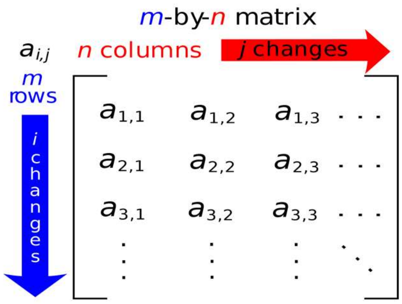
Matrix Quick Revision Notes

Matrix: A matrix is a **rectangular array of numbers** (or other mathematical objects).

Example -



Types of Matrices:

1) Column Matrix: A type of matrix that has only **one column.**

Example-

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \cdots \\ a_m \end{bmatrix} = a_i \quad (m \times 1)$$

2) Row Matrix: A type of matrix that has only **one row.**

Example-

$$N = \begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$$

3) Horizontal Matrix: A matrix in which **the number of columns is more that number of rows.**

$$P = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

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4) Vertical Matrix: A matrix in which **the number of rows is more that number of columns.**

Example:

$$Q = \begin{bmatrix} -1 & -2 \\ 3 & 4 \\ 5 & 6 \\ -7 & 8 \end{bmatrix}$$

5) Square Matrix: A matrix in which **the number of rows and columns are same.**

Example-

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

6) Diagonal and Scaler Matrix:

a) Diagonal Matrix: If all the **elements of square matrix are equals to zero except diagonal element**.

Example-

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

b) Scaler Matrix: If all the **diagonal elements of diagonal matrix are same**.

Example- $$\mathbf{B} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

7) Identity and Unit Matrix: A diagonal matrix whose **all diagonal elements are equals to unit (or 1)**.

Example- $$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- When a matrix is multiplied by a unit matrix, it remains same.
  AI=A=IA

8) <u>Null Matrix</u>: A matrix is said to be null matrix, when **all the elements of the that matrix is equals to zero**.

Example- $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

- When a matrix is added or subtracted to a null matrix, it remains same.

9) <u>Triangle Matrix</u>: If all elements above or below the leading/principle diagonal are zero in a square matrix, it is called triangular matrix.

a) <u>Upper Triangular Matrix</u>: A square matrix whose all **elements below the leading diagonal is zero** (i.e., $a_{ij} = 0$, $I > j$).

<u>Example</u>-

$$\begin{matrix} 1 & 2 & 4 & 5 \\ 0 & 2 & 3 & 6 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 4 \end{matrix}$$

b) <u>Lower Triangular Matrix</u>: A square matrix whose all **elements above the leading diagonal is zero** (i.e., $a_{ij} = 0$, $I < j$).

<u>Example</u>-

$$\begin{matrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 6 & 5 & 1 \end{matrix}$$

10) <u>Symmetric and Skew-Symmetric Matrix</u>:

a) Symmetric <u>Matrix</u>: A matrix is called a symmetric matrix if its **transpose is equal to the matrix itself**.

b) <u>Skew-Symmetric Matrix</u>: A matrix is called a symmetric matrix if its **transpose is equal to the negative of matrix itself**.

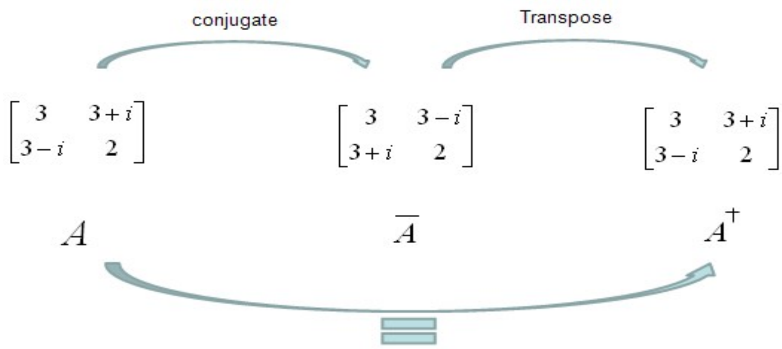
<u>Example</u>-

Symmetric	Skew-symmetric
$A^T = A$	$A^T = -A$
$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 5 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$

11) <u>Hermition and Skew-Hermiton Matrix</u>:

a) <u>Hermition Matrix</u>: A square matrix is said to hermition matrix, **if the transpose of its conjugate is equals to square matrix itself.**

- $A = (\bar{A})^T$.
- <u>Example-</u>



b) <u>Skew-Hermition Matrix</u>: A square matrix is said to skew-hermition matrix, **if the transpose of its conjugate is equals to negative of square matrix itself.**

- $A = -(\bar{A})^T$.

12) <u>Orthogonal Matrix</u>: The **matrix** is said to be an **orthogonal matrix** if the product of a **matrix** and its transpose gives an identity value.

- $A * A^T = I$.

<u>Transpose Matrix</u>: The transpose of the matrix can be found **by interchanging rows and columns.**

- Denoted by $\mathbf{A^T}$.

