \equiv \left(\frac{\cancel{2}\left(\frac{x_2+x_3}{\cancel{2}}\right) + 1(x_1)}{2+1}, \ldots, \right)$$



Distance of a point from Coordinate Planes

Distance of Point P from xy plane = | z |
Distance of Point P from yz plane = | x |
Distance of Point P from xz plane = | y |

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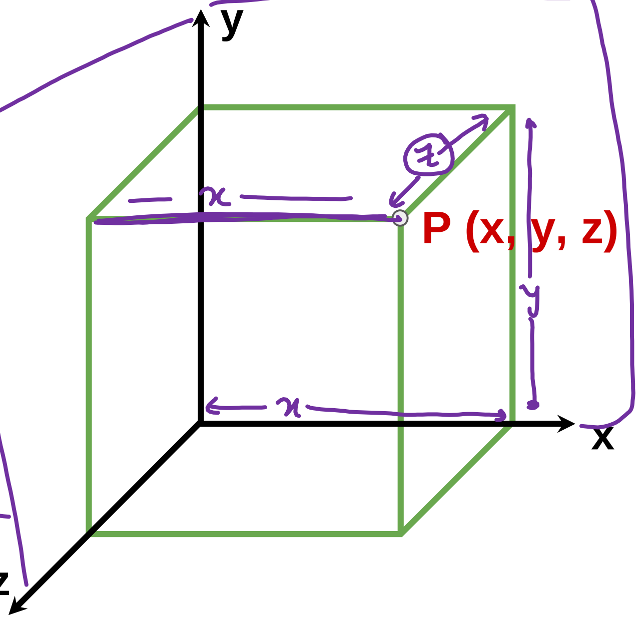
P(2, -3, -1) dis xy plane ⇒ 1

" yz plane ⇒ 2

" xz plane ⇒ 3



Distance of a point from Coordinate axes

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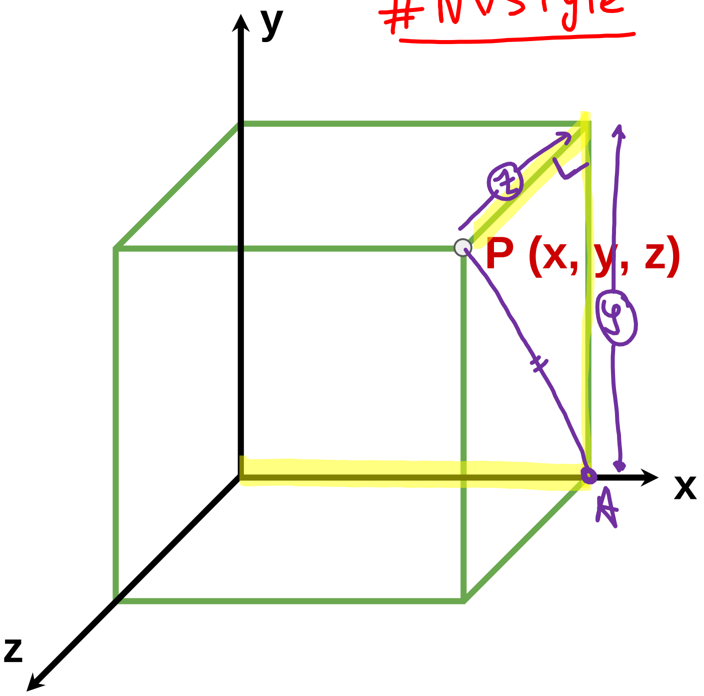


$$PA = \sqrt{y^2 + z^2}$$
$$PB = \sqrt{z^2 + x^2}$$
$$PC = \sqrt{x^2 + y^2}$$

P (1, -2, 3)

dis of P from x axis $\Rightarrow \sqrt{y^2+z^2} \Rightarrow \sqrt{2^2+3^2}$

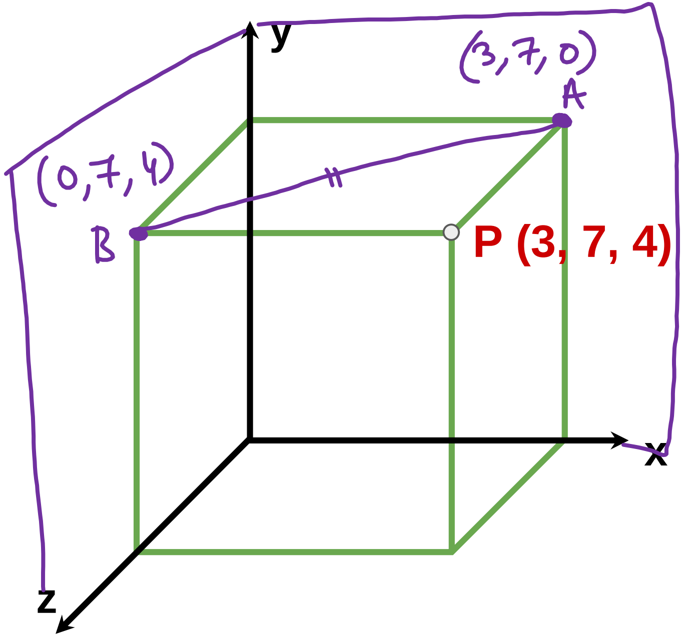
" " " " y axis $\Rightarrow \sqrt{x^2+z^2} = \sqrt{10}$

If A and B are foot of perpendicular from **(3, 7, 4)** on the **XY** plane and **YZ** plane respectively, then find the length of **AB**.

A (3, 7, 0)

B (0, 7, 4)

$$AB = \sqrt{(3-0)^2 + 0^2 + 4^2} = 5$$



Direction Ratios and Direction Cosines

Direction Cosines (DCs)

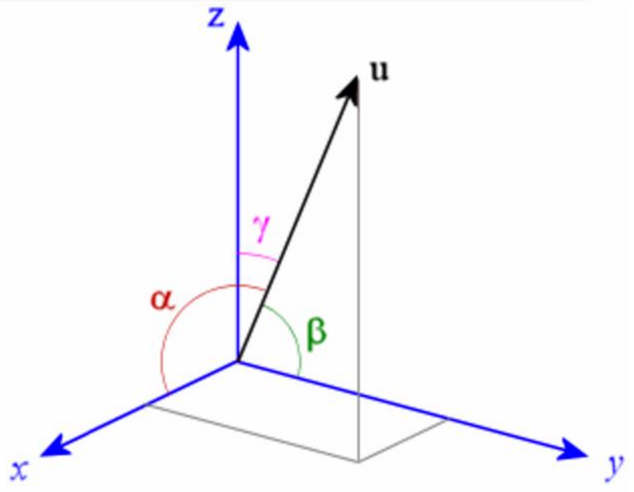
If α, β, γ are the angles which vector makes with positive direction of the x, y, z axes respectively then their cosines $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called the direction cosines of the vector and are generally denoted by l, m, n respectively.

Thus $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

α , β , γ

$\underline{\cos\alpha}$ $\underline{\cos\beta}$ $\underline{\cos\gamma}$

l m n



Important Results

1. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

2. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$$\boxed{l^2 + m^2 + n^2 = 1}$$

$l\hat{i} + m\hat{j} + n\hat{k} \Rightarrow$ unit vector

$$1 - \sin^2\alpha + 1 - \sin^2\beta + \cancel{1} - \sin^2\gamma = \cancel{1}$$

$$\boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$$

Direction Ratios (DRs)

Direction ratios are multiples of direction cosines

Q $DC = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ $l^2+m^2+n^2=1$

$DR = \lambda\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

Direction Vector

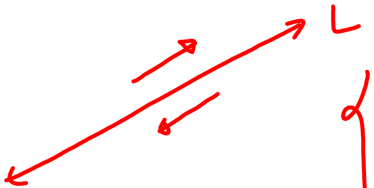
$= (2, 2, 1)$ ✓ $\Longrightarrow \boxed{2\hat{i}+2\hat{j}+\hat{k}}$

$= (-2, -2, -1)$ ✓

$= (4, 4, 2)$ ✓

Important Results

1 Direction ratios of a line is not unique but infinite in number but direction cosines will be for a line will be only two.
(l, m, n or -l, -m, -n)



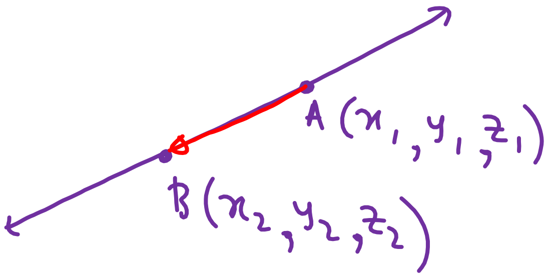
$$\begin{cases} (l, m, n) \\ (-l, -m, -n) \end{cases}$$

Important Results

2 A vector along the line with direction ratios a, b, c can be $a\hat{i} + b\hat{j} + c\hat{k}$

Important Results

3 Direction ratios of a line joining two points A and B are proportional to $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$



$\overrightarrow{AB}$ OR $\overrightarrow{BA}$

$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

DV

$DR = (x_2 - x_1, \; y_2 - y_1, \; z_2 - z_1)$

A line AB in three-dimensional space makes 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals

A. 45° **B.** 60° **C.** 75° **D.** 30°

2010

$\alpha = 45°$ $\beta = 120°$ $\gamma = ?$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 45 + \cos^2 120 + \cos^2\gamma = 1$

$\frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$

$\boxed{\cos\gamma = \frac{1}{2}}$

Equation of Straight Line

Equation of Line: Vector Form

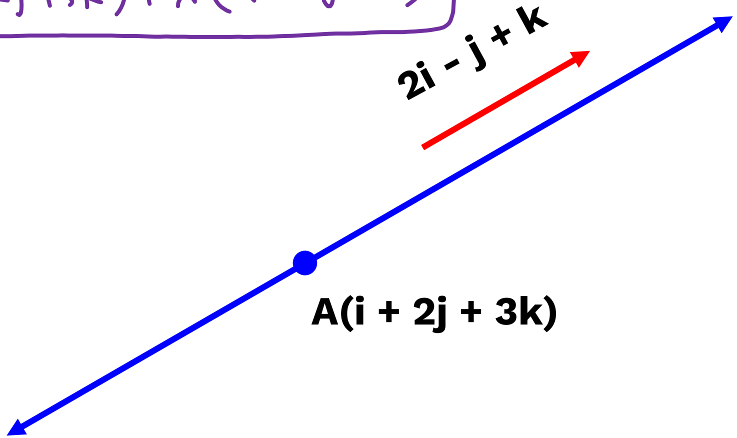
1 Passing through a point and parallel to given direction

point :- ✓
dir^n :- ✓

$$\boxed{\vec{r} = \vec{a} + \lambda \vec{b}}$$

point dir^n

$$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(2\hat{i}-\hat{j}+k)$$



Equation of Line: Cartesian Form

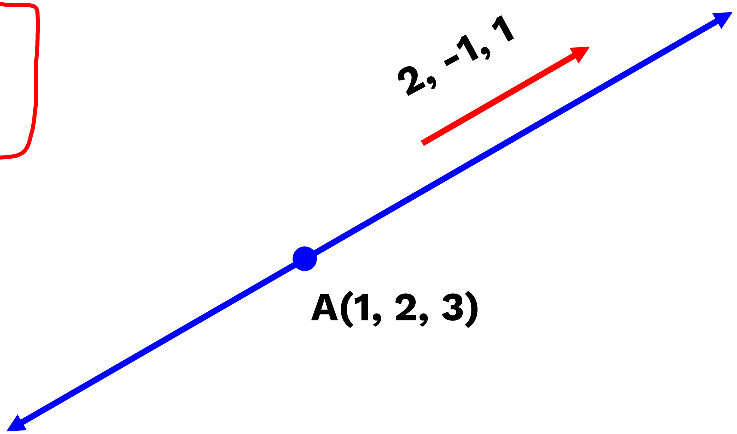
1 Passing through a point and given DRs or DCs

