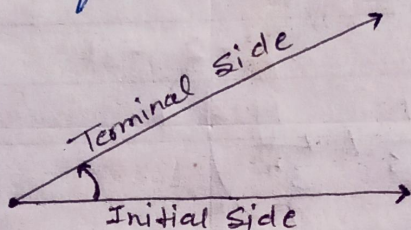


Trigonometric Function

Angle: Angle is measure of rotation of a given ray about its initial point.

- The original ray is called initial side of angle.
- The final position of ray after rotation is called final side of angle.
- The point of rotation is called the vertex.
- Direction of rotation \rightarrow clockwise \rightarrow angle is (-ve)
- Direction of rotation \rightarrow anti-clockwise \rightarrow angle is (+ve)



Degree measure :- If the rotation of initial side to terminal side is $(\frac{1}{360})^{\text{th}}$ of a revolution then the angle is said to be have measure of 1 degree, written as 1° .

$$1^\circ = 60'$$

$$60' = 60''$$

Radian measure :- Angle subtended by 1 unit length of arc at centre of 1 unit circle is called radian measure.

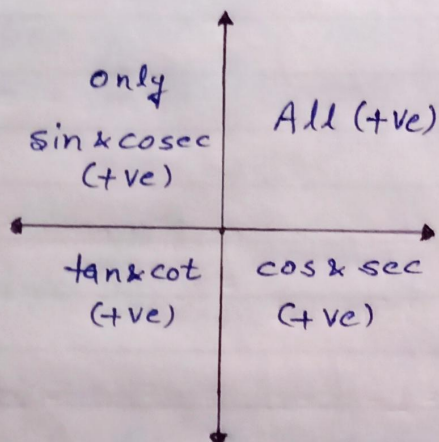
$$\theta = \frac{l}{r}$$

Relation b/w degree & Radian :-

$$\text{Radian measure} = \frac{\pi}{180} \times \text{degree measure}$$

$$\text{Degree measure} = \frac{180}{\pi} \times \text{Radian measure.}$$

Sign of trigonometric function :-



Trigonometric Formulas

Pythagorean Identity

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta.$$

Reciprocal Identity

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}.$$

Even - Odd Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

Sum & difference formula

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$1 \mp \tan x \tan y$$

$$\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

No-function identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$$

$$\sin(\pi - x) = \sin x$$

$$\cos(\pi - x) = -\cos x$$

$$\sin(\pi + x) = -\sin x$$

$$\cos(\pi + x) = -\cos x$$

$$\sin(2\pi - x) = -\sin x$$

$$\cos(2\pi - x) = \cos x$$

Double-angle formula

$$\sin 2\theta = 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Triple-angle formula :-

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

~~$$\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$~~

~~$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$~~

~~$$\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x}$$~~

Half-angle formula:-

$$\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{(1 - \cos x)}{\sin x}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\bullet 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2} = \sin x$$

$$\bullet 1 - \cos x = 2 \sin^2 \frac{x}{2}$$

$$\bullet \cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\bullet 1 - \cos 2x = 2 \sin^2 x$$

$$\bullet 1 + \cos 2x = 2 \cos^2 x$$

	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined	0	not defined	0