<u>Adjoint Matrix</u>: The adjoint of the matrix can be defined as **the transpose of the cofactor of that particular matrix.**

- Denoted by **Adj.A**.
- Here positive and negative sign depends upon the sum of the indices of the matrix if the sum is an odd number, then sign will be negative else remains positive.

$$A_{2\times2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$A_{11} = |d| \quad A_{12} = -|c| \quad A_{21} = -|b| \quad A_{22} = |a|$

- Cofactor of A will be,

$$\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$$

- Now transpose of this matrix.

$$Adj.\,A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

<u>Inverse Matrix</u>: Inverse matrix is obtained by **dividing the adjugate of the given matrix by the determinant of the given matrix.**

- Denoted by **A^{-1}.**

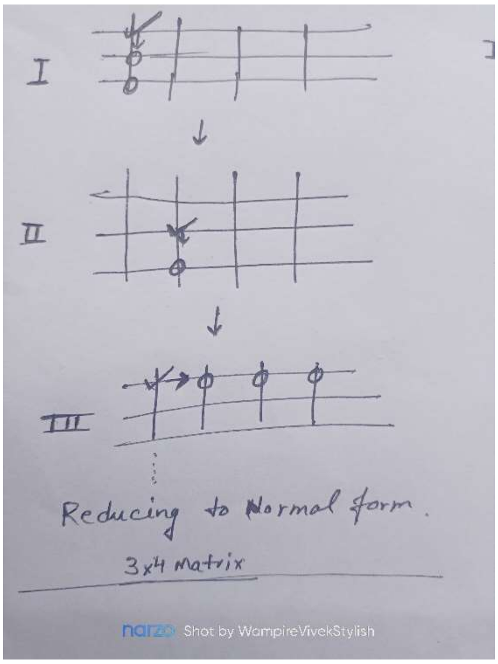
$$A^{-1} = \frac{1}{|A|}\,\text{adj}\,A$$

Unitary Matrix: A matrix is said to be unitary matrix, **when product of the It's conjugate transpose with itself is equals to a unit matrix**.

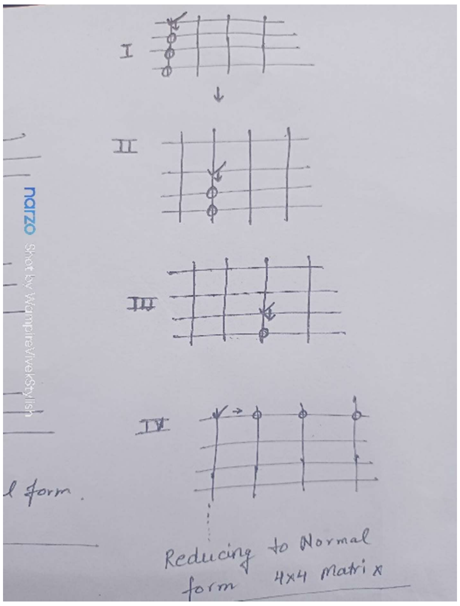
- $A.A^{Q} = I$
- $A^{Q} = (\bar{A})^{T}$

Problems Solving Methods:

1. **Linear Equation:** Use **AX = B**, where A is the given matrix and X is the unknown matrix and B is the value of linear equation.
2. **Linear Dependence and Independence:**
   a. Convert the given vectors into linear equations by multiplying with a_1, a_2, a_3, … (scalars).
   b. Now using Cramer's Rule, find the value of a_1, a_2, a_3, …
   c. Then put these values in one of all equations.
   d. If the **final value becomes 0 then it's a linearly dependent system else it's a linearly independent system**.
3. **Rank of a Matrix:**
   a. **Rank by E-transform:** After transformation **number of non-zero rows will be the rank of that matrix.**
   b. **Reducing to Normal Form:** After reducing to normal form **the dimension or size of the identity matrix will be rank of that matrix.**
      Steps reducing a matrix is discussed below:
      (i) For 3*4 Matrix:



(ii) For 4*4 Matrix:



c. **Rank by Determinant: If determinant of matrix is not equals to zero then the size of that matrix will be the rank of that matrix else again find the determinant by decreasing the size of the matrix by 1.**

4. Eigen Value:

a. Make a characterstic equation by using following formula –
L^3 – [Sum of elements of principle diagonal] L^2 + [Sum of minors of principle diagonal] L + |A| = 0.

b. The values come after solving the above equation is known as Eigen Value.

5. Eigen Vector:

a. Calculate Eigen Value first.

b. $(A - L_1 I) X_1 = 0$
$(A - L_2 I) X_2 = 0$
$(A - L_3 I) X_3 = 0$

c. Values of the unknown matrix are the Eigen Vector.

6. Cayley-Hamilton Theorem:

a. Replacing L of the characterstic by A where A is given Matrix and I is identity matrix.
L^3 – [Sum of elements of principle diagonal] L^2 + [Sum of minors of principle diagonal] L + |A| = A^3 – [Sum of elements of principle diagonal] A^2 + [Sum of minors of principle diagonal] A + |A|I = 0.