$(x_1, y_1, z_1) \rightarrow$ point on S L

$$\boxed{\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}}$$

$(a, b, c) = DR$

$$\boxed{\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{1}}$$

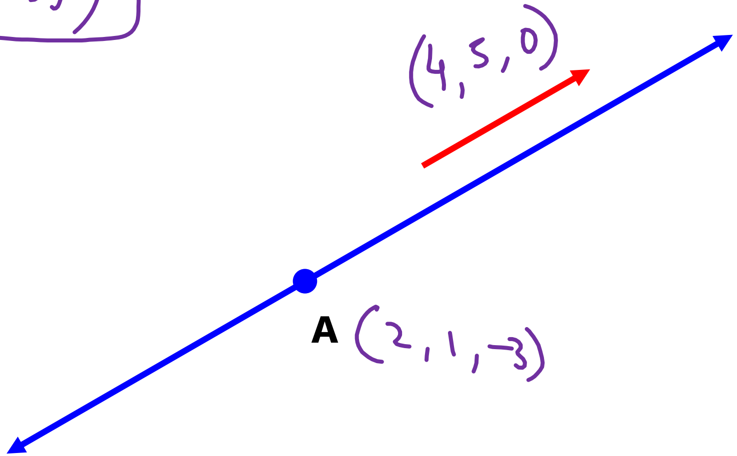


Find the equation of the line through the point **2i + j - 3k** and parallel to the vector **4i + 5j** in

I. Vector Form
II. Cartesian Form

(I) $$\boxed{\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda(4\hat{i} + 5\hat{j})}$$

(II) $$\boxed{\frac{x-2}{4} = \frac{y-1}{5} = \frac{z+3}{0}}$$



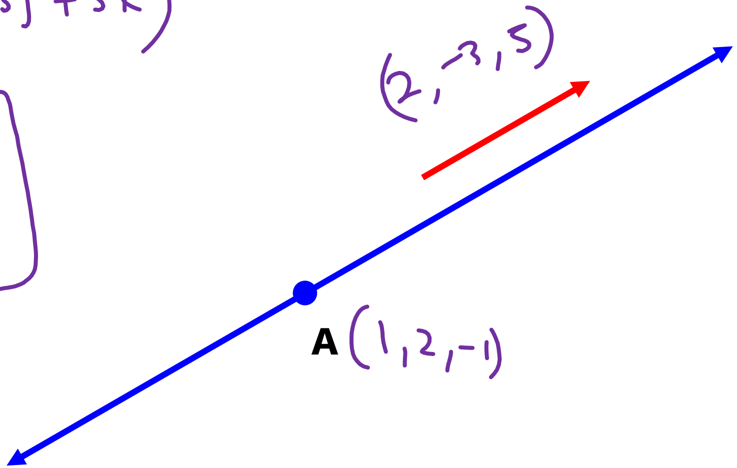
Find the equation of the line through the point (1, 2, -1) and having DRs 2, -3 and 5

I. Vector Form

II. Cartesian Form

(I) $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(2\hat{i} - 3\hat{j} + 5\hat{k})$

(II) $$\boxed{\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+1}{5} = \lambda}$$



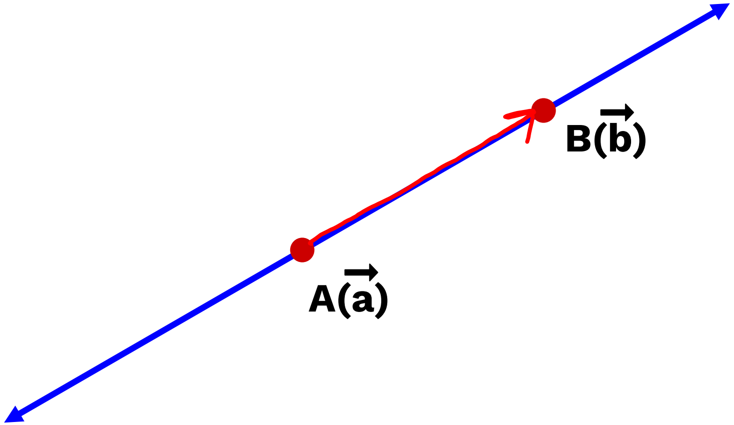
Equation of Line: Vector Form

2 Passing through a two points

point :- $\vec{a}$ / $\vec{b}$

dirⁿ :- $\vec{AB} = \vec{b} - \vec{a}$

$$\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$$



Find the equation of straight line passing through points A (6, -7, -1) and B (2, -3, 1) in

I. Vector Form

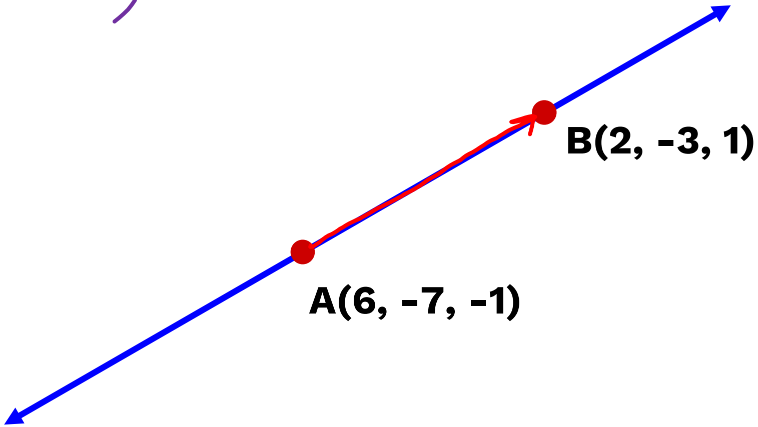
II. Cartesian Form

point = (2, -3, 1)

dir^n = $\overrightarrow{AB} = (-4, 4, 2)$

(I) $\vec{r} = 2\hat{i} - 3\hat{j} + \hat{k} + \lambda(-4\hat{i} + 4\hat{j} + 2k)$

(II) $\dfrac{x-2}{-4} = \dfrac{y+3}{4} = \dfrac{z-1}{2}$



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Angle Between Two Straight Lines

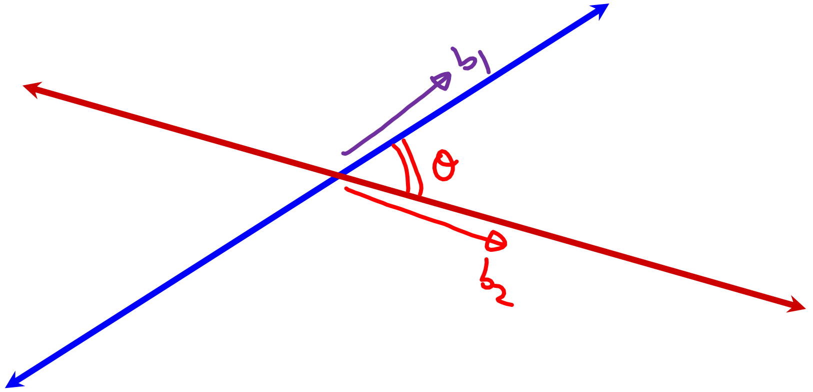
Angle Between Two Lines

Angle between two lines = Angle between their Direction Vectors

$L_1 \quad \vec{r} = \vec{a_1} + \lambda \vec{b_1}$

$L_2 \quad \vec{r} = \vec{a_2} + \mu \vec{b_2}$

$$\boxed{\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|b_1||b_2|}}$$



Find the angle between the pair of lines given by

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) \qquad L_1$$

$$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k}) \qquad L_2$$

$$\boxed{\begin{array}{l} \Rightarrow (1,2,2) \\ \Rightarrow (2,4,4) \end{array}}$$

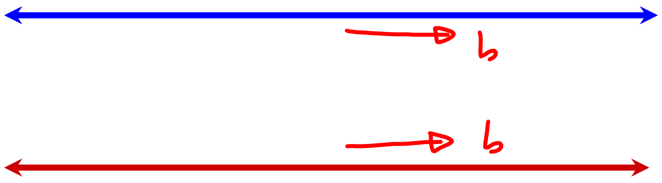
$$\frac{1}{2} = \frac{2}{4} = \frac{2}{4}$$

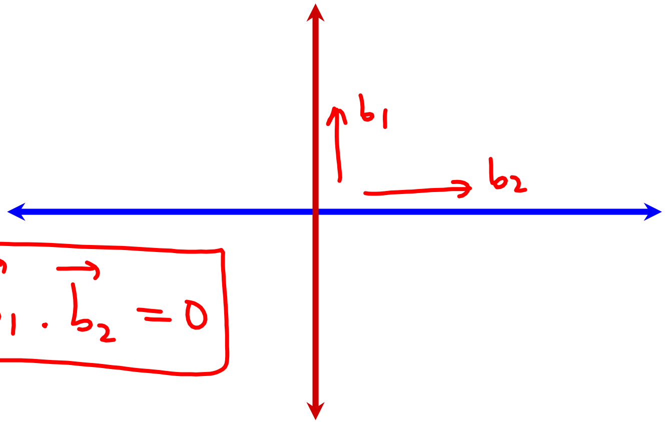
$$\cos\theta = \frac{(1,2,2)\cdot(3,2,6)}{\sqrt{1+4+4}\sqrt{9+4+36}}$$

$$= \frac{3+4+12}{3\times 7}$$

$$\cos\theta = \frac{19}{21} \Longrightarrow \boxed{\theta = \cos^{-1}\left(\frac{19}{21}\right)}$$

Angle Between Two Lines





$$\boxed{\vec{b_1} \cdot \vec{b_2} = 0}$$

Find the value of λ so that the following lines are perpendicular

$$\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1}$$

$$\frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}$$

$$\frac{x-5}{5\lambda+2}=\frac{y-2}{-5}=\frac{z-1}{1}$$

$$\frac{x}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3}$$

$$1(5\lambda+2)-5(2\lambda)+3=0$$

$$5\lambda+2-10\lambda+3=0$$

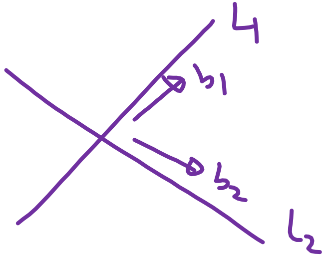
$$\boxed{\lambda=1}$$

Consider the lines $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ $\quad L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$

The unit vector perpendicular to both L_1 and L_2 is

A. $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ **B.** $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$ ✓ **C.** $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ **D.** $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

[JEE Adv. 2018]



$$= \vec{b_1} \times \vec{b_2}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = \frac{\hat{i}(-1) - \hat{j}(7) + k(5)}{\sqrt{1^2+7^2+5^2}}$$

Parametric Coordinates

Line

Find parametric coordinates of straight lines

CF I. $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4} = \lambda$ PC $(3\lambda - 3, 5\lambda + 1, 4\lambda - 3)$

VF II. $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

PC $(1 + 2\lambda, 1 - \lambda, \lambda)$

Point of Intersection of Two Lines

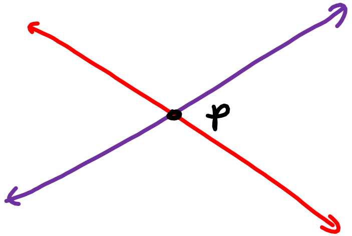
Point of Intersection of two Lines

Find the point of intersection of L_1 and L_2

$$L_1 : \frac{x-2}{6} = \frac{y-1}{-8} = \frac{z+2}{10} = \lambda = \frac{1}{2}$$

$$L_2 : \boxed{\frac{x+1}{6} = \frac{y+10}{7} = \frac{z-5}{-2} = \mu = 1}$$

POI
$(5, -3, 3)$



$6\lambda + 2 = 6\mu - 1$ | $-8\lambda + 1 = 7\mu - 10$ | $\underbrace{10\lambda - 2} = \underbrace{-2\mu + 5}$

$LHS = 3$

$RHS = 3$

$LHS = RHS$

L_1 and L_2

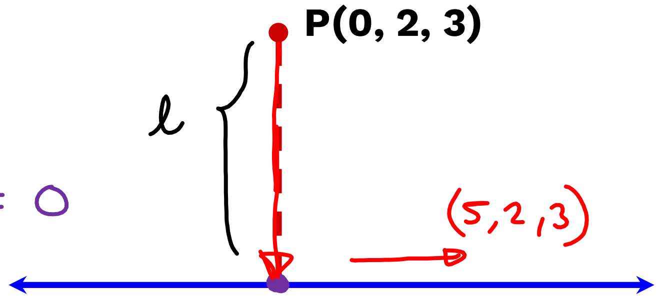
intersecting

$\lambda = \frac{1}{2} \quad \mu = 1$

Foot of Perpendicular/ Image from Point to Line

Find the foot of perpendicular from the point (0, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Find the length of perpendicular.

Method :- $\underline{\overrightarrow{PQ}}\cdot(5,2,3)=0$



$Q(5\lambda-3,\ 2\lambda+1,\ 3\lambda-4) = \underline{(2,3,-1)}$

$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda$

$(5\lambda-3,\ 2\lambda-1,\ 3\lambda-7)\cdot(5,2,3)=0$

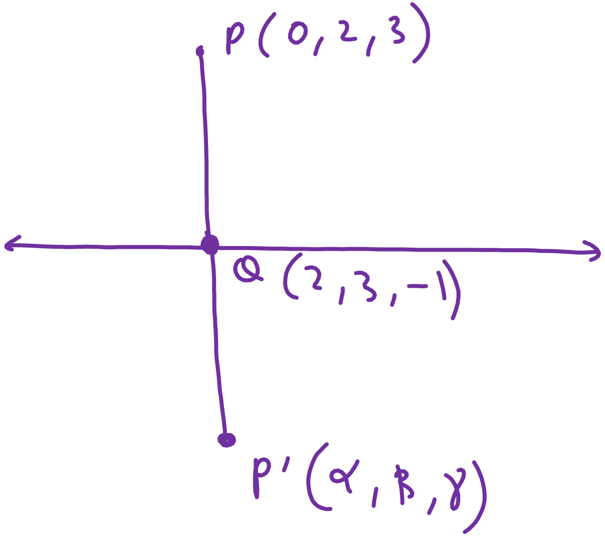
$5(5\lambda-3)+2(2\lambda-1)+3(3\lambda-7)=0$

$25\lambda+4\lambda+9\lambda-15-2-21=0$

$38\lambda-38=0$

$\boxed{\lambda=1}$

Image of Point in Line



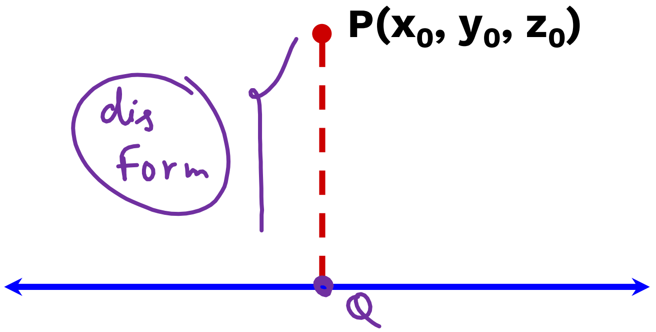
$$\left(\frac{\alpha+0}{2}, \frac{\beta+2}{2}, \frac{\gamma+3}{2}\right) \equiv (2, 3, -1)$$

$$\boxed{\alpha = 4 \quad \beta = 4 \quad \gamma = -5}$$

Perpendicular Distance of Point from Line

Perpendicular Distance of Point from Line

M-1 Find Foot of Perpendicular then use Distance Formula



$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Perpendicular Distance of Point from Line

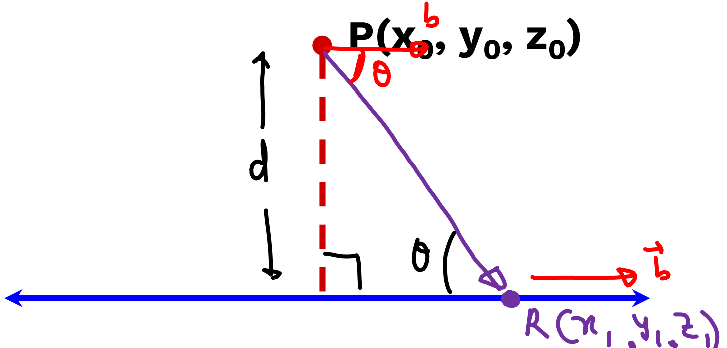
M-2 Perpendicular Distance = $\dfrac{|\vec{PR} \times \vec{b}|}{|\vec{b}|}$

$\underline{\vec{b} \wedge \vec{PR}} = \theta$

$\dfrac{d}{PR} = \sin\theta$

$d = |PR| \sin\theta$

$= \cancel{|PR|} \dfrac{\vec{PR} \times \vec{b}}{\cancel{|PR|}\,|b|}$



$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Find distance between point P(0, 2, 3) and line $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z+1}{2}$

$P(0,2,3)$

$R(3,1,-1)$

$\overrightarrow{PR}=(3,-1,-4)$

$\vec{b}=(2,1,2)$

$$\overrightarrow{PR}\times\vec{b}=\begin{vmatrix} i & j & k \\ 3 & -1 & -4 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= i(2)-j(14)+k(5)$$

$|PR\times b|=\sqrt{2^2+14^2+5^2}=\sqrt{4+196+25}$

$= 15$

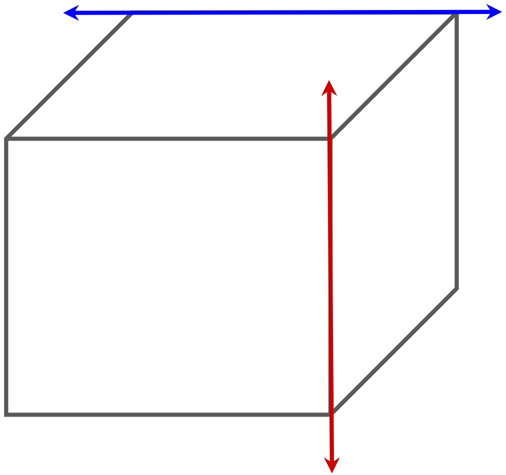
$|b|=\sqrt{2^2+1^2+2^2}=3$

$$\boxed{p\cdot d=\frac{15}{3}=5 \text{ units}}$$

Skew Lines

Skew Lines

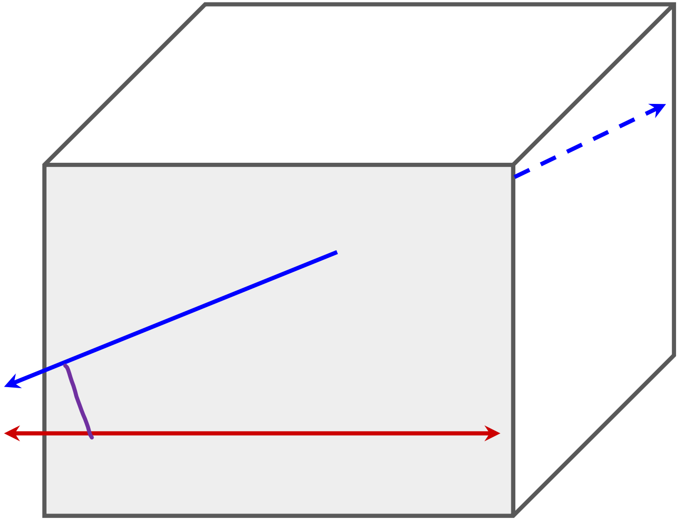
Non parallel, non intersecting lines are called skew lines



Shortest Distance between two Skew Lines

Shortest distance = $\dfrac{\left|(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})\right|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$

$$\boxed{\begin{aligned} \vec{r} &= \vec{a}_1 + \lambda \vec{b}_1 \\ \vec{r} &= \vec{a}_2 + \mu \vec{b}_2 \end{aligned}}$$



Consider the lines $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ $L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$

The shortest distance between L_1 and L_2 is

A. 0   **B.** $17 / \sqrt{3}$ **C.** $41 / 5\sqrt{3}$ **D.** $17 / 5\sqrt{3}$

[JEE Adv. 2018]

$\vec{a}_2 = (2, -2, 3)$

$\vec{a}_1 = (-1, -2, -1)$

$\vec{a}_2 - \vec{a}_1 = (3, 0, 4)$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

$= \hat{i}(-1) - \hat{j}(7) + \hat{k}(5)$

$= (-1, -7, 5)$

$$S d = \frac{(a_2 - a_1) \cdot \vec{b_1} \times \vec{b_2}}{|b_1 \times b_2|}$$

$$= \frac{(3, 0, 4) \cdot (-1, -7, 5)}{\sqrt{1 + 49 + 25}}$$

$$= \left| \frac{-3 + 20}{5\sqrt{3}} \right|$$

$$= \frac{17}{5\sqrt{3}}$$

If the shortest distance between the lines

$\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}), \lambda \in \mathbf{R}, \alpha > 0$ and

$\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$, $\mu \in R$ is 9, then α is equal to _

[20 July 2021 Shift 1]

$S d = 9$

$\vec{a}_2 = (-4, 0, -1)$

$\vec{a}_1 = (\alpha, 2, 2)$

$\vec{a}_1 - \vec{a}_2 = (\alpha+4, 2, 3)$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$= \hat{i}(8) - \hat{j}(-8) + \hat{k}(4)$

$= (8, 8, 4)$

$= (2, 2, 1)$

$$S.d = \frac{(\vec{a_1} - \vec{a_2}) \cdot (\vec{b_1} \times \vec{b_2})}{|b_1 \times b_2|}$$

$$= \left| \frac{(\alpha+4, 2, 3) \cdot (2, 2, 1)}{3} \right| = 9$$

$$|2(\alpha+4) + 4 + 3| = 27$$

$$2\alpha + 13 = \pm 27$$

$$2\alpha + 13 = 27$$

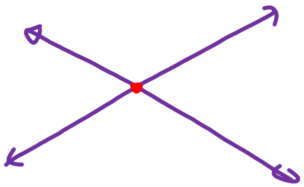
$$\boxed{\alpha = 6} \checkmark$$

$$\cancel{2\alpha + 13 = -27}$$

$$\cancel{\alpha = \text{Rej}}$$

Condition for 2 Lines to Intersect

If shortest distance between 2 lines = 0 ⇒ they are intersecting



$$\boxed{\text{if } sd = 0 \Rightarrow \text{intersecting}}$$

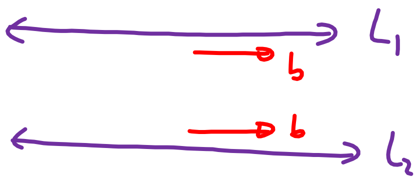
$$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$$

$$\boxed{[\vec{a}_2 - \vec{a}_1 \quad \vec{b}_1 \quad \vec{b}_2] = 0}$$

Shortest Distance between two Parallel Lines

Shortest Distance between two Parallel Lines

$$\textit{Shortest distance between parallel lines} = \frac{\left|(\vec{a_2} - \vec{a_1}) \times \vec{b}\right|}{|\vec{b}|}$$



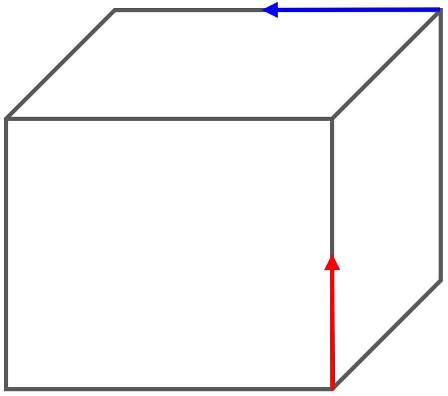
$$\boxed{\begin{aligned} \vec{r} &= \vec{a_1} + \lambda \vec{b} \\ \vec{r} &= a_2 + \mu \vec{b} \end{aligned}}$$

$$\frac{|(a_2 - a_1) \times b|}{|b|}$$

Condition for Coplanarity

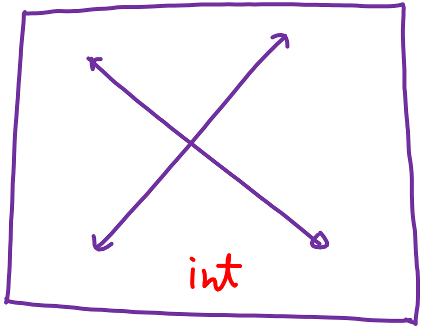
Condition of Coplanarity (for vectors)

1. Two non-zero vectors are always coplanar
2. Three non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

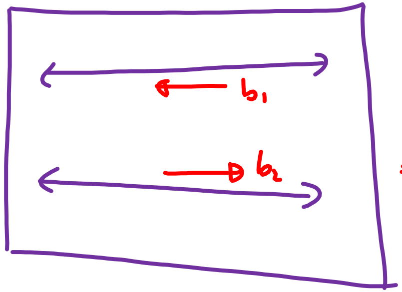


Condition of Coplanarity (for lines)

Two lines are COPLANAR if either they are Intersecting or Parallel



$\boxed{Sd = 0}$



$\vec{b}_1 \parallel \vec{b}_2$

$\boxed{\text{Ratio} = \text{same}}$

$$\frac{x-k}{1}=\frac{y-2}{2}=\frac{z-3}{3} \text{ and } \frac{x+1}{3}=\frac{y+2}{2}=\frac{z+3}{1}$$

are co-planar, then the value of k is

$\Downarrow$

int OR ~~parallel~~

$$\left.\begin{array}{l} a_2=(-1,-2,-3) \\ a_1=(k,\ 2,\ 3) \end{array}\right\} \vec{a}_1-\vec{a}_2=(k+1,4,6)$$

[25 July 2021 Shift 2]

$$[a_2-a_1 \quad b_1 \quad b_2]=0$$

$$\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix}=0 \Longrightarrow k=1$$

Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$

are coplanar. Then **α** can take value(s)

AD

A. 1 B. 2 C. 3 D. 4

[JEE Adv. 2013]

$L_1 \quad \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$

$L_2 \quad \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$

$\vec{a}_2 = (\alpha, 0, 0)$

$\vec{a}_1 = (5, 0, 0)$

$\vec{a}_2 - \vec{a}_1 = (\alpha - 5, 0, 0)$

$$S_d = 0$$

$$[a_2 - a_1 \quad b_1 \quad b_2] = 0$$

$$\begin{vmatrix} \alpha - 5 & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$$(\alpha - 5)\{(3-\alpha)(2-\alpha) - 2\} = 0$$

$$(\alpha - 5)\{\alpha^2 - 5\alpha + 4\} = 0$$

$$(\alpha - 5)(\alpha - 1)(\alpha - 4) = 0$$

$$\boxed{\alpha = 1, 4, 5}$$

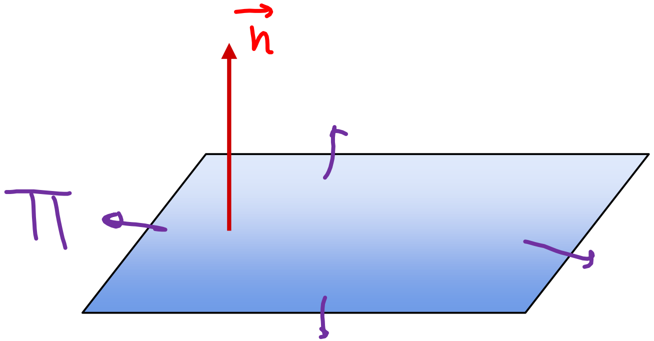
Equation of Planes

Definition of Plane

Flat surface perpendicular to a fixed vector.

This flat surface is called **Plane** and fixed vector is called **Normal**

Representation of Plane : P, $\prod$, $\boldsymbol{\sigma}$



General Equation of Plane

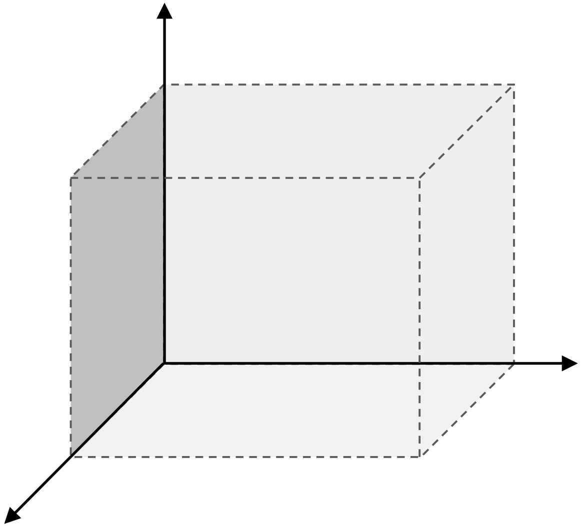
$$\boxed{ax + by + cz + d = 0}$$

$$\boxed{2x + 3y + 5z - 3 = 0}$$

Plane

Note

1. Equation of xy plane is $z=0$
2. Equation of yz plane is $x=0$
3. Equation of zx plane is $y=0$



Form

If point lying on plane is $A(\vec{a})$

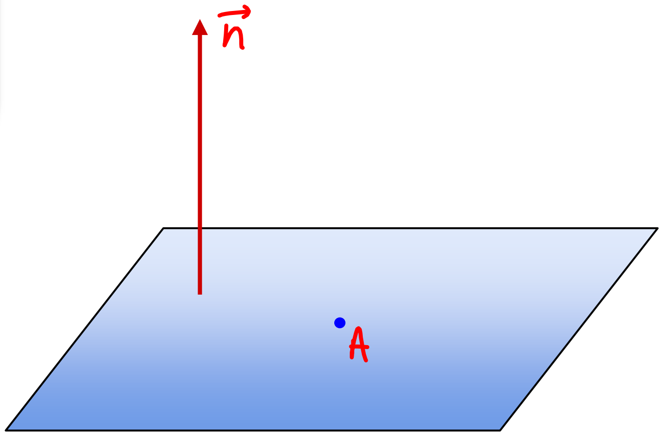
and normal vector to plane is $\vec{n}$

then Equation of Plane is given by $\boxed{\vec{r}.\vec{n} = \vec{a}.\vec{n}}$

① point $= \vec{a}$

② $\vec{n} = \vec{n}$

$\boxed{\vec{r}\ \vec{n} = \vec{a}\ n}$



Find the equation of plane passing through ***(2,0,1)*** and normal to ***2i + j - k*** in

I. Vector Form

II. Cartesian Form

Point $\vec{a} = (2,0,1)$

$\vec{n} = (2, 1, -1)$

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n} \rightarrow$ Normal vector

VF $\boxed{\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) = 3}$

CF $\boxed{2x + y - z = 3}$ (given)

$\boxed{\vec{n} = (2, 1, -1)}$

Note

If $ax + by + cz + d = 0$ is the equation of plane, then

a, b, c are direction ratios of normal or

$a\hat{i} + b\hat{j} + c\hat{k}$ is the normal vector

If the Equation of Plane is ***2x + y + 2z = 5***, then find

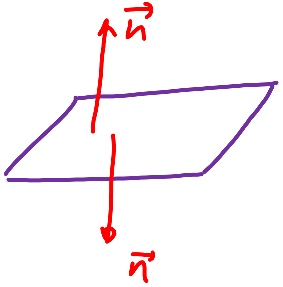
I. Normal Vector

II. DRs of Normal $\Rightarrow \lambda(2, 1, 2)$

III. DCs of Normal $\Rightarrow \pm\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

IV. Equation of plane in Vector Form

$$\boxed{\vec{r} \cdot (2\hat{i}+\hat{j}+2\hat{k}) = 5}$$



$$2x + y + 2z = 5$$

$$\sqrt{4+1+4}$$

$$\vec{n} = \pm(2\hat{i}+\hat{j}+2\hat{k})$$

$$DC = \pm\left(\frac{2\hat{i}+\hat{j}+2\hat{k}}{3}\right)$$

2. Three Points Form

Equation of plane passing through $A(\vec{a})$, $B(\vec{b})$, $C(\vec{c})$ is given by

$[\vec{r}-\vec{a} \quad \vec{b}-\vec{a} \quad \vec{c}-\vec{a}]=0$

Eqn of plane

① point $= \vec{a}$

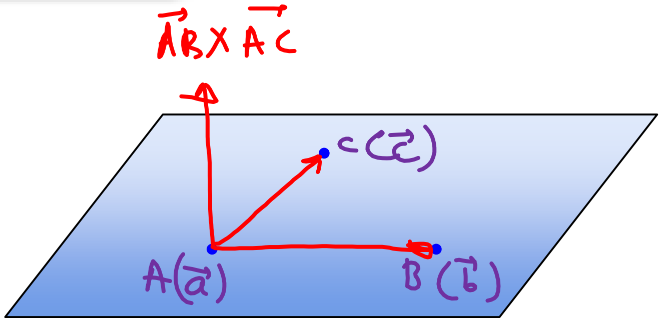
② $\vec{n} = \vec{AB} \times \vec{AC}$

$\vec{r} \cdot \vec{n} - \vec{a} \cdot \vec{n} = 0$

$(\vec{r}-\vec{a}) \cdot \vec{n} = 0$

$(\vec{r}-\vec{a}) \cdot (\vec{AB} \times \vec{AC}) = 0$

$[r-a \quad \vec{AB} \quad \vec{AC}] = 0$



3. Plane Containing Two Intersecting Lines

Equation of plane containing

$L_1 : \vec{r} = \vec{a_1} + \lambda \vec{b_1}$

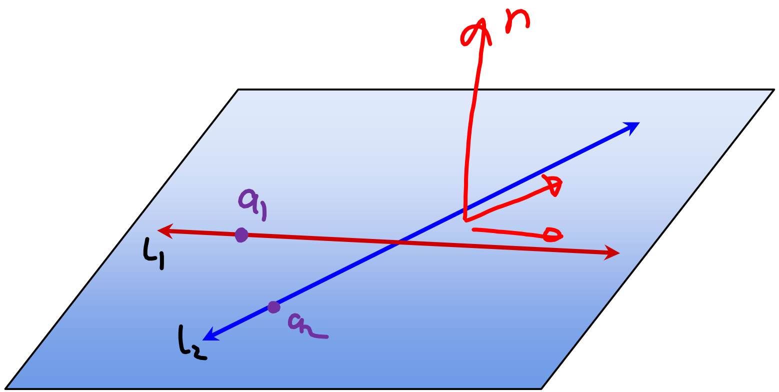
$L_2 : \vec{r} = \vec{a_2} + \lambda \vec{b_2}$

is given by $[\vec{r} - \vec{a_1} \quad \vec{b_1} \quad \vec{b_2}] = 0$

Eqn of plane

① point = $\vec{a}_1$ / $\vec{a}_2$

② normal = $\vec{b}_1 \times \vec{b}_2$

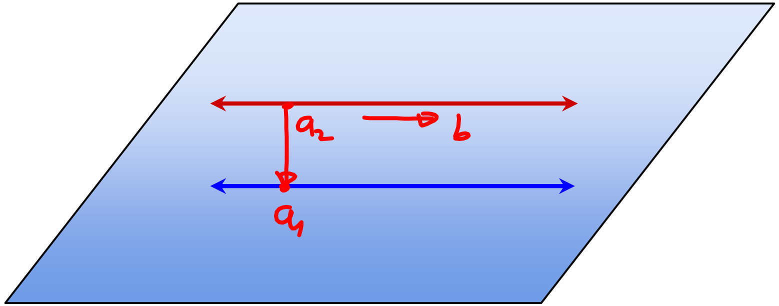


4. Equation of Plane Containing Two Parallel Lines

$$\boxed{\begin{aligned} \vec{r} &= \vec{a}_1 + \lambda \vec{b} \\ \vec{r} &= a_2 + \mu \vec{b} \end{aligned}}$$

point $= \vec{a}_1 \,/\, \vec{a}_2$

$$\vec{n} = \underline{(a_2 - a_1) \times \vec{b}}$$

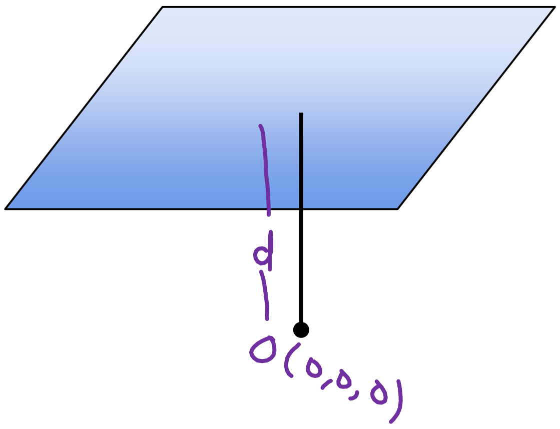


5. Equation of Plane: Normal Form

If unit vector normal to the plane is $\hat{n}$, and perpendicular distance of plane from origin is d then equation of plane is given by $\vec{r}.\hat{n} = d$

$\hat{n}$ d

$$\boxed{\vec{r}\,.\,\hat{n} = d}$$



Find the vector equation of plane which is at a distance of 8 **units** from the origin and which is normal to the vector **2i + j + 2k**.

$d = 8$

$\vec{n} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\hat{n} = \left(\dfrac{2\hat{i} + \hat{j} + 2\hat{k}}{3}\right)$

Eqn of Plane $\vec{r} \cdot \hat{n} = d$

$$\boxed{\vec{r} \cdot \left(\frac{2\hat{i} + \hat{j} + 2\hat{k}}{3}\right) = 8}$$

6. Intercept form

If x, y and z intercept of a plane is a, b, c

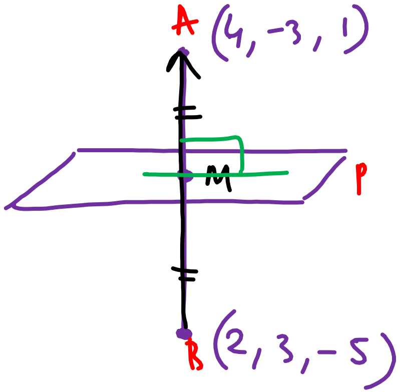
then Equation of Plane is given by $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$

$$\frac{x}{2}+\frac{y}{3}+\frac{z}{5}=1$$

Let the plane $ax + by + cz + d = 0$ bisect the line joining the points (4, -3, 1) and (2, 3, -5) at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

[18 Mar 2021 shift 1]



Eqn of Plane

① point = $(3, 0, -2)$

② $\vec{n} = \overrightarrow{BA} = (2, -6, 6)$

$= (1, -3, 3)$

$$\boxed{\vec{r} \cdot \vec{n} = \vec{a}\,\vec{n}}$$

$$(x, y, z) \cdot (1, -3, 3) = (3, 0, -2) \cdot (1, -3, 3)$$

$$\boxed{x - 3y + 3z = -3}$$

$$ax + by + cz + d = 0$$

$$\boxed{x - 3y + 3z + 3 = 0} \checkmark$$

$a = 1 \quad b = -3 \quad c = 3 \quad d = 3$

$$\boxed{a^2 + b^2 + c^2 + d^2 = 28}$$

Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2 + \beta^2 + \gamma^2 \neq 0$ and $\boxed{\alpha + \gamma = 1}$. Suppose the point $(3, 2, -1)$ is the mirror image of the point $(1, 0, -1)$ with respect to the plane $\alpha x + \beta y + \gamma z = \delta$. Then which of the following statements is/are TRUE?

[JEE Adv. 2020]

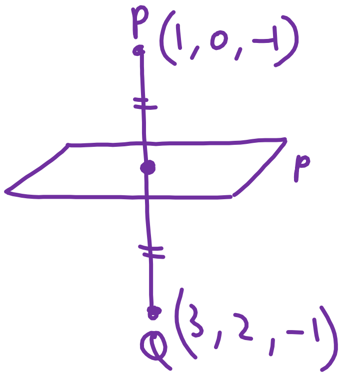
(A) $\overset{1}{\alpha} + \overset{1}{\beta} = 2$ ✓

(B) $\delta - \gamma = 3$ ✓

(C) $\overset{3}{\delta} + \overset{1}{\beta} = 4$ ✓

(D) $\alpha + \beta + \gamma = \delta$ ✗

ABC



Eqn of Plane

① point = (2, 1, -1)

② $\vec{n} = (2, 2, 0)$

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$(x, y, z) \cdot (2, 2, 0) = (2, 1, -1) \cdot (2, 2, 0)$

$\Rightarrow 2x + 2y = 6$

$$x + y = 3$$

$$\alpha x + \beta y + \gamma z = \delta$$

$$\alpha = 1 \quad \beta = 1 \quad \gamma = 0 \quad \delta = 3$$

$$\alpha + \gamma = 1$$

The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is

(a) $14x + 2y - 15z = 1$ (b) $14x - 2y + 15z = 27$

(c) $14x + 2y + 15z = 31$ (d) $-14x + 2y + 15z = 3$

[JEE Adv. 2017]

point = (1, 1, 1)

normal = $\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = \hat{i}(-14) - \hat{j}(2) + \hat{k}(-15)$

$= (-14, -2, -15)$

$= (14, 2, 15)$

$\vec{r}\,\vec{n} = \vec{a}\,\vec{n}$

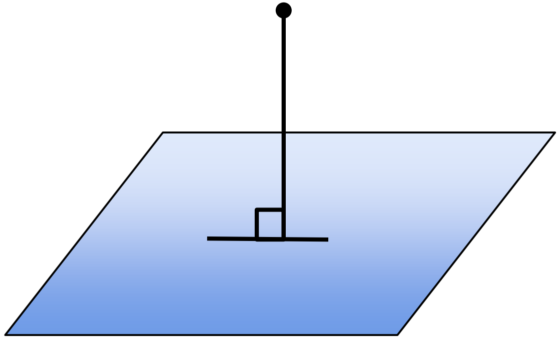
$\boxed{14x + 2y + 15z = 31}$

Perpendicular Distance of Point from Plane

Perpendicular Distance Of A Point From A Plane

Perpendicular distance of (x_1, y_1, z_1) from $ax + by + cz + d = 0$ is

$$\boxed{\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}}$$



A plane which is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, passes through $(1, -2, 1)$. The distance of the plane from the point $(1, 2, 2)$ is

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

[JEE Adv. 2006]

plane

① point $= (1, -2, 1)$

② Normal $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(-3) - \hat{j}(3) + \hat{k}(0)$

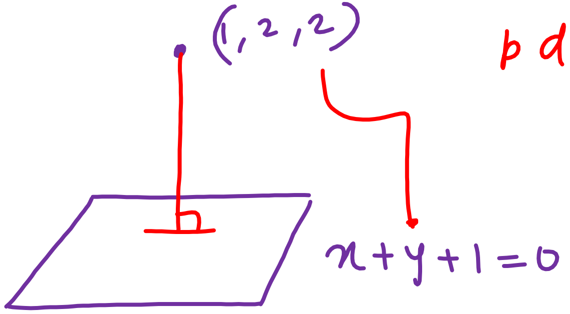
$= (-3, -3, 0)$

$= (1, 1, 0)$

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$(x, y, z) \cdot (1, 1, 0) = (1, -2, 1) \cdot (1, 1, 0)$

$\boxed{x + y = -1}$

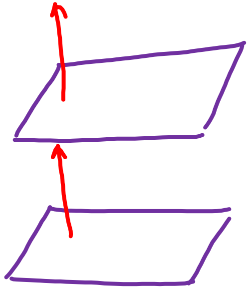


$$p\,d = \left|\frac{1+2+1}{\sqrt{1^2+1^2+0^2}}\right| = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Distance Between Two Parallel Planes

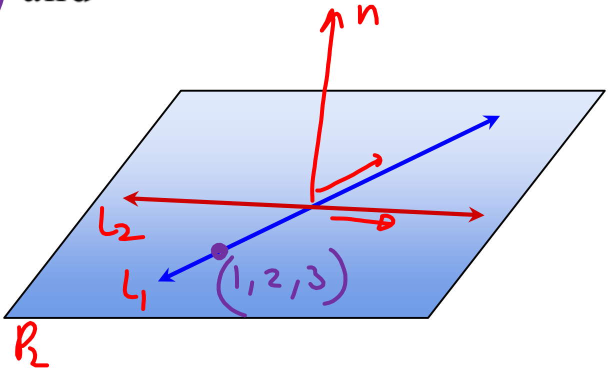
Distance between Two Parallel Planes

Distance between two parallel planes $ax + by + cz + d_1 = 0$ *and* $ax + by + cz + d_2 = 0$ is given by $\boxed{\dfrac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}}$



If the distance between the plane $\underbrace{Ax - 2y + z = d}_{P_1}$ and the plane containing the lines (P_2) L_1: $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and L_2: $\dfrac{x-2}{3}=\dfrac{y-3}{4}=\dfrac{z-4}{5}$ is $\sqrt{6}$, then find $|d|$.

P_1 & $P_2 \rightarrow \sqrt{6}$



Eqn of P_2 :-

① Point = (1, 2, 3)

② Normal = $\begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \hat{i}(-1) - \hat{j}(-2) + k(-1)$

$= (-1, 2, -1)$

[JEE Adv. 2010]

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$(x, y, z) \cdot (-1, 2, -1) = (1, 2, 3) \cdot (-1, 2, -1)$

P_2 $\boxed{-x + 2y - z = 0}$

P_1 $\boxed{Ax - 2y + z = d}$

P_2 $\boxed{x - 2y + z = 0}$

$A = 1$

$\underbrace{P_1 \text{ and } P_2}$

int OR Parallel

$$\left| \frac{d - 0}{\sqrt{1^2 + 2^2 + 1^2}} \right| = \sqrt{6}$$

$$\boxed{|d| = 6}$$

The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point $(1, 2, 3)$ is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is

[18 Mar 2021 Shift 1]

given plane $\quad x - 2y + 2z - 3 = 0$

$P : x - 2y + 2z + \lambda = 0 \qquad (1, 2, 3)$

$$pd = \left| \frac{1 - 4 + 6 + \lambda}{\sqrt{1^2 + 2^2 + 2^2}} \right| = 1$$

$|3 + \lambda| = 3$

$3 + \lambda = \pm 3$

$\boxed{\lambda = 0 ; \lambda = -6}$

$x - 2y + 2z = 0$

$x - 2y + 2z - 6 = 0$

$ax + by + cz + d = 0$

$a = 1 \quad b = -2$

$c = 2 \quad d = 0$

$$K = \frac{(b-d)}{(c-a)} = \frac{-2}{1}$$

Rej

$a = 1 \quad b = -2$

$c = 2 \quad d = -6$

$$\boxed{K = \frac{b-d}{(c-a)} = \frac{4}{1}}$$

$$\boxed{K = 4}$$

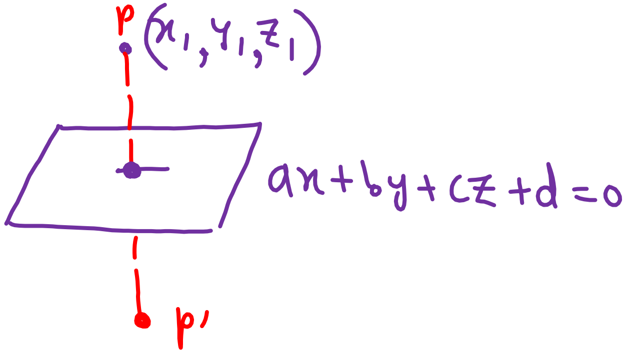
S C

Foot of Perpendicular / Image of Point in Plane

Image of Point in Plane

Image of the point (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$

is given by $\boxed{\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = -2\left(\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}\right)}$



Foot of Perpendicular from Point on the Plane

Foot of perpendicular of point (x_1, y_1, z_1) on the plane $ax + by + cz + d = 0$

is given by $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = -\left(\frac{ax_1+by_1+cz_1+d}{a^2+b^2+c^2}\right)$

If the mirror image of the point (1,3,5) with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals:

A. 47 B. 39 C. 43 D. 41

[26 Feb 2021 Shift 2]

$$\frac{x-1}{4} = \frac{y-3}{-5} = \frac{z-5}{2} = -2\left(\frac{4-15+10-8}{16+25+4}\right)$$

$$\frac{4}{5}+5$$

$$\frac{x-1}{4} = \frac{y-3}{-5} = \frac{z-5}{2} = \frac{2}{5}$$

$$(\alpha, \beta, \gamma) = \left(\frac{13}{5}, 1, \frac{29}{5}\right)$$

If the distance of the point $P(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is

(a) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (b) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

(c) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (d) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

[JEE Adv. 2010]

$P(1, -2, 1)$

$x + 2y - 2z - \alpha = 0$

$$\left|\frac{1 + (-4) - 2 - \alpha}{\sqrt{1^2 + 2^2 + 2^2}}\right| = 5 \Rightarrow \left|\frac{+5 + \alpha}{3}\right| = 5$$

$|5 + \alpha| = 15$

$5 + \alpha = \pm 15$

$\boxed{\alpha = 10}$ $\boxed{\cancel{\alpha = -20}}$

$P(1, -2, 1)$

$x + 2y - 2z - 10 = 0$

$\frac{15}{9} = 5/3$

$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{-2} = -1\left(\frac{1-4-2-10}{9}\right)$$

$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{-2} = \frac{5}{3}$$

$$x = \frac{8}{3}$$

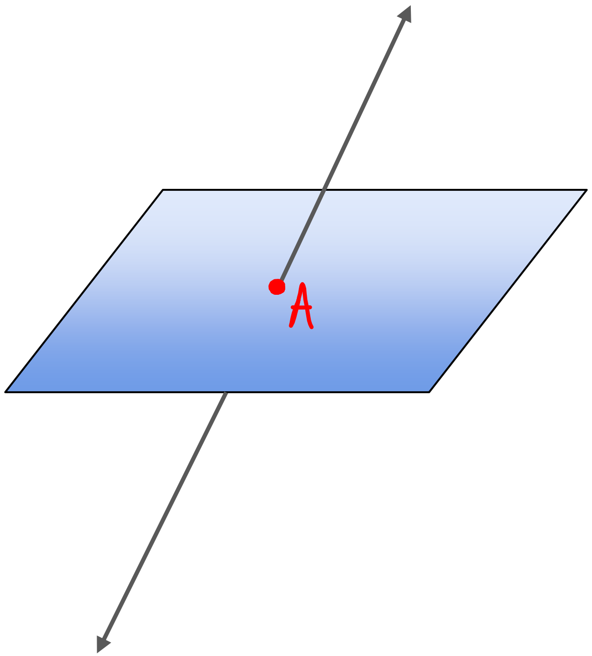
Unacademy Atoms

Solving Line and Plane

Solving Line and Plane

Method:

Take a general point on line and substitute it in plane



The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane x + y + z = 17 is :

JEE M 2024

(1) $\sqrt{38}$

(2) $19\sqrt{2}$

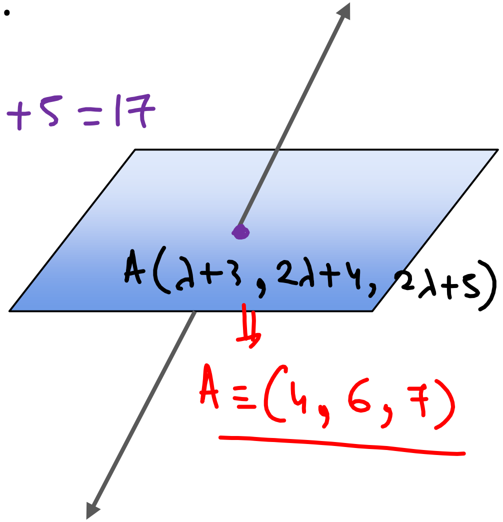
(3) $2\sqrt{19}$

(4) 38

$\lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$5\lambda = 5$

$\lambda = 1$



$A = (4, 6, 7)$

(1, 1, 9)
(4, 6, 7)

$d = \sqrt{3^2 + 5^2 + 2^2}$

$= \sqrt{9 + 25 + 4}$

$= \sqrt{38}$ //

Projection of Line Segment on Plane

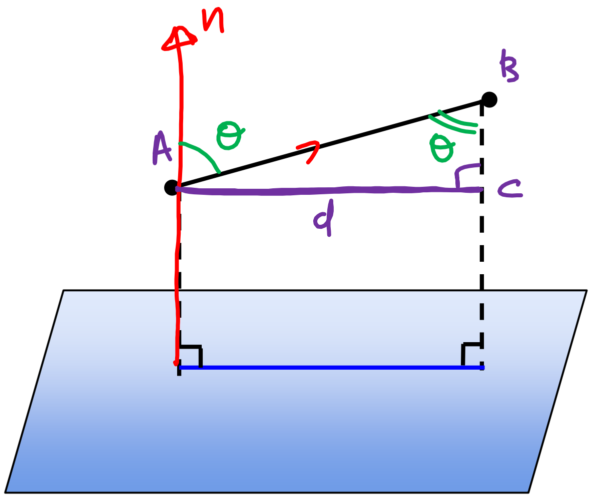
Projection of Line Segment on Plane

$$\frac{d}{|AB|} = \sin\theta$$

$$d = |AB| \sin\theta$$

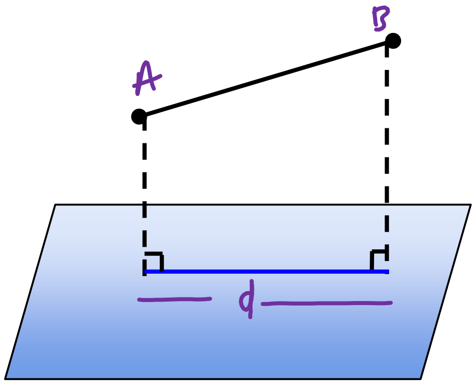
$$= \cancel{|AB|} \frac{\vec{AB} \times \vec{n}}{\cancel{|AB|}\,|\vec{n}|}$$

$$\boxed{d = \frac{|\vec{AB} \times \vec{n}|}{|\vec{n}|}}$$



The length of the projection of the line segment joining the points $\underset{A}{(5,-1,4)}$ and $\underset{B}{(4,-1,3)}$ on the plane, $x+y+z=7$ is:

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) ✓ $\sqrt{\frac{2}{3}}$ (d) $\frac{2}{\sqrt{3}}$



[JEE M 2018]

$\vec{AB} \equiv (-1, 0, -1)$

$\vec{n} = (1, 1, 1)$ $\boxed{|\vec{n}| = \sqrt{3}}$

$$\vec{AB} \times \vec{n} = \begin{vmatrix} i & j & k \\ -1 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(1) - \hat{j}(0) + k(-1)$$

$$= (1, 0, -1)$$

$$|\vec{AB} \times \vec{n}| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$$

Angle between Plane-Plane, Line-Line and Line-Plane

Angle Between Two Planes

Angle between two planes is same as angle between their normal

$$\boxed{\cos\theta = \frac{\overrightarrow{n_1}.\overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|}}$$

Angle Between Two Planes

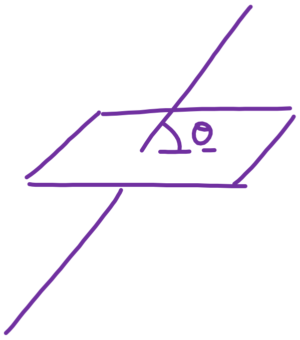
Angle between two lines is same as angle between direction vectors

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$$

Angle Between Line and Plane

$$\boxed{\sin\theta = \frac{\vec{n}.\vec{b}}{|\vec{n}||\vec{b}|}}$$

$$\boxed{\sin\theta = \frac{\vec{n}\cdot\vec{b}}{|n|\,|b|}}$$



Parallel and Perpendicular Planes

Two plane $a_1x + b_1y + c_1z + d_1 = 0$ *and* $a_2x + b_2y + c_2z + d_2 = 0$ *are*

1. *Parallel if* $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} \neq \dfrac{d_1}{d_2}$
2. *Identical if* $\boxed{\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} = \dfrac{d_1}{d_2}}$
3. *Perpendicular if* $\boxed{a_1a_2 + b_1b_2 + c_1c_2 = 0}$

$P_1 \perp P_2$

$\boxed{90^\circ}$

$\boxed{\vec{n}_1 \cdot \vec{n}_2 = 0}$

Consider the three planes

$P_1 : 3x + 15y + 21z = 9$

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

$$\boxed{\frac{3}{2} = \frac{15}{10} = \frac{21}{14} \neq \frac{9}{5}}$$

Then, which one of the following is true ?

A. P_1 and P_3 are parallel.

B. P_2 and P_3 are parallel.

C. P_1 and P_2 are parallel.

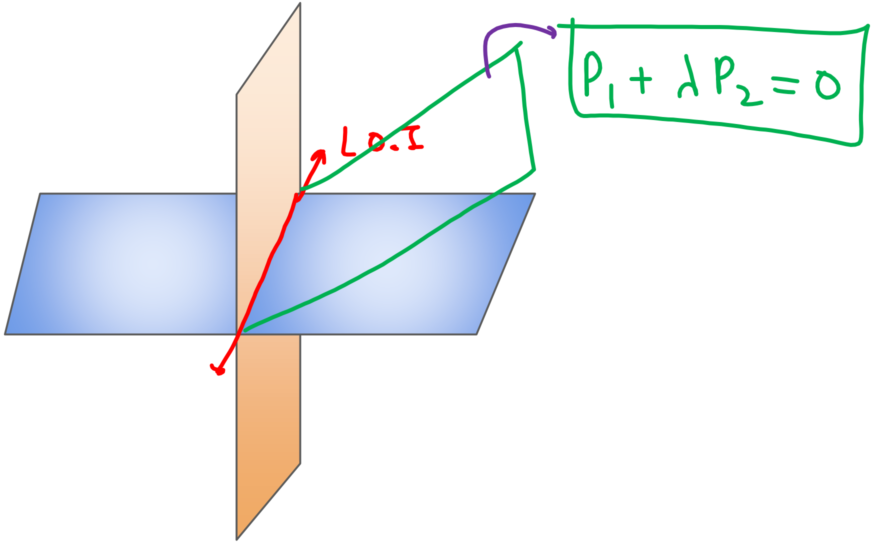
D. P_1, P_2 and P_3 all are parallel.

[26 Feb 2021 Shift 1]

Family of Planes

Family of Planes

Equation of Plane passing through the line of intersection of planes P_1 and P_2 is given by $P_1 + \lambda P_2 = 0$



The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point (1, 1, 0) is :

[08 Apr 2019 Shift 1]

A. $x - 3y - 2z = -2$

B. $2x - z = 2$

C. $x - y - z = 0$

D. $x + 3y + z = 4$

$(2x - y - 4) - 1(y + 2z - 4) = 0$

$(2x - y - 4) + \lambda(y + 2z - 4) = 0$

$2x + (-1+\lambda)y + (2\lambda)z + (-4-4\lambda) = 0$

$2(1) + (-1+\lambda)1 + 0 - 4 - 4\lambda = 0$

$2 - 1 + \lambda - 4 - 4\lambda = 0$

$-3\lambda - 3 = 0$

$\boxed{\lambda = -1}$

If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0$, $3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$ is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is

[17 Mar 2021 Shift 1]

$$(2x - 7y + 4z - 3) + \lambda(3x - 5y + 4z + 11) = 0$$

$$(-4 - 7 + 12 - 3) + \lambda(-\cancel{6} - \cancel{5} + 12 + \cancel{11}) = 0$$

$$(-2) + \lambda(12) = 0$$

$$\boxed{\lambda = 1/6}$$

$$6\left\{P_1 + \frac{1}{6}P_2 = 0\right\}$$

$$6P_1 + P_2 = 0$$

$$6(2x - 7y + 4z - 3) + (3x - 5y + 4z + 11) = 0$$

$$15x - 47y + 28z - 7 = 0$$

$$a = 15 \quad b = -47 \quad c = 28$$

$$2a + b + c - 7$$

$$= 30 - 47 + 28 - 7 = 4$$

Angle Bisector

Bisector of Angle between Two Lines

$L_1 \quad \vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ ✓

$L_2. \quad \vec{r} = \vec{a}_2 + \mu \vec{b}_2$ ✓

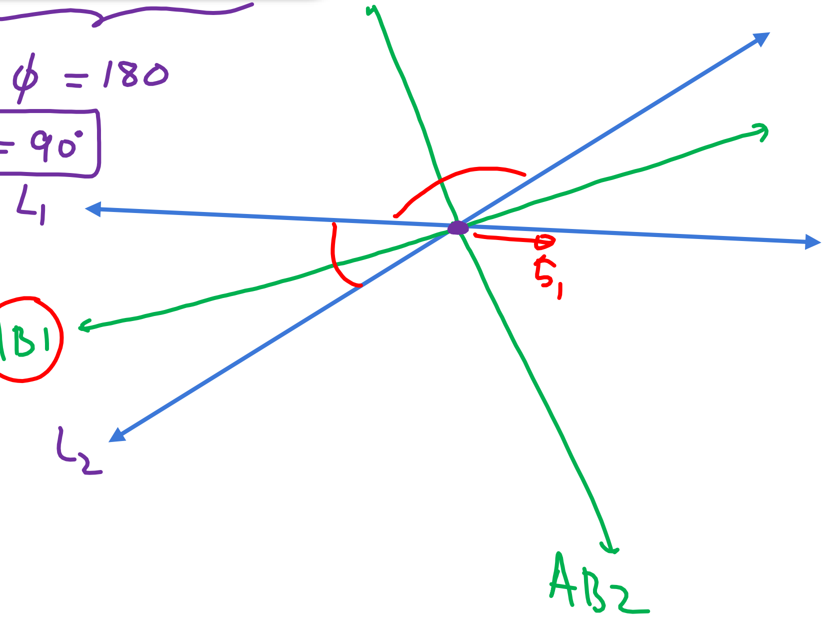
$2\theta + 2\phi = 180$

$\boxed{\theta + \phi = 90^\circ}$

$\boxed{AB1 \perp AB2}$

AB1 P.O.I $\hat{b}_1 + \hat{b}_2$

AB2 P.O.I $\hat{b}_1 - \hat{b}_2$



Bisector of Angle between Two Lines

	Acute Angle Bisector	Obtuse Angle Bisector
$\boxed{b_1 . b_2 > 0}$ +	$\widehat{b_1} + \widehat{b_2}$	$\widehat{b_1} - \widehat{b_2}$
$\boxed{b_1 . b_2 < 0}$	$\widehat{b_1} - \widehat{b_2}$	$\widehat{b_1} + \widehat{b_2}$

$b_1 \quad b_2 = +$

Unacademy Atoms

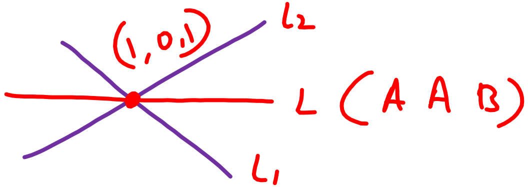
Let L_1 and L_2 be the following straight lines.

$$\boxed{L_1: \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{3}} \text{ and } \boxed{L_2: \frac{x-3}{-3} = \frac{y}{-1} = \frac{z-1}{1}}$$

$\boxed{\text{POI } (1, 0, 1)}$

Suppose the straight line

$$\boxed{\text{AAB } L_1 \,\&\, L_2} \Leftarrow \boxed{L: \frac{x-\alpha}{l} = \frac{y-1}{m} = \frac{z-\gamma}{-2}}$$



AB

lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE?

A. $\alpha - \gamma = 3$

B. $l + m = 2$ (1+1)

C. $\alpha - \gamma = 1$

D. $l + m = 0$

[JEE Adv. 2020]

$\vec{b}_1 \cdot \vec{b}_2 = (1, -1, 3) \cdot (-3, -1, 1)$

$= -\cancel{3} + 1 + \cancel{3}$

$= +$

$$\boxed{\frac{1-\alpha}{1} = \frac{-1}{1} = \frac{1-\gamma}{-2}}$$

$\alpha = 2 \quad \gamma = -1$

dirⁿ of L : $\hat{b}_1 + \hat{b}_2$

$$= \frac{(1, -1, 3)}{\sqrt{11}} + \frac{(-3, -1, 1)}{\sqrt{11}}$$

$$= \left(\frac{-2}{\sqrt{11}}, \frac{-2}{\sqrt{11}}, \frac{4}{\sqrt{11}}\right)$$

$l = 1$
$m = 1$

$$= (-2, -2, 4)$$

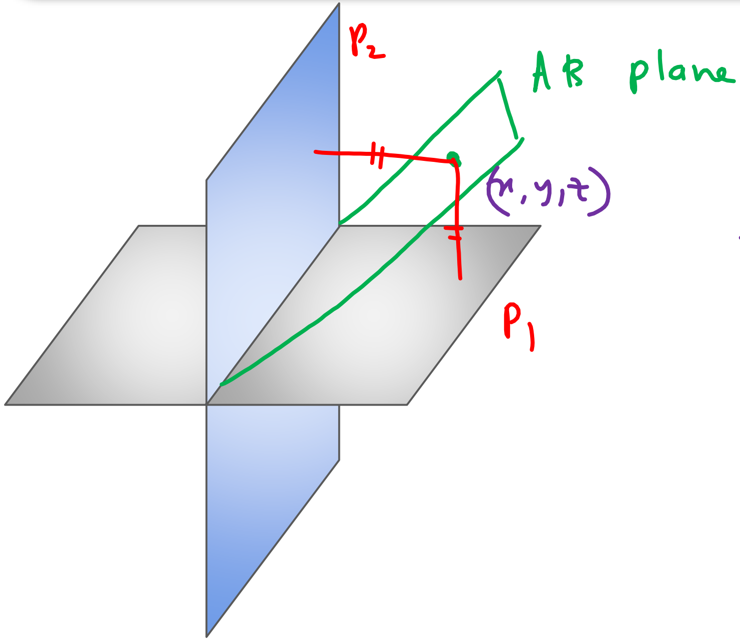
$$= (1, 1, -2)$$

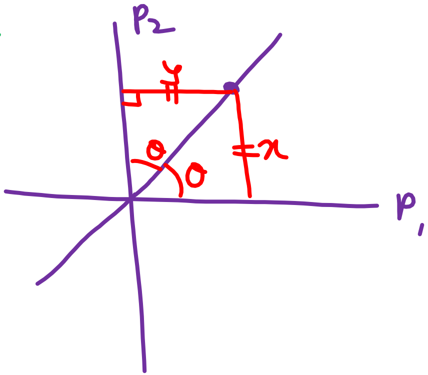
$$= (l, m, -2)$$

Bisector of Angle between Two Planes

SL

$$\left(\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}\right)=\pm\left(\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}\right)$$





Bisectors

1. Make constant terms positive

2.

	Obtuse Angle Bisector	Acute Angle Bisector
$a_1a_2 + b_1b_2 + c_1c_2 > 0$	+	-
$a_1a_2 + b_1b_2 + c_1c_2 < 0$	-	+

$\vec{n}_1 \cdot \vec{n}_2 = ?$

Equation of plane bisecting the acute angle between the planes $x - y + z - 1 = 0$ and $x + y + z - 2 = 0$

A. $x + z = 3/2$

B. $2y = 1$

C. $x - y - z = 3$

D. $x + 2z = 3$

$\vec{n}_1 \cdot \vec{n}_2 = +$

Acute Angle Bisector

1. $\boxed{-x + y - z + 1 = 0 \quad \boxed{P_1} \\ -x - y - z + 2 = 0 \quad \boxed{P_2}}$

$(-1, 1, -1) = \vec{n}_1$

$(-1, -1, -1) = \vec{n}_2$

$$\left(\frac{-x + y - z + 1}{\cancel{\sqrt{1^2 + 1^2 + 1^2}}}\right) = -\left(\frac{-x - y - z + 2}{\cancel{\sqrt{1^2 + 1^2 + 1^2}}}\right)$$

$$-x+y-z+1 = -(-x-y-z+2)$$

$$-x+\cancel{y}-z+1 = x+\cancel{y}+z-2$$

$$\boxed{x+z = 3/2}$$

$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}}$$

Equation of Straight Line in Unsymmetrical Form

Equation of Straight Line in Symmetrical form

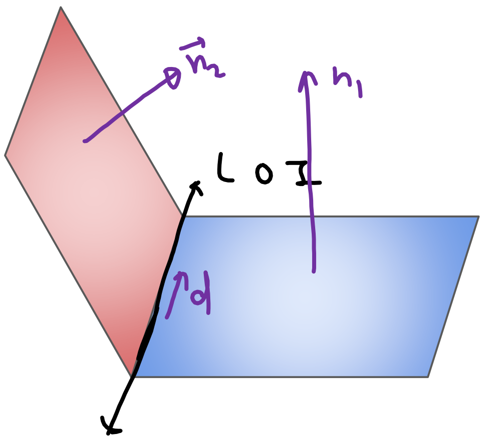
$$\boxed{\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}}$$

(x_1, y_1, z_1) point

$(a, b, c) =$ d v. / d R

Equation of Straight Line in Unsymmetrical form

$$\underline{a_1x + b_1y + \cancel{c_1z} + d_1} = \underline{a_2x + b_2y + \cancel{c_2z} + d_2} = 0$$



$\underline{\text{L o I}}$

① point. $\boxed{z = 0 \quad x = \checkmark \quad y = \checkmark}$

② $\boxed{\text{dir}^n = \vec{n}_1 \times \vec{n}_2}$

Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$

F **Statement-1 :** The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$, $y = 1 + 2t$, $z = 15t$. Because

T **Statement-2 :** The vector $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the lines of intersection of given planes.

[JEE Adv. 2007]

$$\left.\begin{array}{l} 3x - 6y - 2z = 15 \\ 2x + y - 2z = 5 \end{array}\right\} \text{L.O.I.}$$

① Point $z = 0$

$$3x - 6y = 15$$
$$6 \times (2x + y = 5)$$
$$12x + 6y = 30$$
$$15x = 45$$

$x = 3$

$y = -1$

Point $(3, -1, 0)$

$$\text{dir}^n = n_1 \times n_2$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(14) - \hat{j}(-2) + \hat{k}(15)$$

$$= \underline{14\hat{i} + 2\hat{j} + 15\hat{k}}$$

LOI

$$\frac{x-3}{14} = \frac{y+1}{2} = \frac{z-0}{15} = t$$

$$(14t+3,\ \underline{2t-1},\ 15t)$$

Equation of Sphere

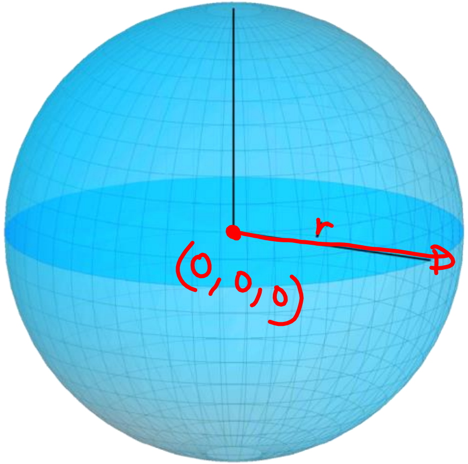
(JEE M)

Equation of Sphere

Equation of Sphere with centre at origin and radius r is given by

$$x^2 + y^2 + z^2 = r^2$$

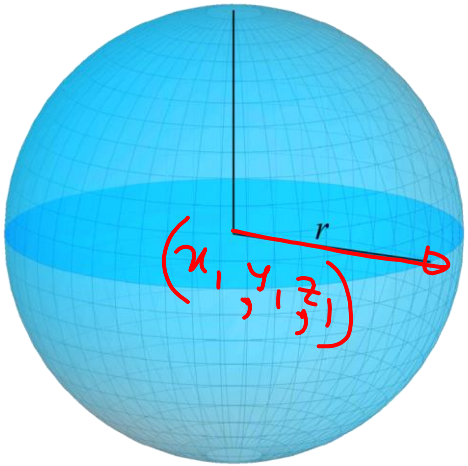
$$\boxed{x^2 + y^2 + z^2 = r^2}$$



Equation of Sphere

Equation of Sphere with centre at (x_1, y_1, z_1) and radius r is given by

$$\boxed{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r^2}$$



General Equation of Sphere

$x^2+y^2+2gx+2fy+c=0$

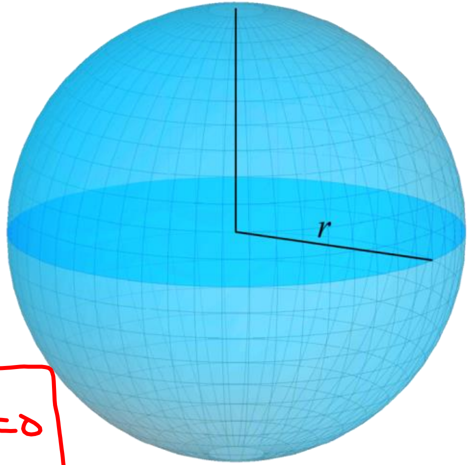
$c(-g,-f)$

$\text{rad}=\sqrt{g^2+f^2-c}$

$x^2+y^2+z^2+2gx+2fy+2hz+c=0$

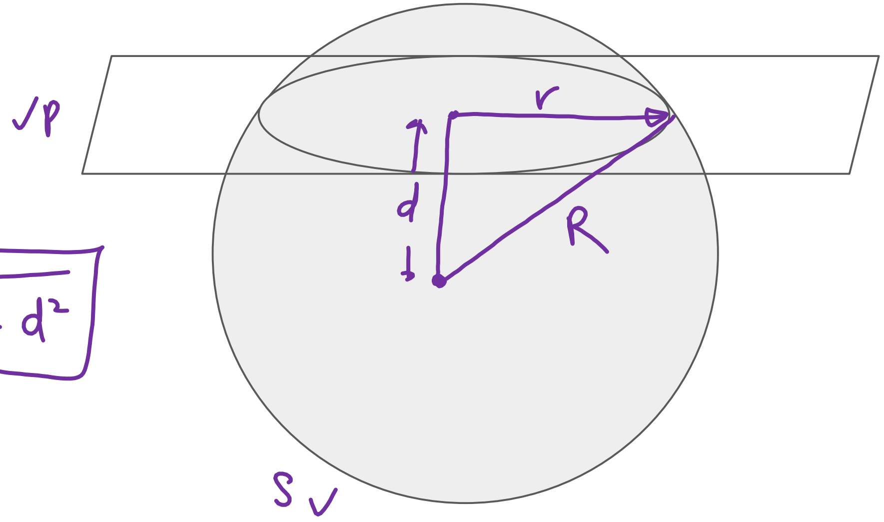
$c(-g,-f,-h)$

$\text{rad}=\sqrt{g^2+f^2+h^2-c}$



Plane and Sphere

JEE m



$$\boxed{r = \sqrt{R^2 - d^2}}$$

The plane x + 2y - z = 4 cuts the sphere $x^2 + y^2 - x + z - 2 = 0$
In a circle of radius

[2005]

A. 3

B. 1

C. 2

D. $\sqrt{2}$

$\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$

$\boxed{C\left(\frac{1}{2}, 0, \frac{-1}{2}\right)}$

$rad = \sqrt{\left(\frac{1}{2}\right)^2 + 0^2 + \left(\frac{1}{2}\right)^2 - (-2)}$

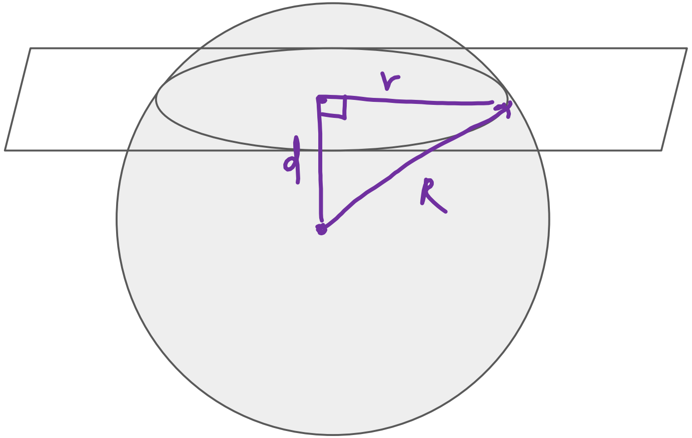
$R = \sqrt{\frac{1}{2} + 2} = \sqrt{\frac{5}{2}}$

$d = \left| \frac{\frac{1}{2} + 0 + \frac{1}{2} - 4}{\sqrt{1^2 + 2^2 + 1^2}} \right|$

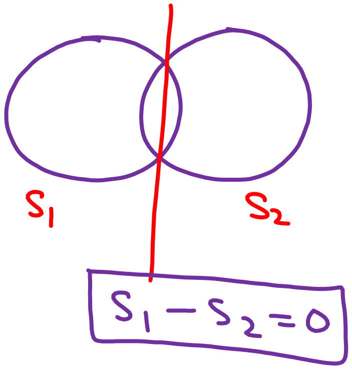
$\boxed{d = \frac{3}{\sqrt{6}}}$

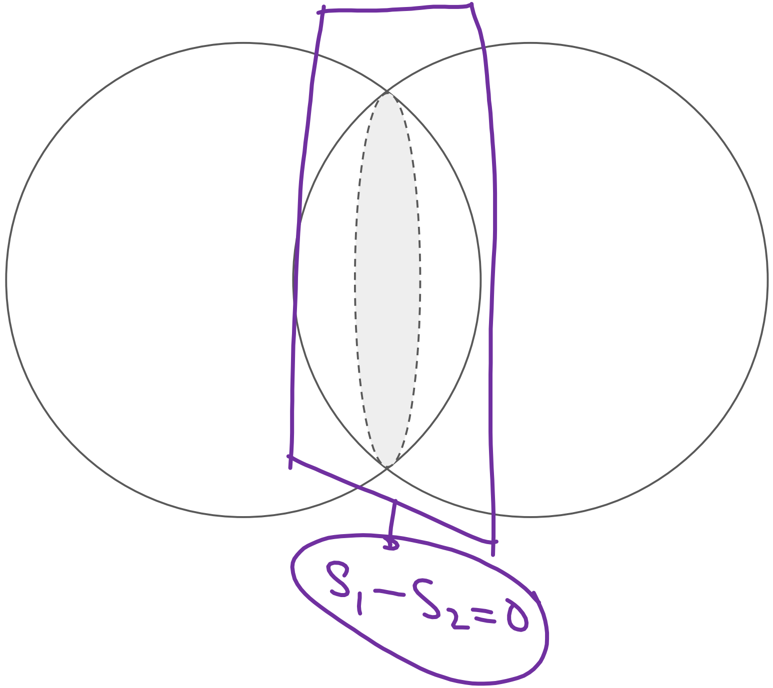
$r = \sqrt{R^2 - d^2}$

$= \sqrt{\frac{5}{2} - \frac{9}{6}} = 1$



Intersection of Two Spheres





The intersection of the spheres

$x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and

$x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane

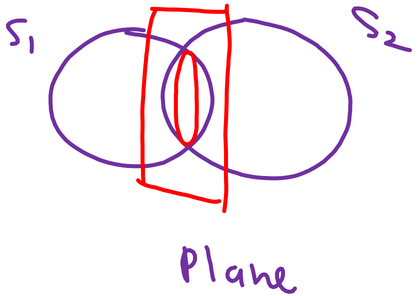
[2004]

A. $2x - y - z = 1$

B. $x - 2y - z = 1$

C. $x - y - 2z = 1$

D. $x - y - z = 1$



$10x - 5y - 5z = 5$

$2x - y - z = 1$

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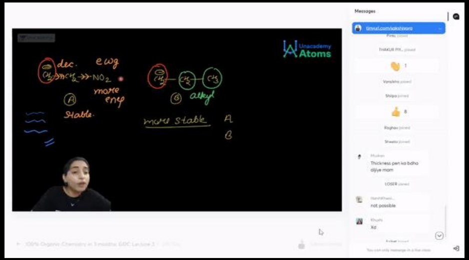

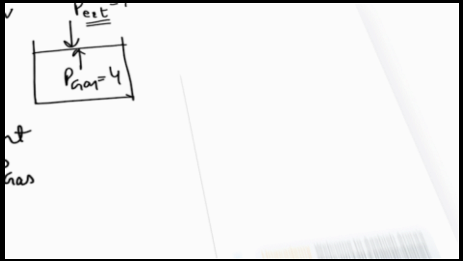

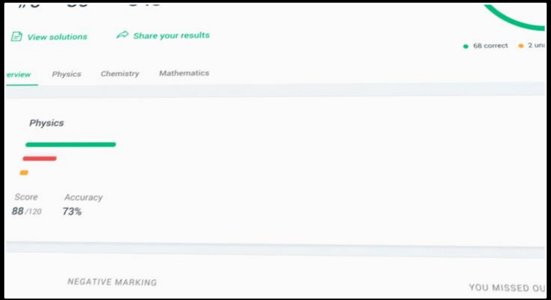

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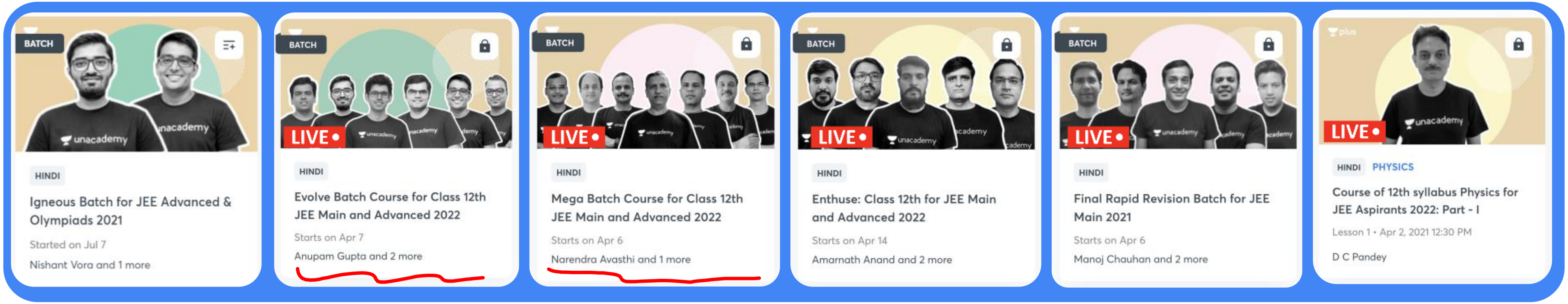

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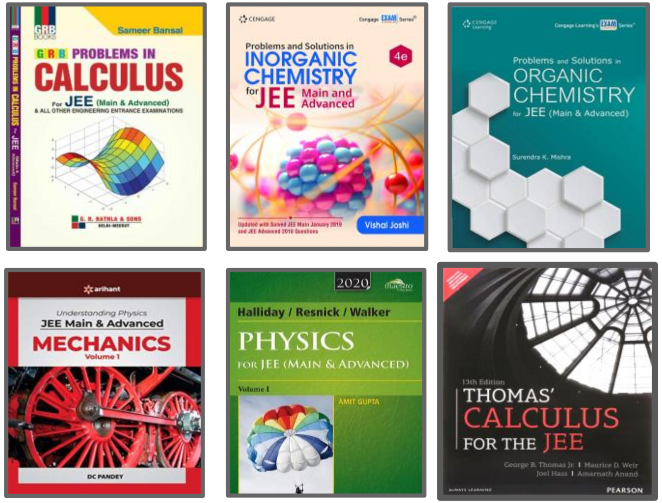


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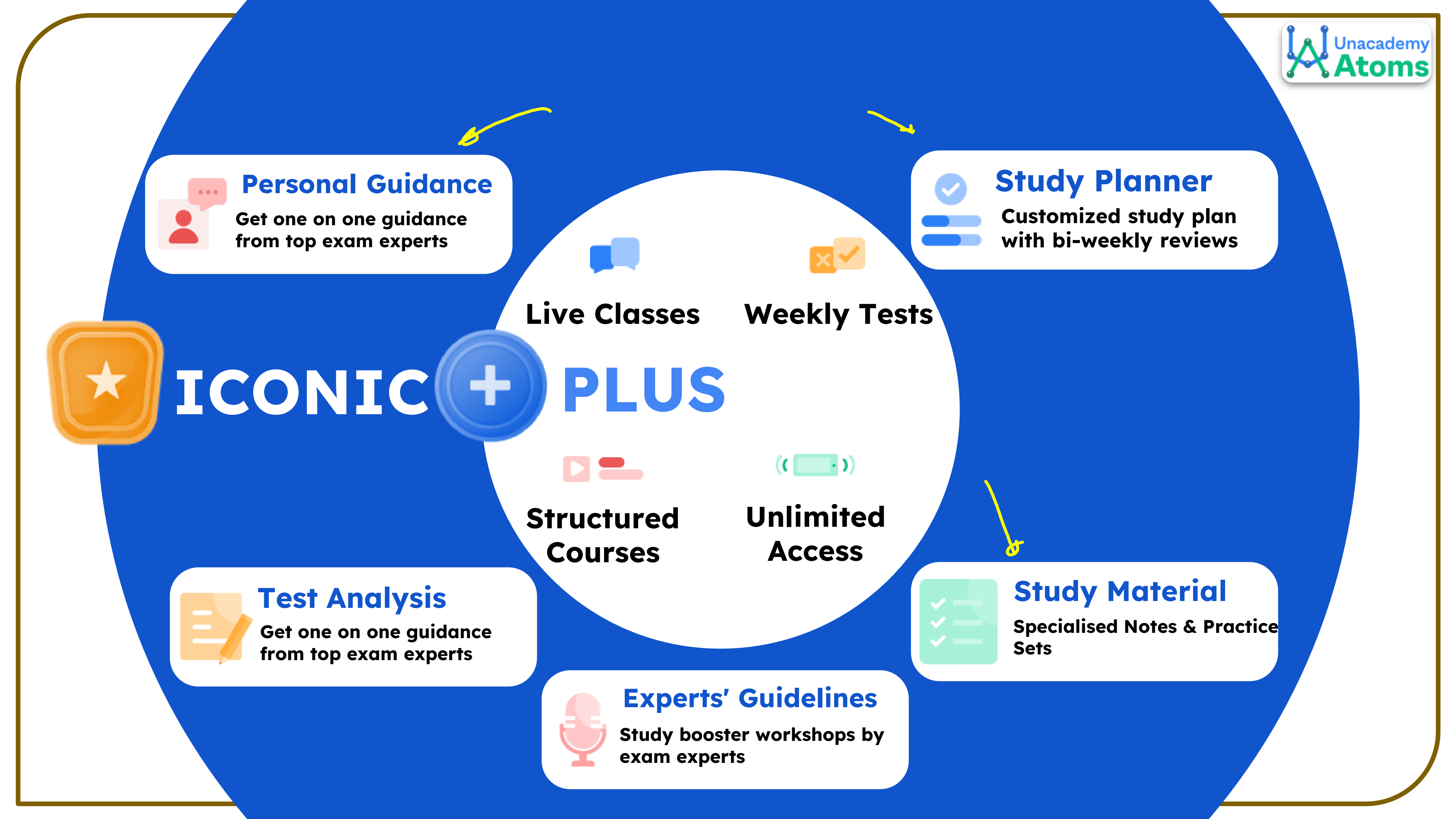

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